

Modelling the Japanese Exchange Rate in Terms of $I(d)$ Statistical Models with Parametric and Semiparametric Techniques

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Abstract

In this article we model monthly data on the Japanese nominal exchange rate in relation to the US dollar by means of fractionally integrated statistical models. For this purpose, we use both parametric and semiparametric techniques proposed by P.M. Robinson in a number of papers. The results indicate that the order of integration of the series is higher than 1 and thus the standard approach of taking first differences to get series which are integrated of order 0 (which is required, for example, in the context of cointegration) may lead to spurious results, the series still having a component of long memory behaviour.

Key words: fractional integration; long memory; exchange rates

JEL classification: C22

1. Introduction

Modelling macroeconomic time series is a matter that still remains controversial. Initially, deterministic approaches based on linear (or quadratic) functions of time were proposed but they were shown to be inappropriate in many cases, especially after the seminal paper of Nelson and Plosser (1982) which showed, following the work and ideas of Box and Jenkins (1970), that many US macroeconomic series could be specified in terms of stochastic trends or unit root models. They showed that by taking first (or sometimes second) differences of the original series, the resulting series was integrated of order 0, defined for the purposes of the present paper as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. Note that the differenced series may still be “weakly autocorrelated” (e.g., autoregressive), with the correlation structure describing its short run dynamics. Following this approach, a large variety of test statistics were proposed during the 80s and 90s for testing unit roots (e.g., Dickey and Fuller, 1979; Phillips and Perron, 1988; Kwiatkowski et al.,

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1992), and they have been widely applied to many economic and financial time series.

In the last few years, however, a growing literature has emerged trying to model the stochastic behaviour of the series in terms of fractionally integrated (denoted $I(d)$) processes. This type of process was initially proposed by Granger and Joyeux (1980), Granger (1980, 1981), and Hosking (1981), though earlier work by Adenstedt (1974) and Taquu (1975) show an awareness of its representation; it was theoretically justified in terms of aggregation of ARMA series by Granger (1980) and Robinson (1978) and more recently in terms of the duration of shocks by Parke (1999). Also, Cioczek-Georges and Mandelbrot (1995), Taquu et al. (1997), Chambers (1998), and Lippi and Zaffaroni (1999) also used aggregation to motivate long memory processes, while Diebold and Inoue (2001) reported another source of long memory based on structural change/regime-switching.

Let us consider, for example, the following process:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

for any real value d . Clearly, if $d = 0$ in (1), $x_t = u_t$ and a “weakly autocorrelated” x_t is allowed for. If $d > 0$, x_t is said to have long memory or to be “strongly dependent” or “strongly autocorrelated”, so-named because of the strong association between observations widely separated in time. This may also be seen by expressing the polynomial in (1) in terms of its binomial expansion; that is, for all real d ,

$$(1 - L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \dots$$

such that

$$(1 - L)^d x_t = 1 - d x_{t-1} + \frac{d(d-1)}{2!} x_{t-2} - \frac{d(d-1)(d-2)}{3!} x_{t-3} + \dots$$

Empirical applications of fractional models like (1) can be found in Diebold and Rudebusch (1989), Baillie and Bollerslev (1994), and Gil-Alana and Robinson (1997), and recent surveys of long memory processes are in Beran (1994) and Baillie (1996).

The estimation and testing of the fractional differencing parameter d plays a crucial role from both statistical and economic viewpoints. In particular, if $d \in (0, 0.5)$, x_t is covariance stationary and mean-reverting, i.e., with the effect of shocks dying away in the long run. If $d \in [0.5, 1)$, x_t is nonstationary but mean-reverting, while $d \geq 1$ implies x_t is neither stationary nor mean-reverting.

In this paper, we analyse monthly observations of the Japanese exchange rate by means of fractionally integrated techniques. Traditionally, it has been assumed that the exchange rates have a unit root implying that shocks have permanent effects on the series. (e.g., Taylor, 1995; Breuer, 1996; Rogoff, 1996), though other authors (e.g., Abuaf and Jorion, 1990; Glen, 1992; Lothian and Taylor, 1996) argued in the

opposite direction, suggesting mean-reverting behaviour. Most of these articles concentrate on real rather than nominal exchange rates, though Cheung (1993) reported evidence of long memory in the nominal exchange rates. Other articles, suggesting that the exchange rates are mean-reverting and, in particular, that they can be specified in terms of $I(d)$ statistical models include Diebold et al. (1991), Cheung and Lai (1993), and more recently Gil-Alana (2000).

