Interference Effect of UDP Mixed with TCP-RED

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Abstract

Multimedia applications are gaining momentum as the principal service on the Internet. Because most of multimedia service use the UDP protocol to carry the traffic over the Internet, it is very important therefore to explore the interaction between UDP and TCP protocol in the routers. In this paper, we apply the matrix-analytic approach to explore the interactive behavior between UDP and TCP protocol under RED scheme. In order to reduce the analysis complexity of RED mechanism, we chose to bypass the interactions between TCP sessions and RED mechanisms and constructed a simple queuing model for RED mechanism with UDP and TCP traffic which follow a continuous-time Markovian arrival process (MAP). The queueing model of the router with RED scheme is modelled as MAP/M/1/K. With the numerical results, we find that as the arrival rate of UDP traffic increases, the drop probability of TCP traffic will also increase under RED scheme. In practice, this effect causes TCP congestion control protocol to reduce the transmission rate of TCP traffic. This is an unfair effect for providing multimedia applications on the Internet. Based on this observation, the router in the Internet must implement some mechanisms to suppress this effect.

I. INTRODUCTION

Recall that the Internet provides two distinct transportlayer protocol to the application layer. One of these protocols is UDP (User Datagram Protocol), which provides an unreliable, connectionless service to the invoking application. The second of these protocols is TCP (Transmission Control Protocol), which provides a reliable, connectionoriented service to the invoking application. UDP, like IP, is an unreliable service-it does not guarantee that data sent by one host will arrive intact at the destination. TCP, on the other hand, is designed to provide a reliable data transport service between end-systems. TCP congestion control prevents any one TCP connection from swamping the links and switches between communicating hosts with an excessive amount of traffic. In principle, TCP permits TCP connections traversing a congested network link to equally share that link's bandwidth. This is done by regulating the rate at which the sending-side TCPs can send traffic into the network. UDP traffic, on the other hand, is unregulated. An application using UDP transport can send at any rate, for as long as it wants.

So far, most of all services on the Internet use the TCP protocol, much research attention has been focused on performance and behavior of the TCP congestion control schemes. Firoiu and Borden [4] proposed a method for configuring RED congestion control scheme, based on a model of RED as the feedback control system with TCP sessions. Chiu and Jain [3] formulated a set of basic principles of the additive-increase and multiplication-

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decrease congestion avoidance to achieve efficiency. In [8], Kelly, Maulloo, and Tan showed that a network deploying the additive-increase and multiplication-decrease congestion avoidance tends to distribute rate according to the proportional fairness. Vojnovic, Le Boudec, and Boutremans [19] showed that in a network employing additiveincrease and multiplication-decrease, the source rates tend to distribute in order to maximize the objective function of fairness. In [13], Misra, Gong, and Towsley used jump process driven stochastic differential equations to model the interactions of TCP sessions and RED routers in the network setting. In order to reduce the analysis complexity of RED mechanism, some researches chose to bypass the interactions between TCP sessions and RED mechanisms and constructed a simple queuing model for RED mechanism. For example, May and Bolot [12] proposed a simple analytic model with Poisson input process for RED, and used these models to quantify the properties of RED. Wang [20] proposed the MAP/M/1/K queueing system with RED scheme to derive the loss information of RED mechanism. The adoption of the world-wide-web technology on the Internet contributes to the dynamic growth of Internet users. This in turn not only creates a huge bandwidth demand, but also reveals the fact that the Internet must provide multimedia services in the near future. Multimedia services using the UDP protocol will soon become a popular trend on the Internet. It is therefore important to examine in advance the interaction between UDP and TCP protocol in the router before the multimedia applications jam the world internet.

The essence of TCP congestion control scheme is that a TCP sender can adjust its sending rate according to the probability of packets being dropped in the Internet router. In traditional implementations of router buffer management, the packets are dropped when the buffer becomes full. Under this circumstance this mechanism is called "Drop-Tail". In order to overcome the synchronization of TCP sessions problem encountered in "Drop-Tail", random early detection (RED) buffer management mechanism has been proposed [5]. Analyzing the drop behavior of UDP traffic mixed with TCP traffic under RED sechme in the router has become imperative as the Internet multimedia applications are gaining increasing popularity. In order to reduce the analysis complexity of RED mechanism. We chose to bypass the interactions of TCP sessions and RED mechanism and constructed a simple queuing model for RED mechanism.

