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Second-Degree Monopoly Wholesaler with Variable Ordering Costs

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Abstract

Ordering costs are usually considered as fixed costs. For the case of one seller and two identical buyers, we examine ordering charges that are proportional to the number of units ordered. We find that no extra profits will be generated, neither for the producer nor for the retailers. Thus, proportional ordering costs are not economically justified.

Key words: retailers; ordering charges; excess demand; excess supply

JEL classification: C44; L12; L16; M11

1. Introduction

The issue of ordering costs is very popular in the literature of inventory management where most papers deal with cases where the buyer is charged a fee per order irrespective as to the actual quantity ordered. This charge per order covers issuing ordering certificates, forms, correspondences, and other expenses.

When we review the economic, operational research, and management science literature, we find the term "ordering cost" (or in some cases "setup cost") to refer to fixed costs that are independent of the size of the units purchased, or "replenishment costs" in the case of buying inventory. For example in a recent book, Silver et al. (1998) say "For a merchant it is called an ordering cost and it includes the cost of order forms, postage, telephone calls, authorization typing of orders, receiving, (possibly) inspection, following up on unexpected situations, and handling of vendor invoices" (p. 46).

The literature on optimal inventory policy uses in several academic papers and

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books the term “ordering cost” per unit as equivalent to the term “purchasing cost” per unit without distinguishing between the two terms (see for example Gallego and Moon, 1993; Lau, 1980). In models dealing with the “newsboy problem,” see Scarf (1959), who defines three types of cost (p. 196) and Clark and Scarf (1960).

Chen et al. (2006) assume that each replenishment incurs a fixed ordering (setup) cost while the variable part of the cost is proportional to the procurement quantity. They further assume that holding costs are proportional to the ordered quantities. They follow several other researchers who deal with ordering costs and assume that they are fixed per order regardless of the size of the order, and even assume an absence of fixed ordering costs (e.g., Parlar, 1988; Avsar and Baykal-Gursoy, 2002). This approach of a fixed ordering cost combined with a variable holding cost per unit of inventory brings us back to the square root formula of Whitin (1957) of minimizing total cost of inventory management, recently cited by Wagner and Whitin (2004).

We develop a different approach towards ordering costs where we consider them as variable costs that are proportional to the units ordered and not as a fixed cost that is independent of the units ordered. We consider these costs as revenues of the monopoly seller who may create another tool to generate more profits where the buyers not only pay for the units actually purchased but for ordering those units.

Our approach is therefore unique since we distinguish between two kinds of costs that are spent or may be spent at different time intervals: at the time of ordering where the producer charges the buyers for ordering inventories regardless of whether they will later actually purchase them or not. In addition, a further charge is imposed on the buyers who must pay for the actual delivery of the quantity ordered when purchasing those units from the producer.

The strategy of imposing a charge for placing orders in addition to a price per unit sold can be beneficial to the monopolist in certain cases. First, it can be applied to agricultural products where uncertainty as to the actual crops exists. The monopoly farmer can argue that he plans to produce items such as vegetables, fruit, or dairy products, but he also wants to guarantee that his efforts do not go unrewarded. He therefore charges his buyers for their committed orders and in return the producer commits himself towards the buyers to supply a quantity that is proportional to their orders from his total available output.

Another similar realistic example is in the stock or bond market. A company issues a certain number of units of new stocks and allocates it between new shareholders proportionally to their orders. Often the total orders are significantly larger than the quantity supplied. However, since the potential buyers must put up enough money to cover their entire order until the actual allocation takes place, this requires many shareholders to take out very short-term loans and to pay an interest charge over and above the value of their final stock purchase. The only difference between the two examples above is that in the former both sources of revenues are gained by the monopolist, while in the latter example the revenues from ordering go to the commercial bank, while the revenues from the sale of the new stock go to the issuing company.

We know from microeconomic theory that a monopolist can increase profits by (first-, second-, and third-degree) price discrimination, by use of a two-part tariff policy, or other ways. We suggest yet another way the monopolist can achieve higher profit levels: by paying for ordered units the monopolist also generates a commitment on the part of the buyers towards the producer/seller that can be considered by the latter as a kind of insurance policy that he does not produce in vain, since the buyers are now committed to buying.

These ordering charges are especially useful in an environment where the buyers compete with each other and there is a limited quantity supplied by the monopolist that is rationed by the seller either on purpose or due to natural reasons (e.g., crop failures).