A motivation for this work is as follows: Given interest and inflation rates in two countries and a constant risk premium, it could be argued that the nominal exchange rate should follow a pure random walk on the grounds that the market would immediately react to incorporate any expected future appreciation of the exchange rate. This would imply that future returns are unforecastable, i.e., a martingale difference sequence. However, and more generally, the exchange rate return ($E_t\Delta y_t$) can be written as the sum of the interest rate differential (or forward premium $i_t - i_t^*$) and a risk premium (rp_t): $E_t\Delta y_t = i_t - i_t^* + rp_t$ (see, e.g., Engel, 1996). Baillie and Bollerslev (1994) reported evidence of fractional integration in $i_t - i_t^*$, and, in theory, this raises two possibilities: long memory in rp_t or long memory in Δy_t . On the other hand, it might also be reasonable to look for evidence of mean reversion in the nominal exchange rates if it was thought that governments or central banks had an informal policy of smoothing overly large exchange rate movements, as possibly suggested by the 1985 Plaza Accords.

The structure of the paper is as follows. Section 2 briefly presents several ways of testing and estimating $I(d)$ statistical models using both parametric and semiparametric techniques. These methods are applied in Section 3 to the Japanese exchange rate, and Section 4 contains concluding comments.

2. Testing and Estimating $I(d)$ Statistical Models

We divide this section into two parts. In Section 2.1, we present a testing procedure of Robinson (1994a) that permits us to test $I(d)$ statistical models in a fully parametric way. In Section 2.2, several semiparametric estimation procedures, due to Robinson (1994b, 1995a, b), are briefly explained.

2.1 Testing $I(d)$ Models with Parametric Techniques

Following Bhargava (1986), Schmidt and Phillips (1992), and others on parameterization of unit root models, Robinson (1994a) considered the following regression model:

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, \dots, \quad (2)$$

where y_t is the time series we observe, β is a $k \times 1$ vector of unknown parameters, z_t is a $k \times 1$ vector of deterministic regressors that may include an intercept (i.e., $z_t \equiv 1$) or an intercept and a linear time trend (i.e., $z_t = (1, t)'$), and x_t are the regression errors, which are of the form described in (1). Robinson (1994a) proposed a Lagrange Multiplier (LM) test of the null hypothesis

$$H_o : d = d_o, \quad (3)$$

for (1) and (2) for any real value d_o . Specifically, the test statistic is given by:

$$\hat{r} = \left(\frac{T}{\hat{A}} \right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \quad (4)$$

where T is the sample size and:

$$\begin{aligned} \hat{a} &= \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); & \hat{\sigma}^2 &= \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \\ \hat{A} &= \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right); \\ \psi(\lambda_j) &= \log \left| 2 \sin \frac{\lambda_j}{2} \right|; & \hat{\varepsilon}(\lambda_j) &= \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); & \lambda_j &= \frac{2\pi j}{T}; \\ \hat{u}_t &= (1-L)^{d_o} y_t - \hat{\beta}' w_t; & w_t &= (1-L)^{d_o} z_t; & \beta &= \left(\sum_{t=1}^T w_t w_t' \right)^{-1} \sum_{t=1}^T w_t (1-L)^{d_o} y_t; \end{aligned}$$

$I(\lambda_j)$ is the periodogram of \hat{u}_t ; and the function g above is a known function coming from the spectral density function of \hat{u}_t , $f(\lambda; \tau) = (\sigma^2/2\pi) \cdot g(\lambda; \tau)$, with $\hat{\tau}$ obtained by minimising $\sigma^2(\tau)$. Note that if u_t is white noise, then $g \equiv 1$, and if u_t is an AR process of the form $\phi_p(L)u_t = \varepsilon_t$, then $g = |\phi_p(e^{i\lambda})|^{-2}$, so that the AR coefficients are functions of τ .

Robinson (1994a) established that under certain very mild regularity conditions,

$$\hat{r} \rightarrow_d N(0,1) \quad \text{as } T \rightarrow \infty. \quad (5)$$

Thus, an approximate one-sided test of H_o in (3) against the alternative $H_a: d > d_o$ rejects H_o if $\hat{r} > z_{\alpha}$, where α is the probability that a standard normal variate exceeds z_{α} , and conversely a one-sided test of H_o (3) against $H_a: d < d_o$ rejects H_o if $\hat{r} < -z_{\alpha}$. As these rules indicate, we are in a classical large-sample testing situation for reasons described in Robinson (1994a), who also showed that the above test is efficient in the Pitman sense against local departures from the null. In other words, if the test is implemented against local departures of the form: $H_a: d = d_o + \delta T^{-1/2}$ for $\delta \neq 0$, the limiting distribution is normal with variance 1 and mean that cannot be exceeded by that of any rival statistic.

