A NEW METHOD FOR B-SPLINE SHAPE REPRESENTATION

Jia-Rui Zhang, Shih-Yu Chiu, and *Leu-Shing Lan*

Department of Electronics Engineering
National Yunlin University of Science and Technology, Taiwan

ABSTRACT

B-spline curves are well-known utilities for shape modeling and analysis. They have been used in shape design and coding using least-squares (LS) approaches for years. The key disadvantage of LS approaches is its incapability to deal with corners. To reduce this effect, in this work we propose a weighted least-squares technique that puts more emphasis on the data points with larger errors. Through this mechanism, significant reduction of peak reconstruction errors can be achieved. The weight matrix is chosen by first detecting error peaks and then assigning more weight to only those few points with extraordinarily high approximation errors. The effectiveness of the proposed approach is demonstrated through experiments on two real image examples with considerably different characteristics.

Keywords: weighted least-squares, least-squares, B-spline, shape representation, shape modeling

1. INTRODUCTION

To fulfill shape recognition [1] and coding [2], one of the essential steps is shape representation. There have been quite a variety of methods proposed to describe 2-D shapes, such as polygonal approximation [3, 4], chain coding [5], Fourier descriptor [6], B-spline representation [7], wavelet descriptor [8], to name some of them. The B-spline representation method has some particularly attractive properties, including smoothness and continuity, shape invariance, local controllability, and so forth. There have been many application examples of B-splines, for instance, in [2], Schuster and Katsaggelos proposed a method that applies the B-spline technique toward shape coding, which is aimed to be adopted in MPEG-4. Cohen *et al.* [9] used B-splines for the matching and identification of objects. B-spline-related methods also find applications in video-based target tracking. As another practical example, in a recent work [10], B-spline curves are used to record shapes of pedestrians so that people monitoring and tracking can be performed.

Traditionally, B-spline method approximates the original contour of an object in a least-squared (LS) error sense which is common in engineering practice. However, the weakness in any LS approach is that at certain points the peak errors may become overly large. We are thus motivated to investigate a different approach using the weighted least-squared (WLS) error criterion. Larger weighting values are given to data points with larger amounts of errors so that these errors can be reduced. We have derived closed-form formulas for the WLS B-spline shape representation problem. Some shape examples with sharp corners were tested to illustrate the usefulness of the proposed technique.

The rest of this paper is organized as follows. In Section 2, we give a brief introduction to conventional LS B-spline shape representation. Then in Section 3, we present the proposed WLS design approach. A procedure to determine the optimal weight is described in Section 4. Section 5 explains the peak detection method used in weight determination procedure. Experimental results are then given in Section 6. Finally Section 7 concludes this paper.

2. CONVENTIONAL LEAST-SQUARES (LS) B-SPLINE SHAPE REPRESENTATION

Given a set of N points, $(x_0, y_0), (x_1, y_1), \ldots, (x_{N-1}, y_{N-1})$, the design goal is to fit a real-valued B-spline model to this contour data set. The two-dimensional discrete contour curve is described by two parametric curves, i.e., $x(n)$ and $y(n)$, respectively, where n is a parameter variable. $x(n)$ and $y(n)$ are then approximated by a set of B-spline basis curves. This approach has a significant advantage of being able to be extended to multidimensional cases easily. In mathematical form, the approximation [7] is given by

$$\hat{x}(n) = \sum_{k=0}^{M-1} p_{xk} B_k(n) \quad (1)$$

$$\hat{y}(n) = \sum_{k=0}^{M-1} p_{yk} B_k(n) \quad (2)$$

where p_{xk} is the x coordinate of the kth control point, p_{yk} is the y coordinate of the kth control point, $B_k(n)$ is the kth B-spline curve, and M is the number of control points. The traditional least-squares (LS) B-spline shape representation approach minimizes the total squared-error defined by

$$\varepsilon \triangleq \sum_{n=0}^{N-1} |x(n) - \hat{x}(n)|^2 + |y(n) - \hat{y}(n)|^2 \quad (3)$$

$$= \sum_{n=0}^{N-1} \left| x(n) - \sum_{k=0}^{M-1} p_{xk} B_k(n) \right|^2 + \left| y(n) - \sum_{k=0}^{M-1} p_{yk} B_k(n) \right|^2 \quad (4)$$