In this paper we address whether it would be in the interest of a monopoly producer to charge retailers two kinds of separate tariffs: (a) a charge for each unit ordered and (b) a charge for the quantity that is actually purchased. Doing so may enable the monopolist to squeeze more revenues from the retailers than in the case where the monopolist only charges for units purchased. Yet retailers may respond by reducing their orders. We analyze these different cases and develop conditions under which each pricing system is preferable for both retailers and the monopoly producer.

2. The Model

A monopoly producer sells quantity Q according to the price he can charge from two marketers (retailers). Those marketers are price takers, where they know that the quantity produced and available to them depends on the total supply by the monopoly wholesaler, Q, and on the order O_i marketer i offers relative to the order O_j offered by the rival marketer j. If the total orders by the two marketers $O_1 + O_2$ is larger than total quantity supplied by the monopoly wholesaler, Q, then each marketer receives his relative share from Q. More precisely, the assumed simplest rationing rule is that marketer i will receive an actual quantity of $(O_i / O_1 + O_2)Q$ and the rival will receive $(O_j / O_1 + O_2)Q$. Those quantities will be sold to marketers at a given price determined by the monopoly producer.

In the event that $O_1 + O_2 < Q$, the monopolist will sell the surplus at a lower price determined by the monopolist so as to be left without any unintended inventory, which is once again sold in proportion to the original orders. In this sense we can view the monopoly pricing as second-degree price discrimination.

We therefore have a three-player, two-stage game: In the first stage, the monopolist determines the price and whether or not to charge an ordering fee. In the second stage, the marketers respond to the price offered by the monopoly and accordingly determine their orders. (This may lead to a further response on the part of the monopolist who further adjusts his price in response to the last order, and so on; here we focus on the two-stage game.) We start with the reaction functions of both marketers towards the monopoly action. The latter desires to maximize profits based on the responses of the marketers he faces.

Before proceeding, we define the following notation: P_M is the market price charged by marketers to consumers, P_S is the price charged by the monopolist to the marketers, r is the cost per unit ordered paid by the marketers to the monopolist when ordering, and α is the price discount the monopolist offers for leftover units supplied in case of excess supply.

3. Monopoly Marketers' Behavior with No Ordering Charges

At this stage we analyze the case of a monopoly wholesaler (or producer) and two marketers, for the case that ordering costs are not charged by the former, either because it is against the law or because it is not desired by the wholesaler. Below we discuss the case of charging ordering costs and compare solutions with and without order charges. We examine which of the solutions is preferred by the various parties.

The profit function of marketer 1 in the case of no ordering charges is:

$$\begin{aligned}\pi_1 &= (P_M - P_S)\cdot \min\left[\frac{O_1}{O_1+O_2}\cdot Q,\ O_1\right] \\ &\quad + \left[P_M - (1-\alpha)P_S\right]\cdot \max\left[0,\ \frac{O_1}{O_1+O_2}\cdot (Q - O_1 - O_2)\right].\end{aligned} \tag{1}$$

In this case two scenarios are investigated. First assume $O_1 + O_2 > Q$, i.e., excess demand exists. Thus, the profit is:

$$\pi_1 = (P_M - P_S)\cdot \frac{O_1}{O_1+O_2}\cdot Q\,, \tag{2}$$

and the first-order condition is:

$$\frac{\partial \pi_1}{\partial O_1} = (P_M - P_S)\left[\frac{1}{O_1+O_2} - \frac{O_1}{(O_1+O_2)^2}\right]Q > 0\,. \tag{3}$$

Since profit increases with each additional order of each marketer, the values O_1 and O_2 increase until the excess demand disappears, i.e., $O_1 + O_2 = Q$. Because of symmetric marketer behavior, we can assume that $O_1 = O_2$, thus:

$$O_1 = O_2 = \frac{Q}{2}\,. \tag{4}$$

Thus the profit of each marketer is equal to:

$$\pi_1 = \pi_2 = \frac{(P_M - P_S)Q}{2}\,. \tag{5}$$

The second case is where excess supply exists, i.e., $O_1 + O_2 < Q$. Then the profit function is:

$$\pi_1 = (P_M - P_S)O_1 + \left[P_M - (1-\alpha)P_S\right]\frac{O_1}{O_1 + O_2}(Q - O_1 - O_2)\,, \tag{6}$$

and the first-order condition is:

$$\frac{\partial \pi_1}{\partial O_1} = -\alpha P_S + \left[P_M - (1-\alpha)P_S\right]\left[\frac{1}{O_1 + O_2} - \frac{O_1}{(O_1 + O_2)^2}\right]Q = 0\,. \tag{7}$$