This version of the test in Robinson (1994a) was used in empirical applications in Gil-Alana and Robinson (1997) and Gil-Alana (2000), and other versions of his tests based on seasonal (quarterly and monthly) and cyclical models can be found respectively in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001).

2.2 Estimating $I(d)$ Models with Semiparametric Techniques

Several methods of estimating semiparametrically the fractional differencing parameter d were examined in a number of papers by Robinson (1994b, 1995a, 1995b), which we now describe. The estimates of these methods, based on the frequency domain, are the log-periodogram regression estimates (LPEs) initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995a), the averaged periodogram estimate (APE) proposed by Robinson (1994b), and the quasi-maximum likelihood estimate (QMLE, Robinson, 1995b). The first of these estimates is based on the regression model:

$$\log I(\lambda_j) = c - 2d \log \lambda_j + \varepsilon_j, \quad (6)$$

where:

$$I(\lambda_j) = (2\pi T)^{-1/2} \left| \sum_{t=1}^T x_t e^{i\lambda_j t} \right|^2; \quad \lambda_j = \frac{2\pi j}{T}, \quad j = 1, \dots, m, \quad \frac{m}{T} \rightarrow 0;$$

$$C \sim \log \left(\frac{\sigma^2}{2\pi} f(0) \right); \quad \varepsilon_j = \log \left(\frac{I(\lambda_j)}{f(\lambda_j)} \right);$$

and the estimate is just the OLS estimate of d in (6). Unfortunately, it has not been proven that this estimate is consistent for d , but Robinson (1995a) modified the former regression introducing two alterations: the use of a pooled periodogram instead of the raw periodogram, and a trimming number q so that frequencies λ_j , $j = 1, 2, \dots, q$, are excluded from the regression, where q tends to infinity slower than J , so that q/J tends to zero. Thus, the final regression model is:

$$Y_K^{(J)} = C^{(J)} - 2d \log \lambda_K + U_K^{(J)}, \quad \text{with} \quad Y_K^{(J)} = \log \left(\sum_{j=1}^J I(\lambda_{k+j-J}) \right),$$

where $k = q+J, q+2J, \dots, m$, J controls the pooling, and q controls the trimming. The estimate of d is:

$$d_1 = -\frac{1}{2} \frac{\sum_{j=q+1}^J \left(\log \lambda_j - J^{-1} \sum_{j=q+1}^J \log \lambda_j \right) \log I(\lambda_j)}{\sum_{j=q+1}^J \left(\log \lambda_j - J^{-1} \sum_{j=q+1}^J \log \lambda_j \right)^2}, \quad (7)$$

and assuming normality, Robinson (1995a) proves the consistency and asymptotic normality of d_1 in a multivariate framework.

The averaged periodogram estimate of Robinson (1994b) is based on the average of the periodogram near zero frequency,

$$\overline{F(\lambda_m)} = \frac{2\pi}{T} \sum_{j=1}^m I(\lambda_j),$$

suggesting the estimator:

$$d_2 = \frac{1}{2} - \log \left(\frac{\overline{F(q\lambda_m)}}{\overline{F(\lambda_m)}} \right) / 2 \log q, \quad (8)$$

where $\lambda_m = 2\pi m/T$, $m/T \rightarrow 0$, for any constant $q \in (0, 1)$. Robinson (1994b) proved the consistency of this estimate under very mild conditions, and Lobato and Robinson (1996) showed asymptotic normality for $0 < d < 1/4$ and non-normality of the limiting distribution for $1/4 < d < 1/2$.

Finally, the quasi-maximum likelihood estimate in Robinson (1995b) is basically a “local Whittle estimate” in the frequency domain, based on a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$d_3 = \arg \min_d \left(\log \overline{C(d)} - 2d \frac{1}{m} \sum_{j=1}^m \log \lambda_j \right), \quad (9)$$

$$\text{for } d \in (-1/2, 1/2), \quad \overline{C(d)} = \frac{1}{m} \sum_{j=1}^m I(\lambda_j) \lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \rightarrow 0.$$

Under finiteness of the fourth moment and other conditions, Robinson (1995b) proved the asymptotic normality of this estimate, which is more efficient than the former ones (Robinson, 1995a, 1994b). Multivariate extensions of these estimation procedures can be found in Lobato (1999).

3. Testing and Estimating Order of Integration in the Japanese Exchange Rate

The time series data analysed in this section correspond to the monthly (seasonally adjusted) observations of the Japanese exchange rate in relation to the US dollar for the time period January, 1971, to April, 2001.