We propose a complicated queuing model for router with RED scheme, and use this model to analyze the interference

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effect of UDP mixed with TCP-RED. Packet streams are considered to follow a continuous-time Markovian arrival process (MAP)[11][17]. The queueing model of the router with RED scheme can be modelled as MAP/M/1/K. Many familiar arrival processes, such as Markov-modulated Possion process (MMPP), are obtained as special cases of the MAP [11][15]. Traffic with certain bursty characteristics can be qualitatively modeled by a MAP, as confirmed in [17]. With a threshold level set in the RED scheme, the drop behavior of the MAP/M/1/K queueing system with RED scheme is characterized.

This paper is organized as follows: In Section II, the continuous-time Markovian arrival process as the input traffic model of the queueing system is briefly introduced. In Section III, the drop behavior of the MAP/M/1/K queueing system with RED scheme is analyzed. Experimental numerical results are computed and discussed in Section IV to reveal the computational tractability of our analysis and to get some insight of the RED scheme. Some concluding remarks are given in Section V.

II. TRAFFIC MODEL

In this paper, we consider a single server queue with finite buffer capacity K. The arrival process of the queuing system is modeled by a Markovian arrival process (MAP) and the service time distribution of the server is assumed to be an exponential distribution. We will analyze the MAP/M/1/K queue in this paper. A brief exposition of MAP is given in the rest of this section.

The Markovian arrival process (MAP) is a generalization of Poisson arrival process by allowing for non-exponential inter-arrival times, but still preserving an underlying Markovian structure [11]. It is a marked point process with arrivals (i.e. marks) generated at the transition epochs of a particular type of *m*-state Markov renewal process [17]. A MAP can be more easily described by a two-dimensional continuous-time Markov chain $\{(N(t), J(t)), t \ge 0\}$, on the state space $\{(n, j) | n \ge 0, 1 \le j \le m\}$, with infinitesimal generator Q_a having the structure,

$$Q_a = \begin{bmatrix} D_0 & D_1 & 0 & 0 & \cdots \\ \mathbf{0} & D_0 & D_1 & 0 & \cdots \\ \mathbf{0} & \mathbf{0} & D_0 & D_1 & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & D_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

where N(t) stands for a counting variable, J(t) represents an auxiliary phase variable, and D_k 's are $m \times m$ matrices, called parameter matrices. The Markov chain Q_a then defines an arrival process where transition from a state (n, i)to a state $(n + 1, j), n \ge 0$, and $1 \le i, j \le m$, corresponds to an arrival and a phase change from phase *i* to phase *j*. The matrix D_1 with elements $(D_1)_{i,j}, 1 \le i, j \le m$, governs those state transitions which correspond to an arrival, and the matrix D_0 governs state transitions which correspond to no arrivals as follows: The sojourn time in phase *i* and with *n* accumulated packets, i.e. in the state (n, i), is exponentially distributed with parameter $-(D_0)_{ii}$, which is independent of n. At the end of that sojourn time, a state transition will occur. With probability $-(D_0)_{ij}/(D_0)_{ii}$, there will be a transition to phase j without any new arrival, i.e. to state (n, j), for $1 \le j \le m$ and $j \ne i$. With probability $-(D_1)_{ij}/(D_0)_{ii}$, there will be a transition to phase j with an arrival, i.e. to state (n+1, j), for $1 \le j \le m$. Note that in this case, j may be equal to i. The sum of all parameter matrices

$$D = D_0 + D_1 \tag{1}$$

is an $m \times m$ matrix which is the infinitesimal generator of the underlying Markovian structure $\{J(t), t \ge 0\}$ with respect to the MAP. We assume that the underlying Markovian structure is stable and irreducible. Thus the Markov chain $\{J(t), t \ge 0\}$ has a unique stationary probability vector $\boldsymbol{\pi}$, i.e.