From (7) we can derive the reaction order function of marketer 1 towards the order of the rival (marketer 2) as follows:

$$O_1 = \sqrt{\frac{\left[P_M - (1-\alpha)P_S\right]O_2 Q}{\alpha P_S}} - O_2\,. \tag{8}$$

The optimal order quantities are $O_1 = O_2$:

$$O_1 = O_2 = \frac{\left[P_M - (1-\alpha)P_S\right]Q}{4\alpha P_S}\,, \tag{9}$$

and the profit at equilibrium is:

$$\pi_1 = \pi_2 = \frac{\left[P_M - (1-\alpha)P_S\right]Q}{4}\,. \tag{10}$$

At this stage we investigate the monopoly profit maximization under the precise order policies of both marketers:

$$\pi_S = P_S \cdot \min\left[Q,\, O_1 + O_2\right] + (1-\alpha)P_S \cdot \max\left[0,\, Q - O_1 - O_2\right] - \frac{bQ^2}{2}\,. \tag{11}$$

The first case the monopoly producer faces is $O_1 + O_2 > Q$. The profit function is:

$$\pi_S = P_S Q - \frac{bQ^2}{2}\,. \tag{12}$$

Since in this case $Q = P_S / [b(1+\beta)]$, we can rewrite (12) as:

$$\pi_S = \frac{P_S^2}{b(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2}\,. \tag{13}$$

The first-order condition is:

$$\pi_S = \frac{P_S^2}{b(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2}\ , \tag{14}$$

which again indicates that when no ordering cost is included, the monopolist continues to raise P_S until excess demand approaches zero, and therefore, at equilibrium, $O_1 = O_2 = Q/2$. Moreover the profits of the two marketers are equal: $\pi_1 = \pi_2 = (P_M - P_S)Q/2$. In this case the highest price charged by the monopoly producer is $P_S = P_M$. Therefore the monopolist gains maximum profit that is:

$$\pi_S = \frac{P_M^2}{b(1+\beta)} \cdot \left[1 - \frac{1}{2(1+\beta)}\right]. \tag{15}$$

4. Monopoly Marketers' Behavior with Ordering Charges

In this case the objective function of marketer 1 in the second stage is:

$$\begin{aligned}\pi_1 = (P_M - P_S)\cdot \min\left[\frac{O_1}{O_1+O_2}\cdot Q,\ O_1\right] - rO_1 \\ + \left[P_M - (1-\alpha)P_S\right]\cdot \max\left[0,\ \frac{O_1}{O_1+O_2}\cdot (Q - O_1 - O_2)\right].\end{aligned} \tag{16}$$

This profit maximization function should be solved for two different scenarios. The first case is where the total orders by the marketers exceed the quantity produced and supplied by the monopolist. In this case, the profit maximization includes only the first term of (16):

$$\underset{O_1}{Max}\, \pi_1 = (P_M - P_S)\frac{O_1}{O_1+O_2}\cdot Q - rO_1\ . \tag{17}$$

In this case the first-order condition for profit maximization is:

$$\frac{\partial \pi_1}{\partial O_1} = \left(P_M - P_S\right)\left[\frac{1}{O_1+O_2} - \frac{O_1}{\left(O_1+O_2\right)^2}\right]Q - r = 0\ . \tag{18}$$

From (18) we can derive the reaction function of the ordered quantity by marketer 1 in response to the order of his rival (marketer 2):

$$O_1 = \sqrt{\frac{\left(P_M - P_S\right)O_2 Q}{r}} - O_2\ . \tag{19}$$

Since we assume symmetric behavior of marketer 2, we conclude that the optimal order values are the same, and at equilibrium we find the following orders:

$$O_1 = O_2 = \frac{(P_M - P_S)Q}{4r}\,. \tag{20}$$

From (17) and (20) we find equal profit values at equilibrium:

$$\pi_1 = \pi_2 = \frac{(P_M - P_S)Q}{4}\,. \tag{21}$$

The second case we should investigate is when the total ordered quantities by marketers are smaller than the quantity supplied by the monopoly producer. The profit function of the monopoly producer in this case is:

$$\underset{O_1}{Max}\, \pi_1 = (P_M - P_S - r)O_1 + [P_M - (1-\alpha)P_S]\frac{O_1}{O_1 + O_2}\cdot (Q - O_1 - O_2)\,. \tag{22}$$