Figure 1 contains plots of the original series and first differences along with their corresponding correlograms and periodograms. Looking at the correlogram of the original series, we observe a slow decay in the values, which may be consistent with the presence of unit or fractional roots. This is substantiated by the periodogram where we observe a large peak around the zero frequency. However, in the correlogram of the first differenced series, we still observe significant autocorrelations with apparent slow decay and/or oscillation in some cases, which could be indicative of fractional integration smaller than or greater than a unit root.

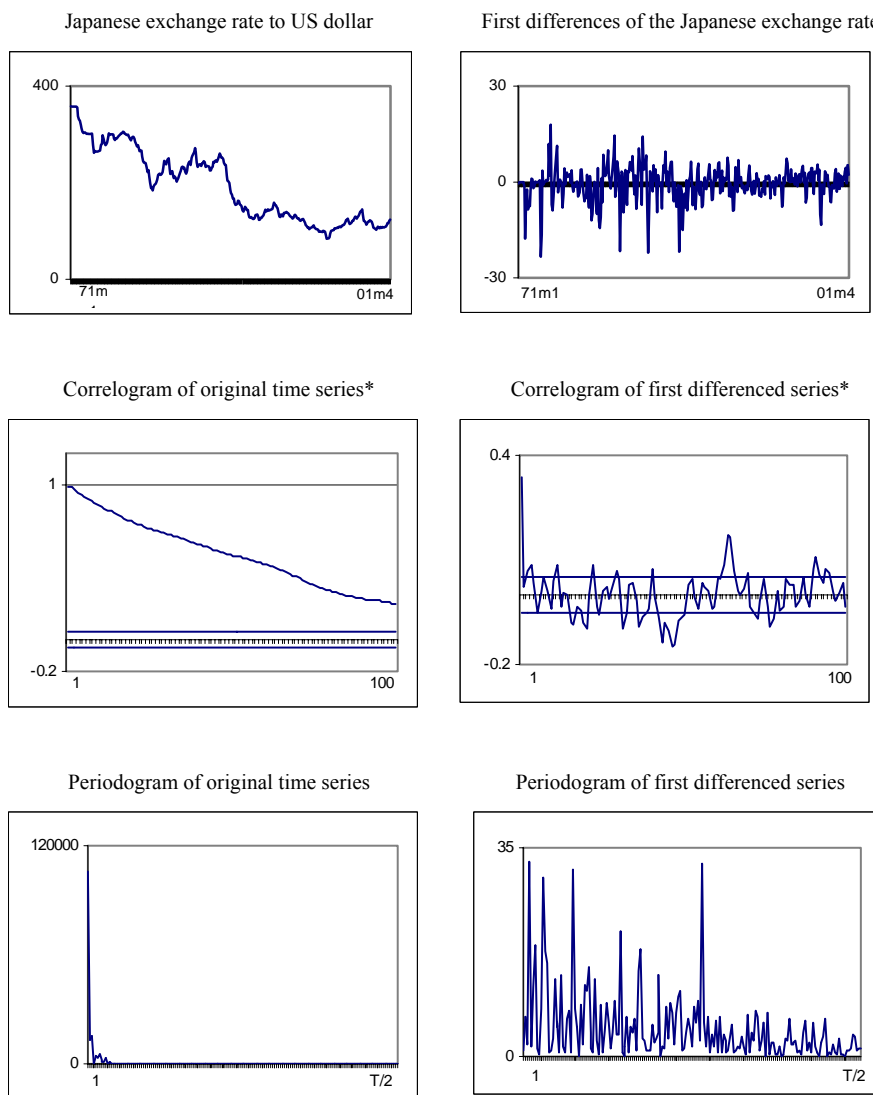
We start with the parametric approach, testing the order of integration of the series in a fully parametric way and using the tests of Robinson (1994a) described in Section 2.1. Denoting the time series y_t , we assume throughout the model in (1) and (2) with $z_t = (1, t)'$ for $t \geq 1$ and $(0, 0)'$ otherwise, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad t = 1, 2, \dots, \quad (10)$$

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots. \quad (11)$$

We test H_0 (3) for values of d_o between 0.5 and 2 in 0.1 increments, i.e., we test for stationarity ($d = 0.5$), unit roots ($d = 1$), I(2) processes ($d = 2$), and other fractionally

Figure 1. Original and Differenced Japanese Exchange Rate with Correlogram and Periodogram



* The large-sample standard error under the hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.052.

integrated possibilities. We treat separately the cases $\beta_0 = \beta_1 = 0$ a priori (i.e., including no regressors in the undifferenced regression (10)), β_0 unknown and $\beta_1 = 0$ a priori (i.e., including an intercept), and finally β_0 and β_1 unknown (i.e., with a linear time trend). The $I(0)$ disturbances are modelled as either white noise or weakly autocorrelated. The reason for the inclusion of a linear time trend is this: If u_t is white noise and $d_o = 1$, the differences $(1 - L)y_t$ behave, for $t > 1$, like a random walk when $\beta_1 = 0$ and a random walk with a drift when both $\beta_0, \beta_1 \neq 0$.

