Irreversibility and Interest Rates

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Abstract

The literature on irreversible investment fails to explore the relationship between the present value of alternative strategies and appropriate risk-adjusted interest rates. We attempt to fill this gap by showing that, to avoid arbitrage opportunities, the real option's rate must be higher than the rate of the immediate strategy. Further, we explain how irreversibility influences the risk-return combination of competing strategies acting as a pure risk factor.

Key words: irreversibility; real option; risk-adjusted rate

JEL classification: G12; D92; E22

1. Introduction

In traditional project evaluation, the net present value (NPV) of an investment plan is computed by discounting profits at some interest rate, and the project is accepted if its current value exceeds the direct cost. There are, however, a number of problems arising from this criterion. In fact, only when returns are certain is there common agreement that the riskless interest rate should be employed to compute the NPV. When returns are uncertain, matters are more complex, and it is natural to adjust expected profit estimates by discounting at an appropriate risk-adjusted interest rate. But several questions remain. For example, does this criterion assure a correct decision when investment is at least partially irreversible? And, since an irreversible project gives the owner a set of real options, what are the appropriate interest rates?

Most investment projects share three important characteristics: the investment may be partially or completely irreversible, the stream of future profits may be uncertain, and an investor may find it advantageous to defer action to get information about the future. These three features interact to determine the value of any investment strategy, and the criterion for asset pricing must be capable of addressing these considerations.

We show that the no-arbitrage principle must be satisfied to figure out the

appropriate interest rates of an irreversible project. Our analysis has two main results. First, the interest rate of the real option is always higher than the rate of the immediate investment. Second, irreversibility affects the risk-return combination of the competing strategies as a pure risk factor.

This problem has received scant attention in the literature. Our analysis starts from empirical evidence discussed by Carnazza and Travaglini (2001). They asked a sample of 4000 Italian manufacturing firms to quantify the minimum interest rate necessary to implement a new project, under the alternative assumptions of reversible or irreversible investment. Their survey reveals that irreversibility increases substantially the expected rate necessary to carry out irreversible projects. Table 1 illustrates this result: with irreversibility, the distribution of answers (expressed in percentage value) is skewed towards the highest rates, implying the existence of a trade-off between irreversibility and interest rates.

Table 1. The Minimum Interest Rate Required by Firms to Implement a New Reversibile or Irreversibile Investment Project.

Interest rate	1 to 5%		5 to 10%		10 to 15%		15 to 20%		20% and up	
Type of project	Revers	Irrevers	Revers	Irrevers	Revers	Irrevers	Revers	Irrevers	Revers	Irrevers
Total firms	19	17	40	22	21	20	13	23	7	18
Traditional	23	17	38	27	19	26	13	17	7	13
High tech	1	3	55	7	22	14	21	23	1	53
Specialized supplier	15	12	31	16	33	24	14	20	7	28
Scale intensive	20	20	43	21	18	12	12	34	7	13

Notes: Numbers are percentages of the total answers; Pavitt taxonomy is used to classify firms by sector.

Unfortunately, the standard literature on investment with irreversibility and uncertainty fails to explore this topic in depth. As an example of the standard way of studying this relationship, let us consider investment models. McDonald and Siegel (1986) employ a simulation to estimate the expected rate of return at which it is optimal to pay a sunk cost to get an irreversible investment. They find that the expected rate of return rises from 4% of the stock to 16% of the real option. Later, Dixit (1992) shows that if the expected interest rate from a reversible project were 5% , the rate for the corresponding irreversible project would be at least 9% .

On theoretical grounds, Baldwin and Trigeorgis (1993), and more recently Trigeorgis (1997), observe that the use of a single risk-adjusted interest rate can lead to significant errors in the evaluation of competing irreversible strategies since asymmetric claims on an asset do not generally have the same discount rate as the asset itself. They conclude that finding the appropriate interest rate via the standard

NPV technique may be difficult in situations involving real options.