The first-order condition is:

$$\frac{\partial \pi_1}{\partial O_1} = P_M - P_S - r + [P_M - (1-\alpha)P_S]\cdot\left[\frac{O_2}{(O_1 + O_2)^2}\cdot Q - 1\right] = 0\,. \tag{23}$$

Thus, the reaction function of the ordered quantity by marketer 1 as a function of the quantity ordered by marketer by 2 is:

$$O_1 = \sqrt{\frac{[P_M - (1-\alpha)P_S]\cdot O_2 \cdot Q}{\alpha P_S + r}} - O_2\,. \tag{24}$$

Since we assume again symmetric behavior of the rival, we conclude that the equilibrium quantity ordered by both marketers is equal to:

$$O_1 = O_2 = \frac{[P_M - (1-\alpha)P_S]Q}{4(\alpha P_S + r)}\,, \tag{25}$$

and the profit value at this equilibrium for each of the two rivals is:

$$\pi_1 = \pi_2 = \frac{(P_M - P_S - r)Q}{4}\,. \tag{26}$$

At this stage we introduce the response of the monopoly wholesaler to the orders of the two marketers. The objective of the monopolist, who faces a simple production cost function $TC = bQ^2/2$, is profit maximization subject to the quantities ordered by the marketers. Furthermore, we assume that P_S, the unit price charged by the monopolist, is proportional to $MC = bQ$, the marginal cost, which increases proportionally to Q, the quantity supplied, as:

$$P_S = (1+\beta)bQ\,,$$

where $\beta > 0$ is the coefficient representing the profit margin ratio. The general profit function of the monopoly producer is:

$$\pi_S = P_S \cdot \min\left[Q,\, O_1 + O_2\right] + r(O_1 + O_2) + (1-\alpha)P_S \cdot \max\left[0,\, Q - O_1 - O_2\right] - \frac{bQ^2}{2}. \tag{27}$$

Again, we introduce the monopoly profit for two specific cases. The first case is $O_1 + O_2 > Q$, i.e., the monopolist faces excess demand. In this case, profit is:

$$\pi_S = P_S Q + r(O_1 + O_2) - \frac{bQ^2}{2}. \tag{28}$$

Assuming $O_1 = O_2 = \left(P_M - P_S\right)Q/4r$ and $Q = P_S/[b(1+\beta)]$, we can rewrite the profit function of the monopolist as:

$$\begin{aligned}\pi_S &= \frac{P_S^2}{b(1+\beta)} + 2r \cdot \frac{(P_M - P_S)P_S}{4rb(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2} \\ &= \frac{P_S^2}{2b(1+\beta)} \cdot \frac{\beta}{1+\beta} + \frac{P_M P_S}{2b(1+\beta)}.\end{aligned} \tag{29}$$

Based on (29), we find that we always have:

$$\frac{\partial \pi_S}{\partial P_S} = \frac{\beta P_S}{b(1+\beta)^2} + \frac{P_M}{2b(1+\beta)} > 0,$$

and this expression is strictly positive, i.e., more profit is accumulated as the monopolist charges a higher price. However, this price cannot increase indefinitely since at some stage excess demand approaches zero, i.e., at some point $Q = O_1 + O_2$. Therefore, at equilibrium:

$$Q = O_1 + O_2 = \frac{2\left(P_M - P_S\right)Q}{4r}. \tag{30}$$

In this case the price at equilibrium is:

$$P_S = P_M - 2r, \tag{31}$$

and the monopoly profit at equilibrium is:

$$\pi_S = \frac{P_M - 2r}{2b(1+\beta)} \cdot \left[\frac{\beta(P_M - 2r)}{1+\beta} + P_M\right]. \tag{32}$$

Based on the monopoly profit and price P_S that we found above, we can derive the quantity ordered and the profit of each marketer as follows:

$$O_1 = O_2 = \frac{(P_M - P_S)Q}{4r} = \frac{P_M - 2r}{2b(1+\beta)}, \tag{33}$$

$$\pi_1 = \pi_2 = \frac{(P_M - P_S)Q}{4} = \frac{r(P_M - 2r)}{2b(1+\beta)}. \tag{34}$$