$$\boldsymbol{\pi} D = 0, \boldsymbol{\pi} \ge 0 \text{ and } \boldsymbol{\pi} \boldsymbol{e} = 1, \tag{2}$$

where e is assumed in this paper to be the all-1 column vector with the designated dimension. We also assume that D_0 is nonsingular such that the sojourn time at any state of the state space $\{(n, j) | n \ge 0, 1 \le j \le m\}$ is finite with probability one, for guaranteeing that the process never terminates. The fundamental arrival rate λ of this MAP is defined as

$$\lambda = \pi D_1 \boldsymbol{e}.\tag{3}$$

where $\boldsymbol{\pi}$ and \boldsymbol{e} are in (2).

The superposition of two independent MAPs with sequences of characterizing parameter matrices $\{D_n^{(1)}\}_{n=0,1}$ and $\{D_n^{(2)}\}_{n=0,1}$ respectively, is also a MAP [17]. The sequence $\{D_n\}_{n=0,1}$ of the defining parameter matrices for the superposed MAP can be obtained by

$$D_n = D_n^{(1)} \oplus D_n^{(2)}, \ \forall \ n = 0, 1,$$
(4)

where \oplus is the Kronecker sum. The Kronecker sum $A \oplus B$ of an $m_1 \times m_1$ matrix A and an $m_2 \times m_2$ matrix B is defined by

$$A \oplus B = A \otimes I_{m_2} + I_{m_1} \otimes B$$

where I_{m_1} and I_{m_2} are identity matrices of dimensions m_1 and m_2 respectively and \otimes is the Kronecker product [2][6].

III. QUEUEING ANALYSIS OF UDP MIXED WITH TCP-RED

As demonstrated in the previous section, traffics will be modeled using MAPs. Determining the characterizing parameter matrices for a MAP is, of course, an essential problem. This obstacle is not dealt with here. However, a large class of traffic modeling of IP network have already been studied in [9].

A. RED Queueing System with UDP and TCP traffic

In this section, we will describe our basic model, and use it to examine the interaction behavior between UDP and TCP protocol in the router with RED scheme. We consider a single server queue with a buffer size K. With the RED buffer management scheme, incoming packets are dropped with probability that is an increasing function of the queue size k. A drop probability is defined by two parameters min_{th} and max_{th} , e.g.

$$q_k = \begin{cases} 0 & k \le \min_{th} \\ \frac{k - \min_{th}}{\max_{th} - \min_{th}} & \min_{th} < k < \max_{th} \\ 1 & k \ge \max_{th}. \end{cases}$$

Please refer to Figure 1. Note that there is no particular reason for choosing $max_{th} < K$ in this case, hence we let $max_{th} = K$.

TCP traffic and UDP traffic will be modeled using a MAP, which is characterized by a sequence $\{D_i^{(t)}\}_{i=0,1}$ of parameter matrices for TCP packets and by another sequence $\{D_i^{(u)}\}_{i=0,1}$ of parameter matrices for UDP packets. $D_i^{(t)}$'s and $D_i^{(u)}$'s are $m_t \times m_t$ and $m_u \times m_u$ matrices respectively. The overall input traffic is their superposition, which is also a MAP with defining sequence $\{D_i\}_{i\geq 0}$ of parameter matrices obtained by (4). Note that each D_i is of dimension $(m_t m_u) \times (m_t m_u)$. The server is assumed to have an exponential distributed service time with service rate μ . And the capacity of the buffer of the queue is assumed to be K. Thus the queuing system can be modeled as a MAP/M/1/K queue with RED scheme and min_{th} , max_{th} and a drop probability function. Note that there is no particular reason for choosing $max_{th} < K$, hence we let $max_{th} = K$.