Further, the certainty equivalent technique—often employed in its refined version of the risk neutral evaluation—introduces two difficulties in the presence of irreversibility. It remains doubtful why it may be optimal to postpone an irreversible investment when its risk is fully diversifiable. Indeed, if a project can be treated as a riskless asset, the decision to postpone the commitment can only produce a loss of profit at the current time without advantages to the firm. Hence, whether or not it is optimal to defer the project, future returns (and the corresponding interest rate) must be higher than those gained by investing immediately, and the single interest rate will not, in general, evaluate the competing strategies correctly.

In the next section we explore a simple framework to explain why it is necessary to use different interest rates when the project is irreversible. Section 3 generalizes this finding, showing how to calculate the value of competing strategies when investment is a commitment. Section 4 analyses the risk structure of interest rates focusing on the risk premium. Section 5 concludes.

2. A Basic Example

This example provides a good baseline case which we extend below. Note that this example differs from others with irreversibility in an important detail (see for example Dixit and Pindyck, 1994): using the arbitrage principle, we derive the appropriate risk-adjusted interest rates of the alternative strategies.

Consider a firm with the monopoly right to start or defer a project. If the commitment is made at the current time, the firm loses the opportunity to resell the project in the future and the chance of deferring the investment decision to some future date. In other words, the investment is irreversible. The firm maximizes the NPV of the project by choosing the most profitable investment schedule. It can invest either at the current time or in the next time period. Let us assume that the investment spending (I) is a sunk cost of 70 units, and that the project is risky with returns governed by a binomial process. This is indicated with $\delta = 0.1$, the dividend rate paid on the stock.

Figure 1. The Binomial Process of V (Stock) and F (Real Option)
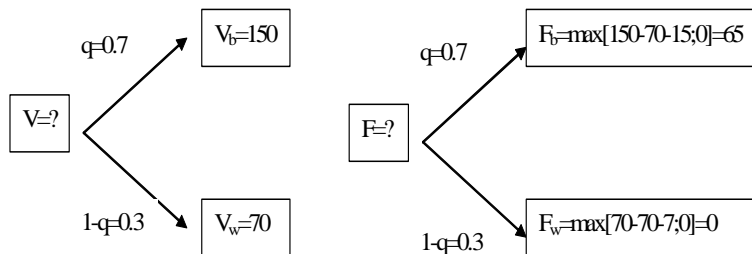
The value of the stock at period t is the present value of all dividends from that

time on. At the end of period t, the value of the project may be $V_b = 150$ with probability $q = 0.7$ or $V_w = 70$ with probability $1 - q = 0.3$; see Figure 1. Of course, the firm does not know the current value V of the project.

The firm can decide to initiate the project immediately or to postpone the commitment until after the state of nature is known, forfeiting the current dividend; see Figure 1. This decision introduces an asymmetry in the pay-offs of the NPV: the deferred strategy has a payoff of $F_b = 65$ in the best state and $F_w = 0$ in the worst state. As before, the present value F of this strategy is unknown at the current time.

2.1 The Value of the Stock

The firm chooses one of the two investment plans by comparing the NPVs. To begin with, let's assume that the stock strategy can be modelled as a twin security traded on the capital market. This security is a risky asset with a current price of $P = 10$. In the next period it will be worth $P_b = 15$ in the best state, with probability $q = 0.7$ or worth only $P_w = 7$ in the worst state. The price will then remain at this new level forever. Then, assume that a riskless security with interest rate equal to $r = 8\%$ is exchanged on the market. Given these two assets what are the appropriate risk-adjusted interest rates of the alternative strategies?