The test statistic reported in Tables 1 to 3 is the one-sided version given by \hat{r} in (4), so that significantly positive values of this ($\hat{r} > z_\alpha$) are consistent with orders of integration higher than the one hypothesized under the null ($d > d_o$), whereas significantly negative ones ($\hat{r} < -z_\alpha$) imply orders of integration smaller than d_o . A notable feature observed across Table 1 (in which u_t is white noise) is the fact that \hat{r} monotonically decreases with d_o . This is something to be expected in view of the previous discussion and since it is a one-sided statistic. Thus, for example, if H_o (3) is rejected with $d_o = 1$ against the alternative $H_a: d > 1$, an even more significant result in this direction should be expected when $d_o = 0.75$ or $d_o = 0.50$ are tested. We

Table 1. Testing Order of Integration with Robinson Tests and White Noise Disturbances

z_t / d_o	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
No regressors	26.09	18.62	12.15	7.03	3.20	0.40*	-1.64	-3.16	-4.31	-5.20	-5.89
An intercept	28.32	21.95	17.11	12.88	9.04	5.74	3.03	0.87*	-0.81*	-2.13	-3.17
A linear time trend	27.59	22.90	18.01	13.32	9.17	5.73	2.98	0.83*	-0.84*	-2.15	-3.18

Note: Values with * (and bolded) are not rejected at the 5% significance level.

Table 2. Testing Order of Integration with Robinson Tests and AR(1) and AR(2) Disturbances

Robinson Tests with AR(1) Disturbances											
z_t / d_o	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
No regressors	—	—	—	—	—	—	1.73	1.72	0.74*	-0.32*	-1.30*
An intercept	—	—	—	0.32*	0.08*	-0.46*	-1.17*	-1.93	-2.65	-3.30	-3.86
A linear time trend	—	—	—	0.39*	0.12*	-0.46*	-1.20*	-1.96	-2.68	-3.32	-3.87
Robinson Tests with AR(2) Disturbances											
z_t / d_o	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
No regressors	—	—	—	—	—	—	—	8.02	6.77	5.03	3.79
An intercept	—	—	—	1.03*	1.02*	0.66*	0.09*	-0.58*	-1.29*	-1.98	-2.62
A linear time trend	—	—	—	1.12*	1.05*	0.65*	0.05*	-0.62*	-1.33*	-2.01	-2.64

Note: Values with * (and bolded) are not rejected at the 5% significance level; “—” means that monotonicity in the value of the test statistic with respect to d_o was not achieved.

see in this table that if we do not include regressors, the only non-rejection value occurs when $d = 1$; however, if an intercept or an intercept and a linear time trend are included in the regression model, the unit root null hypothesis is rejected in favour of higher orders of integration, and the non-rejection values now occur when $d = 1.2$ or 1.3 .

However, the significance of the above results might be in large part due to the unaccounted for $I(0)$ autocorrelation in u_t . Thus, in Tables 2, we perform the same procedure as in Table 1 but allow $AR(1)$ and $AR(2)$ disturbances. Higher AR orders were also tried and, though not reported here, the results were very similar to those given in the table. We observe a lack of monotonicity in \hat{r} for small values of d_o . This may be an indication of model misspecification, as is argued, for example, in Gil-Alana and Robinson (1997). Note that in the event of misspecification, monotonicity is not necessarily to be expected: frequently, misspecification inflates both numerator and denominator of \hat{r} to varying degrees, and thus affects \hat{r} in a complicated way. In order to solve this problem, we perform, in Table 3, the tests of Robinson (1994a), using a non-parametric approach for modelling the $I(0)$ disturbances due to Bloomfield (1973). In his model, the disturbances are exclusively specified in terms of the spectral density, which is given by:

$$f(\lambda; \tau) = \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{l=0}^k \tau_l \cos(\lambda l)\right). \quad (12)$$

Bloomfield (1973) showed that the logarithm of the spectral density function of an $ARMA(p, q)$ process is a fairly well-behaved function and it can thus be approximated by a truncated Fourier series. He showed that the log of (12) approximates well the log of the spectrum of $ARMA$ processes when p and q are small values, which is often seen in economics. Like the stationary $AR(p)$ case, this model has exponentially decaying autocorrelations and thus, using this specification, we do not need to rely on so many parameters as in the $ARMA$ processes, which is often tedious in terms of estimation, testing, and model specification. The results of the tests of Robinson (1994a) based on Bloomfield's (1973) disturbances are given in Table 3. We see that monotonicity is now always achieved and the non-rejection values occur with d ranging between 1.0 and 1.2 in case of $k = 1$ and between 1.0 and 1.3 when $k = 2$. Thus, the unit root null hypothesis cannot be rejected though fractional processes with orders of integration greater than one may also be plausible in this context.