Consider the embedded continuous-time Markov chain $\{(L(t), J(t)), t \geq 0\}$ of the queuing system on the twodimensional state space $(\{0, 1, \ldots, K\} \times \{(1, 1), (1, 2) \ldots, (m_t, m_u)\})$, where L(t), and J(t) denote the buffer occupancy, and the phase of the underlying Markovian structure of the MAP, at time t respectively. For convenience, the queuing system is said to be at a level j if its buffer occupancy is equal to j. Under the RED scheme with a threshold min_{th} to indicate a congestion level of the buffer occupancy, the embedded Markov chain now has an infinitesimal generator of the following block form

B. Packet Drop Probabilities of UDP Mixed with TCP-RED

In this subsection, we will present the long-term packet drop rate when the queuing system is in the steady-state. Let $\boldsymbol{x} = (\boldsymbol{x}_0, \boldsymbol{x}_1, \dots, \boldsymbol{x}_K)$ be the stationary probability

vector of the Markov chain Q, i.e.

$$\boldsymbol{x}Q = \boldsymbol{0}, \ \boldsymbol{x} \ge 0 \text{ and } \boldsymbol{x}\boldsymbol{e} = 1,$$
 (5)

where $\boldsymbol{x}_k = (x_{k,(1,1)}, \ldots, x_{k,(m_t,m_u)}), \forall 0 \leq k \leq K$. Since Q is stable, we have $x_{k,(j_t,j_u)} = \lim_{t\to\infty} P\{L(t) = k, J(t) = (j_t, j_u)\}$, for all $k, (j_t, j_u)$, and the vector \boldsymbol{x}_k corresponds to steady-state probabilities of states of the Markov chain Q at level k. Now let X(t) be the number of packets drop in the interval [0, t] due to RED scheme. Then the expected value of X(t), denoted by E[X(t)], is given by

$$E[X(t)] = \left(\sum_{i=min_{th}+1}^{K-1} \boldsymbol{x}_i q_i D_1 + \boldsymbol{x}_K D_1\right) \boldsymbol{e} t.$$

Consequently, the long-term packet drop rate, denoted by P_{drop} , can be calculated by

$$P_{drop} = \frac{E[X(t)]}{\lambda t} = \frac{\left(\sum_{i=min_{th}+1}^{K-1} \boldsymbol{x}_{i} q_{i} D_{1} + \boldsymbol{x}_{K} D_{1}\right) \boldsymbol{e}}{\lambda}$$

where λ is the fundamental arrival rate of the packets and can be calculated by (3) with the sequence $\{D_i, i = 0, 1\}$ of parameter matrices.

Now let $X^{(t)}(t)$ be the number of TCP packets drop in the interval [0, t] due to RED scheme. Then the expected value of $X^{(t)}(t)$, denoted by $E[X^{(t)}(t)]$, is given by.

$$E[X^{(t)}(t)] = \left(\sum_{i=min_{th}+1}^{K-1} \boldsymbol{x}_i q_i (D_1^{(t)} \otimes I_{m_u}) + \boldsymbol{x}_K (D_1^{(t)} \otimes I_{m_u})\right) \boldsymbol{e}t.$$

where $E_0(q_i) \equiv -\mu \times I + D_0 + q_i D_1$, $0 \leq q_i \leq 1$, $min_{th} + 1 \leq i \leq max_{th}$. Each block in Q is of the dimension $(m_t m_u) \times (m_t m_u)$. The first $min_{th} + 1$ columns of the $(K + 1) \times (K + 1)$ block matrix Q correspond to the transitions to the non-congestion buffer levels from 0 to min_{th} where no packets will be drop. And the last $(K - min_{th})$ columns of Q correspond to the transitions to the congestion buffer levels from $min_{th} + 1$ to K where incoming packets will be dropped with probability. And the incoming packets will be drop due to buffer overflow as indicated in the last column of the block matrix Q.

Consequently, the TCP packet drop rate, denoted by $P_{drop}^{(t)}$, can be calculated by

$$P_{drop}^{(t)} = \frac{E[X^{(t)}(t)]}{\lambda^{(t)}t}$$
$$= \frac{\left(\sum_{i=min_{th}+1}^{K-1} \boldsymbol{x}_{i} q_{i}(D_{1}^{(t)} \otimes I_{m_{u}}) + \boldsymbol{x}_{K}(D_{1}^{(t)} \otimes I_{m_{u}})\right)\boldsymbol{e}}{\lambda^{(t)}}$$

where $\lambda^{(t)}$ is the fundamental arrival rate of the TCP packets and can be calculated by (3) with the sequence $\{D_i^{(t)}, i = 0, 1\}$ of parameter matrices.