Note that returns on P are proportional to returns on the stock strategy, and that the payoffs are governed by the same binomial distribution. We can therefore argue that they are equivalent in risk. Thus, we can use the interest rate of the underlying asset to calculate the NPV of the immediate investment strategy. The required rate μ for the underlying asset is given by the expression:

$$10 = \frac{(0.7)15 + (0.3)7}{(1 + \mu)},$$

Implying that $\mu = 0.26$ is the rate at which the firm will discount the expected payoff from the stock. Employing this risk-adjusted rate, we obtain the present value V:

$$V = \frac{(q)V_b + (1 - q)V_w}{1 + \mu} = \frac{(0.7)150 + (0.3)70}{1.26} = 100,$$

and the corresponding net present value W:

$$W = -I + V = -70 + 100 = 30.$$

As said above, the ability to postpone the commitment alters the payoffs of the strategy, changing the risk attached to the project. Thus, it follows that the rate μ would be inappropriate to evaluate the deferred strategy. However, let us apply the rate μ to the real option. We obtain:

$$F = \frac{(q)F_b + (1 - q)F_w}{1 + \mu} = \frac{(0.7)65}{1.26} = 36.1.$$

Hence $F > W$ and the firm finds it optimal to postpone the investment.

2.2 The Value of the Real Option

In this section we explain why the previous computation of the real option is incorrect. The basic idea enabling the exact pricing of options is that one can construct a portfolio of traded securities which replicates the future payoffs of the deferred strategy. We begin by constructing a portfolio of traded securities calculating the payoffs of the option at time $t+1$. Let z and n denote shareholdings of the risky asset P and the risk-free security, respectively. If shareholdings are to give equivalent returns in the best and the worst states of nature we have:

$$z(15) + n(1.08) = 65,$$
$$z(7) + n(1.08) = 0.$$

Solving this system yields $z^* = 8.125$ and $n^* = -52.662$. Hence, to obtain the same return of the option strategy on the two securities, an investor would have to purchase 8.125 shares in the risky asset and go short on the riskless asset. Of course, these same quantities of securities are in the portfolio at the initial time t. As a result, the NPV of the option strategy is given by:

$$(z^*)(P) + (n^*) = F,$$
$$(8.125)(10) + (-52.662) = 28.588 \equiv F.$$

In short, the value $F = 28.588$ is smaller than it would have been if we had employed the rate μ. Further, note that it is even smaller than the value of the immediate strategy $W = 30$, implying that the best choice is to invest immediately. This result reverses the previous choice.

Now, using the no-arbitrage argument we can see that the appropriate rate γ of the option must be higher than μ. Since $F = 28.588$ and knowing its values at $t+1$, we can calculate the corresponding risk-adjusted interest rate γ:

$$F = \left[\frac{(q)F_b + (1-q)F_w}{1+\gamma} \right],$$
$$28.588 = (0.7)\left[\frac{65}{1+\gamma} \right] \Rightarrow \gamma = 0.59.$$

Thus, this procedure yields two different interest rates: $\mu = 0.26$ for the stock, and $\gamma = 0.59$ for the option. To complete our argument, note that the risk-neutral probability yields the same value for F. The importance of this synthetic probability is that expectations calculated with them equal the NPV of F once discounted by the riskless rate r. We denote this probability $\hat{q}$. To obtain a value for $\hat{q}$, assume the firm is risk neutral. In this scenario, all that the firm requires is the riskless rate on the stock strategy. Hence:

$$(1+r)V = \hat{q}V_b + (1-\hat{q})V_w,$$
$$(1+0.08)100 = (\hat{q})150 + (1-\hat{q})70.$$

Solving, we obtain $\hat{q} = 0.475$. Using this to compute the current value F, we get:

$$F = \frac{(\hat{q})F_b + (1-\hat{q})F_w}{(1+r)},$$
$$F = (\hat{q})\left[\frac{F_b}{1+r}\right] = \frac{(0.475)65}{1.08} = 28.588,$$

which confirms our previous result. This does not mean, of course, that the appropriate rate of interest for the deferred strategy is the riskless rate r.