The second case we want to investigate is when the monopoly wholesaler generates intentionally or due to circumstances beyond his control (e.g., bumper crops) an excess supply in which $Q > O_1 + O_2$. In this case the monopoly profit is:

$$\pi_S = (P_S + r)(O_1 + O_2) + (1-\alpha)P_S(Q - O_1 - O_2) - \frac{bQ^2}{2}. \tag{35}$$

Assuming $O_1 = O_2$ and $Q = P_S / b(1+\beta)$, we can rewrite (35) as:

$$\pi_S = (\alpha P_S + r)(O_1 + O_2) + (1-\alpha)P_S \frac{P_S}{b(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2}. \tag{36}$$

Furthermore, we substitute $O_i = [P_M - (1-\alpha)P_S]Q / 4(\alpha P_S + r)$ for $i = 1,2$ into the reaction function into π_S to get the profit function as a function of the decision variables P_S and α : the regular price for the ordered unit and the discount rate for each extra unit left as excess supply. Thus,

$$\begin{aligned} \pi_S &= 2(\alpha P_S + r)\cdot \frac{[P_M - (1-\alpha)P_S]\frac{P_S}{b(1+\beta)}}{4(\alpha P_S + r)} + (1-\alpha)P_S \frac{P_S}{b(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2} \\ &= \frac{P_S}{2b(1+\beta)}\left[P_M + \left(1 - \alpha - \frac{1}{1+\beta}\right)P_S\right]. \end{aligned} \tag{37}$$

The first-order conditions in this case are:

$$\frac{\partial \pi_S}{\partial P_S} = \frac{P_M}{2b(1+\beta)} + \frac{\left(1 - \alpha - \frac{1}{1+\beta}\right)P_S}{b(1+\beta)} = 0, \tag{38}$$

$$\frac{\partial \pi_S}{\partial \alpha} = -\frac{P_S^2}{2b(1+\beta)} < 0. \tag{39}$$

From (39) we find that monopoly profit decreases with α. Thus, the monopoly wholesaler desires to lower the discount rate α as much as possible. However, we know from the discussion above that the condition for a positive order by each marketer is $\alpha > 1 - [2/3(1+\beta)]$.

At this stage we want to show that the price P_S that the monopoly charges at equilibrium is determined by the price P_M that the marketers charge their customers. To simplify the development let's define $Z \equiv (1+\beta)(1-\alpha)$. Thus,

$$P_S = \frac{P_M(1+\beta)}{2(1-Z)}. \tag{40}$$

The quantity the monopolist offers is:

$$Q = \frac{P_M}{2b(1-Z)}. \tag{41}$$

The last term is the discount price determined as:

$$(1-\alpha)P_S = (1-\alpha)\frac{P_M(1+\beta)}{2(1-Z)} = \frac{P_M Z}{2(1-Z)}. \tag{42}$$

Based on (42), we derive the following proposition.

Proposition 1. The monopoly producer sells his excess supply at a price below his marginal cost of production.

Proof. Since $1/(1+\beta) = MC/P_S$, we find that:

$$MC > (1-\alpha)P_S. \tag{43}$$

The last expression indicates that the discount price excess supply sold by the wholesaler monopoly to the marketers is significantly below the marginal cost or those additional units are sold at a loss.

Using the equilibrium price of (39) in (37), we find that profit of the monopolist is:

$$\pi_S = \frac{P_M^2}{8b(1-Z)}. \tag{44}$$

Based on the equilibrium price determined by the monopolist, we can turn back to the marketers' decisions to order O_1 and O_2 and gain the profit π_1 and π_2:

$$O_1 = O_2 = \frac{[P_M - (1-\alpha)P_S]Q}{4(\alpha P_S + r)} = \frac{(2-3Z)P_M^2}{2b(1-Z)[4\alpha(1+\beta)P_M + 2(1-Z)r]}. \tag{45}$$

Since the denominator of (33) is positive, a necessary condition for a positive order, $O_i > 0$, is $2-3Z > 0$ or that the discount rate satisfies $1-[2/3(1+\beta)] < \alpha$. The profit of each marketer based on (40), (41), and (42) is:

$$\pi_1 = \pi_2 = \frac{(P_M - P_S - r)Q}{4} = \frac{\left[\left(2 - \frac{1+\beta}{1-Z}\right)P_M - 2r\right]P_M}{16b(1-Z)}. \tag{46}$$

5. Comparison between Monopoly with and without Ordering Charges

We summarize results of Sections 3 and 4 in several propositions.