By the way, let $X^{(u)}(t)$ be the number of UDP packets drop in the interval [0, t] due to RED scheme. Then the expected value of $X^{(u)}(t)$, denoted by $E[X^{(u)}(t)]$, is given by.

$$E[X^{(u)}(t)] = \left(\sum_{i=min_{th}+1}^{K-1} \boldsymbol{x}_i q_i (I_{m_t} \otimes D_1^{(u)}) + \boldsymbol{x}_K (I_{m_t} \otimes D_1^{(u)})\right) \boldsymbol{e}t.$$

Consequently, the UDP packet drop rate, denoted by $P_{drop}^{(u)}$, can be calculated by

$$P_{drop}^{(u)} = \frac{E[X^{(u)}(t)]}{\lambda^{(u)}t} \\ = \frac{\left(\sum_{i=min_{th}+1}^{K-1} \mathbf{x}_{i}q_{i}(I_{m_{t}} \otimes D_{1}^{(u)}) + \mathbf{x}_{K}(I_{m_{t}} \otimes D_{1}^{(u)})\right)\mathbf{e}}{\lambda^{(u)}}$$

where $\lambda^{(u)}$ is the fundamental arrival rate of the TCP packets and can be calculated by (3) with the sequence $\{D_i^{(u)}, i = 0, 1\}$ of parameter matrices.

IV. NUMERICAL RESULTS

In this section, we will investigate and discuss an experimental RED queue using the numerical results that are computed by the algorithm developed in the previous sections.

A simple class of MAP, Markov modulated Poisson process (MMPP), will be used as the basic input traffic model in the experimental queue. In MMPP, packets arrive according to a Poisson process whose instantaneous rate is a function of the state of a continuous-time finite Markov chain. Thus an MMPP can be represented by a pair of matrices (P, Λ) , the first matrix being the infinitesimal generator of the Markov chain and the second being a diagonal matrix specifying the arrival intensity associated with each state of the Markov chain. Consider the case where the process is the superposition of k independent and identically distributed two-state MMPPs, each described by the matrices

$$P = \begin{bmatrix} -a & a \\ b & -b \end{bmatrix} \text{ and } \Lambda = \begin{bmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{bmatrix}.$$
 (6)

This superposition can be further represented by a (k+1)state MMPP with an infinitesimal generator P = B(k, a, b)and a rate matrix $\lambda = R(k, \lambda_0, \lambda_1)$, where

$$B(k, a, b) = \begin{bmatrix} -ka & ka & \cdots & 0 & 0 \\ b & -b - (k-1)a & \cdots & 0 & 0 \\ 0 & 2b & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & kb & -kb \end{bmatrix},$$

$$R(k, \lambda_0, \lambda_1) = \text{Diag}(k\lambda_0, \cdots, (k-i)\lambda_0 + i\lambda_1, \cdots, k\lambda_1).$$

Now suppose that in our experimental RED queue, the TCP packet traffic is from a superposition of $(m_t - 1)$ i.i.d. two-state MMPP sources and the UDP packet traffic is from a superposition of $(m_u - 1)$ i.i.d. two-state MMPP sources with the following parameter matrices

$$P^{(t)} = \begin{bmatrix} -a^{(t)} & a^{(t)} \\ b^{(t)} & -b^{(t)} \end{bmatrix}, \Lambda^{(t)} = \begin{bmatrix} \lambda_0^{(t)} & 0 \\ 0 & \lambda_1^{(t)} \end{bmatrix}$$

and

$$P^{(u)} = \begin{bmatrix} -a^{(u)} & a^{(u)} \\ b^{(u)} & -b^{(u)} \end{bmatrix}, \Lambda^{(u)} = \begin{bmatrix} \lambda_0^{(u)} & 0 \\ 0 & \lambda_1^{(u)} \end{bmatrix}$$