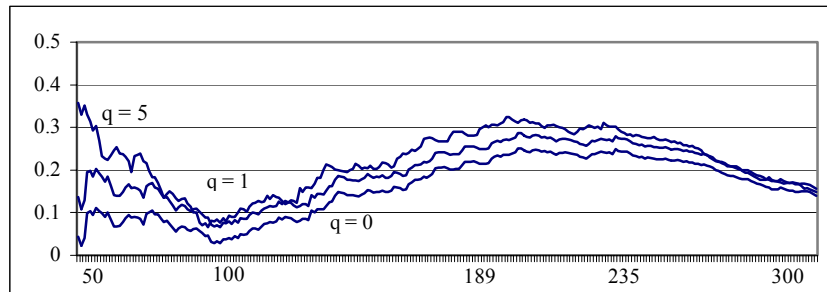
So far, we have tested the degree of integration in the Japanese exchange rate with the tests of Robinson (1994a), which impose a parametric model in the process. (Note that even with Bloomfield's (1973) exponential model for the disturbances, the model is still parametric since it describes the process with a parametric model for the spectral density function of the disturbances.) Next, we present the results based on the semiparametric procedures described in Section 2.2. In all cases, the estimates are based on the first differenced series, so that in order to obtain the proper estimates of d we need to add 1 to the values obtained through the procedures.

Table 3. Testing Order of Integration with Robinson Tests and Bloomfield (1) and (2) u_t

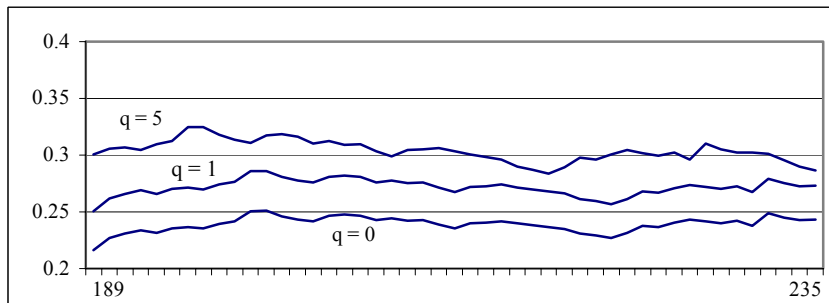
Testing Order of Integration with Robinson Tests and Bloomfield (1) u_t											
z_t / d_o	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
No regressors	12.39	8.97	5.78	3.30	1.99	-0.12*	-1.37*	-1.54*	-2.85	-3.46	-3.92
An intercept	11.39	6.86	4.40	2.22	1.75	-0.42*	-1.52*	-1.59*	-3.25	-3.93	-4.44
A linear time trend	9.26	6.69	4.48	2.72	1.88	-0.43*	-1.57*	-2.57	-3.29	-3.97	-4.47
Testing Order of Integration with Robinson Tests and Bloomfield (2) u_t											
z_t / d_o	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
No regressors	7.45	4.89	4.68	3.66	1.91	0.91*	-0.54*	-0.58*	-1.43*	-4.27	-4.67
An intercept	6.93	4.51	3.09	3.04	1.82	0.71*	-0.95*	-1.62*	-1.58*	-2.43	-3.09
A linear time trend	9.90	5.15	4.35	3.65	1.80	0.71*	-1.02*	-1.63*	-1.78	-2.51	-2.85

Note: Values with * (and bolded) are not rejected at the 5% significance level.