This example provides a simple illustration of the difficulty of applying traditional calculations of NPV when assessing real options. Unless the rate of interest is adjusted upwards to reflect the change in risk due to the firm's investment decisions, the traditional approach over-estimates the value of real options and can induce the firm to choose an investment plan which does not maximize the NPV. This is an important implication for the asset pricing of competing irreversible projects.

But note that the absence of arbitrage does not imply that it is necessary that the choice of the optimal investment strategy must be reversed once plans are correctly priced. Actually, the main point is that no external investor would be willing to finance the investment plan if the interest rate offered by the firm is smaller than the one required by the market. Indeed, the realization of the option strategy cannot be made without knowledge of the appropriate cost of capital.

If the interest rate were equal for both the strategies, the supply of financial funds would be infinitely elastic irrespective of the riskiness of the project. But in a risky environment, the stock and the option have different degrees of risk and, consequently, different risk-adjusted rates. Hence, the example shows how the risk-return combination affects the interest rate, pointing out how to extend the concept of risk-adjusted rate to the situation of irreversibility where opportunities do not all have the same risk.

3. Pricing Competing Strategies: The General Case

In this section we generalize the results presented above by viewing any investment opportunity as offering its owner two facets: expected return and risk. We assume that the operating profit P given by an investment is a stochastic variable obeying geometric Brownian motion:

$$dP = \alpha P dt + \sigma P dz, \tag{1}$$

where α is the constant drift, σ^2 the constant variance, and dz is a normally distributed random variable, with $E(dz) = 0$ and $E(dz)^2 = dt$.

Our approach the problem is to treat immediate and deferred investments as two

distinct assets—a stock and a real option—with values that depend on the more fundamental variable P. When investment is irreversible this approach makes it possible to benchmark alternative plans using the features of the underlying asset P.

3.1 The Stock

We start with the stock, writing the value of the immediate strategy as:

$$V(P) = E_t \left\{ \int_t^\infty e^{-\mu s} \left(P_s \right) ds \mid_{P_s = P} \right\},$$ (2)

where μ is the appropriate rate of interest of the stock. The corresponding NPV is $W(P) = V(P) - I$, where I is the direct cost of the irreversible investment. For the moment we assume μ is given.

It is necessary to show that the stock $V(P)$ has both the same expected growth rate and the instantaneous variance of P. To see this, we use dynamic programming to rewrite equation (2). Dynamic programming breaks a whole sequence of decisions into just two components: the value of the operating profit P over the interval $(t, t + dt)$ and the expected value beyond $t + dt$ which encapsulates the consequences of all subsequent decisions from that time on:

$$V(P) = Pdt + E\left[V(P + dP)e^{-\mu dt} \right].$$

Applying Ito's lemma to relate changes in V to those in P we obtain the expression:

$$V(P) = Pdt + \left[\alpha V_P P + \frac{1}{2} \sigma^2 V_{PP} P^2 \right] dt + \left(1 - \mu dt \right) V(P),$$

where V_P and V_{PP} are the first and the second derivatives of V with respect to P, respectively. This equation can be rewritten as:

$$\mu = \frac{P}{V(P)} + \frac{\alpha V_P P + \frac{1}{2} \sigma^2 V_{PP} P^2}{V(P)}.$$ (3)

This is an arbitrage equation: in equilibrium the expected return μ is equal to the instantaneous rate of dividend $P/V(P)$ plus the expected cash appreciation rate $[\alpha V_P P + (1/2)\sigma^2 V_{PP} P^2]/V(P)$. Thus μ is the actual rate of return for the immediate investment. From (3) we obtain the stochastic partial differential equation:

$$\frac{1}{2} \sigma^2 V_{PP} P^2 + \alpha V_P P - \mu V + P = 0,$$ (4)

which has the general solution $V(P) = A_1 V^{d_1} + A_2 V^{d_2} + (P/\delta)$, where $d_1 > 1$ and $d_2 < 0$ are the roots of the characteristic equation, and where A_1 and A_2 remain to be determined. Given that $V(0) = 0$, we must impose that $A_1 = 0$, and $A_2 = 0$ to avoid