Proposition 2. In a case of excess demand, the monopoly profit that eliminates this excess demand is higher without ordering costs. Moreover, the monopolist exploits its monopoly power and extracts pure economic profits from the marketer.

Proof. Using (46) and (15) we investigate whether the profits that can be generated with ordering costs imposed on the marketers will be larger than in the regular case when no ordering costs are imposed. First we investigate the excess demand case:

$$\pi_{S(\text{no charge order})} = \frac{P_M^2}{b(1+\beta)}\left[1-\frac{1}{2(1+\beta)}\right] \gtrless \frac{P_M - 2r}{2b(1+\beta)}\left[\frac{\beta(P_M-2r)}{1+\beta}+P_M\right] = \pi_{S(\text{with charge order})}. \tag{47}$$

By a simple computation discussed in the Appendix, we conclude that in the case of excess demand, i.e., $O_1 + O_2 > Q$, we get $\pi_{S(\text{no charge order})} > \pi_{S(\text{with charge order})}$, i.e., more profits are gained under simple charging with no ordering costs imposed on marketers. This is true since $P_M / r > 1 > 2\beta/1+3\beta$ always holds true. We also can find the optimal and equal quantities ordered by each marketer:

$$O_1 = O_2 = \frac{Q}{2} = \frac{P_M}{2b(1+\beta)}. \tag{48}$$

This leads to the conclusion that the monopolist extracts all the pure economic profits of both marketers:

$$\pi_1 = \pi_2 = 0. \tag{49}$$

Proposition 3. A case of excess supply will eventually lead to the same level of monopoly profit, regardless of whether ordering costs are imposed.

Proof. Turning now to the case of excess supply, we investigate the monopoly profit maximization for the case where $O_1 + O_2 < Q$. In this case, the profit of the monopolist can be rewritten as:

$$\pi_S = P_S(O_1 + O_2) + (1-\alpha)P_S\left[Q - (O_1 + O_2)\right] - \frac{bQ^2}{2}. \tag{50}$$

Assuming symmetry, i.e., $O_1 = O_2$, and that the quantity supplied is $Q = P_S / b(1+\beta)$, we can rewrite the profit functions:

$$\pi_S = \alpha P_S (O_1 + O_2) + (1-\alpha) P_S \frac{P_S}{b(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2} . \tag{51}$$

Since the marketers are identical in their orders, i.e., $O_i = [P_M - (1-\alpha)P_S]Q/4\alpha P_S$, the profit function can be written as:

$$\begin{aligned} \pi_S &= 2\alpha P_S \frac{[P_M - (1-\alpha)P_S]\frac{P_S}{b(1+\beta)}}{4\alpha P_S} + (1-\alpha)P_S \frac{P_S}{b(1+\beta)} - \frac{bP_S^2}{2b^2(1+\beta)^2} \\ &= \frac{P_S}{2b(1+\beta)}\left[P_M + \left(1-\alpha-\frac{1}{1+\beta}\right)P_S\right]. \end{aligned} \tag{52}$$

The monopolist maximizes its profit with respect to the decision variable, P_S, as follows:

$$\frac{\partial \pi_S}{\partial P_S} = \frac{[P_M - 2(1-\alpha)P_S]}{2b(1+\beta)} + \left[1-\alpha-\frac{1}{2(1+\beta)}\right]\frac{2P_S}{b(1+\beta)} = 0 . \tag{53}$$

Using Z above, we get P_S at equilibrium in terms of P_M as follows:

$$P_S = \frac{P_M(1+\beta)}{2(1-Z)} . \tag{54}$$

Therefore the total profit of the monopolist at the new equilibrium is:

$$\pi_S = \frac{P_M^2}{8b(1-Z)} . \tag{55}$$

Both price and profit with no ordering charges in the case of excess supply are equal to those with ordering charges.

Proposition 4. In the case of excess supply, the marketers gain more profit without ordering costs than with ordering costs.