Figure 2. Log-Periodogram Regression Estimate of d (d_1 in (7))



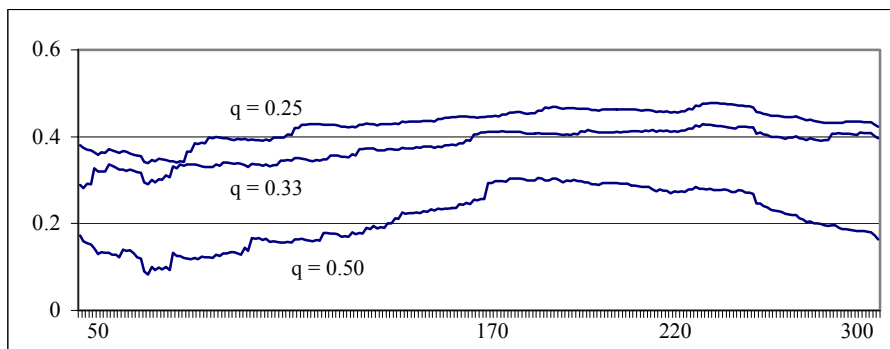
LPE in the Interval (189, 235)



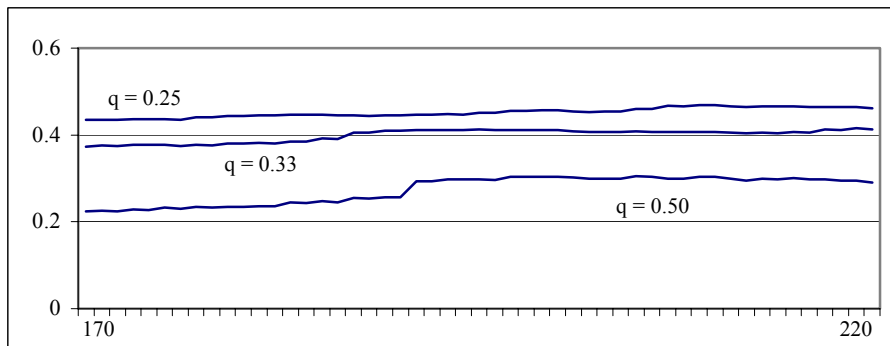
Note: The horizontal and vertical axes refer to the bandwidth parameter m and the estimated values of d .

We start with the log-periodogram regression estimate (LPE) of Robinson (1994b), i.e., d_1 given by (7). The results displayed in Figure 2 correspond to d_1 for trimming values $q = 0, 1,$ and 5 and J initially (in the upper panel) from 50 to 300. We see that if J is between 50 and 100, the estimates are very sensitive to q . However, if $J > 100$, the values behave similarly for the three cases of q . We also observe that the most stable behaviour is obtained when J is between 189 and 235. Thus, in the lower part of the table, we report the same estimates but only for that range of values for J . We see here that d_1 oscillates between 1.22 and 1.25 if $q = 0$, around 1.27 if $q = 1$, and around 1.30 if $q = 5$. These results indicate that according to the LPE, the order of integration of the series is higher than 1, ranging between 1.2 and 1.3 in all cases.

Figure 3. Averaged Periodogram Estimate of d (d_2 in (8))



APE in the Interval (170, 220)



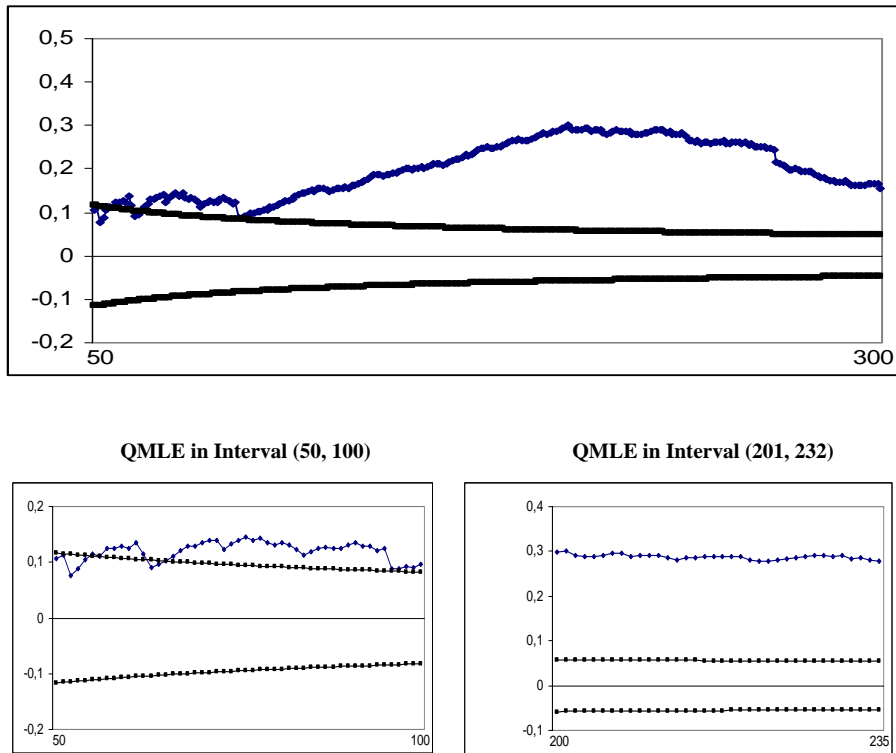
Note: The horizontal and vertical axes refer to the bandwidth parameter m and the estimated values of d .