speculative bubbles. Hence, the solution of this problem is:

$$V(P) = \frac{P}{\delta}, \tag{5}$$

which is the expected present value of the profit obtained by investing at the current time. Further, note that from (5) the dividend rate is given by $\delta = P/V(P)$ and that substituting this into (3) the expected cash appreciation rate reduces to α, so that $\mu = \delta + \alpha$. Finally, a basic consequence is that the change in V can be written as:

$$dV = V_p dp + \frac{1}{2} V_{pp} (dp)^2$$
$$= \frac{1}{\delta} [\alpha P dt + \sigma P dz] + \frac{1}{2} V_{pp} \sigma^2 P^2 dt.$$

Using (5) to solve the previous expression we find that the dynamic of $V(P)$ is:

$$dV = \alpha V dt + \sigma V dz$$

This means that P and V are perfectly correlated and equivalent in risk.

3.2 The Real Option

Let us now turn to the deferred strategy with $F(P)$ representing the value of the real option. So long as nothing has been invested, the life of the investment program remains the same and the firm sacrifices the rewards associated with the investment. This is the opportunity cost of keeping the right to invest in the future. The expected present value of this opportunity is:

$$F(P^*) = E_t \left\{ \left[V(P^*, \tau) - I \right] e^{-\gamma(\tau - t)} \right\}$$
$$= E_t \left[e^{-\gamma(\tau - t)} \int_\tau^\infty \left(e^{-\mu s} P_s ds \big|_{P_\tau = P^*} \right) - e^{-\gamma(\tau - t)} I \right]. \tag{6}$$

The investment decision depends on both the time τ and the critical value P^*, at which the firm finds it optimal to undertake the irreversible project. We use γ to denote the discount rate of the real option.

Now we show that the real option is riskier than the stock. Given that the operating profit P has the same dynamics as V, we can substitute $F(V)$ for $F(P)$ and rewrite equation (6) in the form of the corresponding Bellman equation:

$$\gamma F(V) dt = E_t [dF(V)] . \tag{7}$$

The instantaneous profit P does not appear in this expression because the real option yields no cash flow in the inactive region. As it stands, equation (7) can be seen as an application of Ito's Lemma. The derived dynamic pattern of $F(V)$ is the process:

$$dF = \gamma F dt + sF dz,$$ (8)

where

$$\gamma = \frac{\alpha F_v V + \frac{1}{2}\sigma^2 F_{vv} V^2}{F} = \frac{E(dF)}{dt F}$$ (9)

and

$$s = \sigma F_v \frac{V}{F}$$ (10)

is the corresponding risk, with $F_v(V/F)$ measuring the elasticity of F with respect to V. From (9) we obtain the differential equation:

$$\frac{1}{2}\sigma^2 F_{vv} V^2 + \alpha F_v V - \gamma F = 0,$$ (11)

and to ensure convergence we must have $\gamma > \alpha$. In addition $F(V)$ must satisfy the boundary conditions $F(0) = 0$, $F(V^*) = V^* - I$, and $F_{v^*} = 1$, where V^* is the critical value at which the firm finds it optimal to undertake the irreversible project. Note that from the second boundary condition we have that $V^* = I + F(V^*)$, setting the value of the project equal to the full cost—direct cost plus opportunity cost—of making the commitment. For the positive root $b_1 > 1$ this gives the solution:

$$F(V) = BV^b,$$

where $b_1 \equiv b$. This expression is the value of the real option. Using this solution, we can solve the elasticity term in (10) to find that:

$$F_v \frac{V}{F} = bBV^{b-1} \frac{V}{BV^b} = b > 1$$ (12)

Thus, from (10) we get that $s > \sigma$; that is, the real option is riskier than the stock.