Proof. Next we want to find the ordered quantities and profits of both marketers. We know that the orders of both marketers are:

$$O_1 = O_2 = \frac{[P_M - (1-\alpha)P_S]Q}{4\alpha P_S} = \frac{P_M(2-3Z)}{2\alpha b(1+\beta)(1-Z)} . \tag{56}$$

Thus, the profits are also similar and are equal to:

$$\pi_1 = \pi_2 = \frac{[P_M - (1-\alpha)P_S]Q}{4} = \frac{P_M^2(2-3Z)}{16b(1-Z)^2} . \tag{57}$$

We can compare the profit of the marketer in the case of excess supply with and without an ordering charge using (31), (57), and $2r > 0 > -(1+\beta - Z/1-Z)P_M$; we conclude that in the case of excess supply, i.e., $O_1 + O_2 < Q$:

$$\pi_{1(\text{no charge order})} = \frac{(2-3Z)P_M^2}{16b(1-Z)^2} > \frac{\left[\left(2-\frac{1+\beta}{1-Z}\right)P_M - 2r\right]P_M}{16b(1-Z)} = \pi_{1(\text{with charge order})} . \quad (58)$$

The marketers' profits without an ordering charge are larger than the profits with an ordering charge.

To summarize the propositions above, we conclude that in the case of excess supply, the monopolist is indifferent between charging and not charging ordering costs, while marketers are better off with no ordering charges. However, under an excess demand environment, while marketers have no economic profit, the monopoly producer gains more profit by eliminating ordering costs.

6. Implications and Conclusions

Here we discuss whether a policy of imposing ordering costs would be preferred by any of the parties involved, i.e., producers, marketers, customers, and the social planner. Our results show that we do not find for any of these parties any gain or advantage from imposing ordering costs, and we therefore conclude that none of the parties involved should consider imposing such costs.

We looked at the advantages and disadvantages of imposed costs in the game between the monopolist and the marketers by use of a two-stage game. In the first stage, the monopolist has to decide whether or not to impose an ordering charge in light of the possibility that as a result the marketers would react by deciding whether to maintain an excess demand of orders or an excess supply. We show that it turns out that their best strategy is to reduce their orders and create an excess supply scenario. Therefore in the second stage, we find a stable solution with no ordering charge combined with an excess supply faced by the monopolist.

Similarly, it turns out that no ordering charge is optimal from the point of view of the social planner. The social planner faces a given quantity Q sold to the customer at a price P_M. From the social point of view, a larger Q (which is socially always preferred) can be achieved through an excess supply. However, the quantity Q in this case if there are no ordering costs will be independent of the amount of the excess demand or the excess supply. In contrast, when ordering costs exist, Q under excess demand is smaller than under excess supply, which demonstrates its inferiority from a social welfare point of view.

Appendix

$$\pi_{S(\text{no charge order})} = \frac{P_M^2}{b(1+\beta)}\left[1-\frac{1}{2(1+\beta)}\right] \underset{<}{\overset{>}{-}} \frac{P_M-2r}{2b(1+\beta)}\left[\frac{\beta(P_M-2r)}{1+\beta}+P_M\right] = \pi_{S(\text{with charge order})}$$

$$\pi_{S(\text{no charge order})} = P_M^2\left[\frac{2(1+\beta)-1}{2(1+\beta)}\right] \underset{<}{\overset{>}{-}} \frac{P_M-2r}{2}\left[\frac{\beta(P_M-2r)+P_M(1+\beta)}{1+\beta}\right] = \pi_{S(\text{with charge order})}$$

$$\pi_{S(\text{no charge order})} = P_M^2\left[2(1+\beta)-1\right] \underset{<}{\overset{>}{-}} \left(P_M-2r\right)\left[\beta(P_M-2r)+P_M(1+\beta)\right] = \pi_{S(\text{with charge order})}$$

$$\pi_{S(\text{no charge order})} = P_M^2\left(1+2\beta\right) \underset{<}{\overset{>}{-}} P_M^2\left(1+2\beta\right) - 2rP_M\left(1+3\beta\right) + 4\beta r^2 = \pi_{S(\text{with charge order})}$$

$$\pi_{S(\text{no charge order})} = 2P_M\left(1+3\beta\right) \underset{<}{\overset{>}{-}} 4\beta r = \pi_{S(\text{with charge order})}$$

$$\pi_{S(\text{no charge order})} = \frac{P_M}{r} > 1 > \frac{2\beta}{\left(1+3\beta\right)} = \pi_{S(\text{with charge order})}$$

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