The averaged periodogram estimate of Robinson, (APE, 1994b), i.e., d_2 in (8) was next performed for trimming values $q = 0.25, 0.33,$ and 0.50 . The upper part of Figure 3 displays the results of d_2 with J from 50 to 300. If $J < 170$, we see that the

estimates are very sensitive to q . Thus, similar to the previous figure, we concentrate on those cases where d_2 remains relatively stable across J , i.e., from 170 to 220. We see here that d_2 oscillates between 1.22 and 1.30 if $q = 0.50$, around 1.40 when $q = 0.33$, and slightly above 1.40 when $q = 0.25$. The estimates here are slightly superior to those given in Figure 2 (LPE), being further away from the unit root case.

Finally, the quasi-maximum likelihood estimate (QMLE) of Robinson (1995b) was also computed. The results for a range of values of m from 50 to 300 are displayed in the upper panel of Figure 4. We also include in this figure the 95% confidence intervals corresponding to the I(0) case (I(1) in the original series). We see that practically all estimates of d are strictly above that interval, suggesting orders of integration higher than 1. We see that d_3 ranges between 1.10 and 1.30, and two different sets of values for m are considered in which d_3 remains relatively constant.

Figure 4. Quasi-Maximum Likelihood Estimates of d (d_3 in (9))



Note: The horizontal and vertical axes refer to the bandwidth parameter m and the estimated values of d .

The first corresponds to m between 50 and 100 while the second range is from 210 to 232. In the first case d_3 is around 1.12, and in the second it is around 1.29. Once more, we obtain estimates higher than 1. It should be finally noted that results based

on these semiparametric procedures should be interpreted with care since the most stable behaviour for these estimates is obtained for large m , which includes medium and short cycles and thus might be biased. This may be due in part to the seasonal nature of the series, and though the data are seasonally adjusted, it might be possible that a seasonal structure is still present in the data. In any case, they are completely in line with the results based on the parametric procedure, finding strong evidence against the unit root hypothesis and in favour of $I(d)$ statistical processes with $d > 1$.

5. Concluding Comments

Monthly data of the Japanese exchange rate are examined in this article by means of fractionally integrated models. Using parametric and semiparametric techniques proposed by P.M. Robinson in a number of papers, we show that the series can be well described in terms of an $I(d)$ process with d equal to or greater than 1.

We start by employing a version of one of the tests in Robinson (1994a), which is a parametric procedure for testing $I(d)$ statistical models. The results indicate that if the disturbances are white noise, the unit root null hypothesis cannot be rejected when we do not include regressors, however, if an intercept or an intercept and a linear time trend are included, the order of integration seems to be much higher than one. If the disturbances are autoregressive, we observe a lack of monotonic decrease in the value of the test statistic with respect to d , which might be an indication of model misspecification. If the disturbances follow the Bloomfield's (1973) exponential spectral model, the orders of integration range between 1.0 and 1.3.

Several semiparametric procedures proposed by Robinson (1994b, 1995a, b) for estimating the fractional differencing parameter d are also employed; in particular, the log-periodogram regression estimate (Robinson, 1995a), the averaged periodogram estimate (Robinson, 1994b), and the quasi-maximum likelihood estimate (Robinson, 1995b). In all cases, the estimated values of d are greater than one, suggesting that first differences may not be sufficient to get $I(0)$ differenced series.

This result has strong economic implications. Shocks affecting the series have permanent effects, and strong policy actions are required to return the variable to its original level. Also, the fact that the first differenced series still exhibits a component of long memory behaviour suggests that the standard practise of taking first differences to achieve $I(0)$ stationarity may be erroneous; thus, all the analysis of the real exchange rates, based, for example, on cointegration techniques (at least in its classical sense) should be interpreted with care. A follow-up step in this direction is to examine the possibility of fractional cointegration. Pioneering work in this area are the papers of Cheung and Lai (1993), Baillie and Bollerslev (1994), and Dueker and Startz (1998). More recently, Gil-Alana (2003) proposes a very simple procedure for testing the null hypothesis of no cointegration against the alternative of fractional cointegration, and more elaborate techniques of fractional cointegration were proposed by P.M. Robinson and his co-authors (e.g., Robinson and Marinucci,

2001; Robinson and Yajima, 2002; and Robinson and Hualde, 2003). Any of these approaches can be explored to further examine the behaviour of the Japanese exchange rate.

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