4. Asset Pricing and Interest Rates

To clarify the meaning of the previous result note that in our solution we use the assumption that real and financial assets are exchanged in the market. So, let's assume that the intertemporal CAPM is a valid description of asset returns at equilibrium. Applying this framework, we can explain why optimal investment decisions require $\gamma > \mu > r$. We refer to this relationship as the risk structure of interest rates.

As noted above, the actual interest rate of the stock is μ. In a CAPM framework, this is the required interest rate that satisfies the no-arbitrage condition:

$$\mu = r + \lambda\beta_V . \tag{13}$$

As usual, $\beta_V = \sigma_{Vm}/\sigma_m^2$ is the systematic component of risk, $\lambda = [E(R_m) - r]$ is its price, and m is the market portfolio. Now, since the standard deviation of the real option is $s = b\sigma$, the covariance between the option and the market portfolio is $\sigma_{Fm} = b\sigma_{Vm}$, so that the corresponding beta coefficient for the option is given by:

$$\beta_F = b\beta_V . \tag{14}$$

This last equality is of key importance: since $b > 1$ the option is riskier than the stock, and the deferred strategy will have a higher risk premium. Of course, excluding arbitrage this must imply that $\gamma > \mu$. To prove that this result is a consequence of the arbitrage principle, take the CAPM equation of the real option $\gamma = r + \lambda\beta_F$ and equations (13) and (14) to obtain:

$$\gamma = r + b(\mu - r) . \tag{15}$$

Finally, using equations (9) and (12) and the condition $\mu = \delta + \alpha$, we obtain the following evaluation equation:

$$\frac{1}{2}\sigma^2 F_{VV}V^2 + (r - \delta)F_V V - rF = 0 . \tag{16}$$

This differential equation is identical to (11) with the significant difference that now we adopt an explicit no-arbitrage argument to identify the relationship between interest rates. To show that (16) satisfies the no-arbitrage condition, equate equation (15) to the actual expected rate of the option $F(V) = BV^b$. Ito's lemma gives:

$$\frac{dF}{F} = \left[b\alpha + \frac{1}{2}b(b-1)\sigma^2\right]dt + b\sigma dz , \tag{17}$$

with expected value equal to $E(dF/dtF) = b\alpha + (1/2)b(b-1)\sigma^2$. Equating this expression to equation (15) we get the characteristic equation:

$$b\alpha + \frac{1}{2}b(b-1)\sigma^2 = r + b(\mu - r) , \tag{18}$$

which has the solution:

$$b = \frac{1}{2} + \frac{\delta - r}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}} . \tag{19}$$

But, this expression is the positive root $b_1 > 1$ of equation (16) which ensures that:

$$\gamma > \mu > r .$$

Hence, in a portfolio approach the risk-adjusted rate of the real option incorporates a risk premium which is higher than that of the stock.

5. Concluding Remarks

Irreversibility plays a particularly important role for investment decisions. The firm must look ahead to plan its current choice taking into account irreversibility and uncertainty on future profits. If the firm has the opportunity to postpone an irreversible project, the appropriate interest rate of any single strategy must capture the characteristics of the corresponding investment plan, and the current value of any strategy requires an appropriate rate, reflecting not only the cost of uncertainty but also the market value of irreversibility.

Some of the results presented in this paper contrast with the traditional wisdom of economic modeling. In fact, it is common to investigate the exchange of fixed quantities of risk for a given return, and firms frequently evaluate alternative irreversible strategies for the same project using a single interest rate. The key point of our analysis is that irreversibility introduces peculiar elements into investment decisions, leading to the result that it is necessary to compute the risk structure of the interest rates to correctly evaluate competing irreversible investment opportunities.

This result is based on the general property that an investment is correctly priced only if its market value does not allow opportunities for arbitrage. When this condition is violated, it is likely that incorrect decisions are made. Hence, a given interest rate will not, in general, evaluate alternative strategies of a single irreversible project correctly.

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