respectively. Thus the underlying Markovian structure for the TCP (UDP) traffic is seen to be an $m_t(m_u)$ -state birth-and-death process with infinitesimal generator $D^{(t)} = B(m_t - 1, a^{(t)}, b^{(t)}) \ (D^{(u)} = B(m_u - 1, a^{(u)}, b^{(u)}))$ and rate matrix is $\Lambda^{(t)} = R(m_t - 1, \lambda_0^{(t)}, \lambda_1^{(t)}) \ (\Lambda^{(u)} = R(m_u - 1, \lambda_0^{(u)}, \lambda_1^{(u)}))$. Then the sequences $\{D_i^{(t)}\}_{i=0,1}$ and $\{D_i^{(u)}\}_{i=0,1}$ of parameter matrices for the TCP and the UDP packet traffics are

and

$$D_0^{(u)} = D^{(u)} - \Lambda^{(u)}, D_1^{(u)} = \Lambda^{(u)}$$

 $D_0^{(t)} = D^{(t)} - \Lambda^{(t)}, D_1^{(t)} = \Lambda^{(t)},$

respectively [16]. The mean rate λ of the overall superposed traffic is

$$\begin{aligned} \Lambda &= (m_t - 1) \left(\frac{b^{(t)} \lambda_0^{(t)} + a^{(t)} \lambda_1^{(t)}}{a^{(t)} + b^{(t)}} \right) \\ &+ (m_u - 1) \left(\frac{b^{(u)} \lambda_0^{(u)} + a^{(u)} \lambda_1^{(u)}}{a^{(u)} + b^{(u)}} \right) \end{aligned}$$

The two-state MMPP is often used to model the data traffic generated by a voice user [1]. In our experimental studies, the following numerical values of parameters are used:

$$a^{(t)} = 1.5, \quad b^{(t)} = 2.5, \quad \lambda_0^{(t)} = 0.01, \quad \lambda_1^{(t)} = 0.25, \\ a^{(u)} = 1.25, \quad b^{(u)} = 5.0, \quad \lambda_0^{(u)} = 0.005, \quad \lambda_1^{(u)} = 0.48.$$

The service time distribution of the server is assumed to be exponential with service rate $\mu = 1.5$. The buffer capacity K is taken to be 30. The TCP traffic source is taken to be 5 two state MMPPs and the UDP traffic source will be adjusted such that the queue will have different UDP traffic condition. As shown in Figure 2, when the arrival rate of UDP traffic increases, the drop probability of TCP traffic will also increase under both RED and Drop-Tail mechanisms. In principle, TCP permits TCP connections traversing a congested network link to equally share that link's bandwidth. This is done by regulating the rate at which the sending-side TCPs can send traffic into the network. UDP traffic, on the other hand, is unregulated. An application using UDP transport can send at any rate it pleases, for as long as it wants. This is an unfair problem between UDP and TCP protocol. Based on this observation, the router in the Internet must implement some mechanisms to suppress this effect.

V. CONCLUSION

In this paper, we apply the matrix-analytic approach to explore the interactive behavior between UDP and TCP protocol under RED scheme. In order to reduce the analysis complexity of RED mechanism, we chose to bypass the interactions between TCP sessions and RED mechanisms and constructed a simple queuing model for RED mechanism with UDP and TCP traffic. UDP and TCP traffics are considered to follow a continuous-time Markovian arrival process (MAP). The queueing model of the router with RED scheme can be modelled as MAP/M/1/K. By the numerical results, we find that as the arrival rate of UDP traffic increases, the drop probability of TCP traffic also increase under RED scheme. In practice, this effect will cause TCP congestion control protocol to reduce the transmission rate of TCP traffic. This is an unfair effect for providing multimedia applications on the Internet. Based on this observation, the router in the Internet must implement some mechanisms to suppress this effect.

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Fig. 1. RED buffer management scheme.



Fig. 2. Drop probability for Drop-Tail and RED with $min_{th} = 20$, $max_{th} = K = 30$.

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