Using vector and matrix notations, Eq.(4) can be expressed as

$$\varepsilon = \|\boldsymbol{B}\boldsymbol{p}_x - \boldsymbol{x}\|_2^2 + \|\boldsymbol{B}\boldsymbol{p}_y - \boldsymbol{y}\|_2^2 \quad (5)$$

where $\boldsymbol{B}$ is the B-spline basis matrix that is composed of the B-spline basis vectors, $\boldsymbol{x}$ is a vector formed by the x components of the contour data, $\boldsymbol{y}$ is a vector formed by the y components of the contour data, $\boldsymbol{p}_x$ is a vector composed of all x components of the control points, and $\boldsymbol{p}_y$ is a vector composed of all y components of the control points. To determine the locations of optimal control points, we set the partial derivatives of ε with respect to both $\boldsymbol{p}_x$ and $\boldsymbol{p}_y$ equal to zero, which result in

$$\boldsymbol{p}_x = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{x} \quad (6)$$

$$\boldsymbol{p}_y = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{y} \quad (7)$$

Written more compactly, the control points are given by

$$\boldsymbol{P} = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{X} \quad (8)$$

where

$$\boldsymbol{P} = [\boldsymbol{p}_x \quad \boldsymbol{p}_y] \quad (9)$$

$$\boldsymbol{X} = [\boldsymbol{x} \quad \boldsymbol{y}] \quad (10)$$

Note the dimensions of the key quantities are $\boldsymbol{B}$: N x M, $\boldsymbol{X}$: N x 2, and $\boldsymbol{P}$: M x 2, respectively.

3. B-SPLINE SHAPE REPRESENTATION USING A WEIGHTED LEAST-SQUARES APPROACH

The weighted least-squares (WLS) B-spline shape representation approach attempts to minimize the total weighted squared-error defined by

$$\varepsilon = \|\boldsymbol{W}_x(\boldsymbol{B}\boldsymbol{p}_x - \boldsymbol{x})\|_2^2 + \|\boldsymbol{W}_y(\boldsymbol{B}\boldsymbol{p}_y - \boldsymbol{y})\|_2^2 \quad (11)$$

where

$$\boldsymbol{W}_x \triangleq \mathrm{diag}(w_{x0}, w_{x1}, \cdots, w_{x,N-1}) \quad (12)$$

$$\boldsymbol{W}_y \triangleq \mathrm{diag}(w_{y0}, w_{y1}, \cdots, w_{y,N-1}) \quad (13)$$

with $\mathrm{diag}(\cdot)$ representing a diagonal matrix composed of its arguments. Here, $\boldsymbol{W}_x$ and $\boldsymbol{W}_y$ are weighting matrices for the x-component and y-component, respectively. To minimize the total weighted least-squared error, we first find the partial derivative of ε with respect to $\boldsymbol{p}_x$, which can be shown to be given by

$$\frac{\partial \varepsilon}{\partial \boldsymbol{p}_x} = 2(\boldsymbol{B}^T\boldsymbol{W}_x^2\boldsymbol{x} - \boldsymbol{B}^T\boldsymbol{W}_x^2\boldsymbol{B}\boldsymbol{p}_x) \quad (14)$$

Setting this partial derivative equal to zero, we obtain the optimal $\boldsymbol{p}_x$ as

$$\boldsymbol{p}_x = (\boldsymbol{B}^T\boldsymbol{W}_x^2\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{W}_x^2\boldsymbol{x} \quad (15)$$

Similarly, following the same derivation procedure, the y-part of the optimal control points is given by

$$\boldsymbol{p}_y = (\boldsymbol{B}^T\boldsymbol{W}_y^2\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{W}_y^2\boldsymbol{y} \quad (16)$$

4. CHOOSING THE WEIGHTING MATRIX

To apply the weighted LS technique, weighting matrices in Eqs. (12) and (13) should be chosen first. Heuristically, a wise design is to choose these weights based on the error magnitude of each reconstructed boundary point. Data with larger reconstruction errors should be emphasized with larger weighting factors. In our implementation, we use the following guidelines for choosing the weighting matrices:

1. Only diagonal weighting matrices are considered.
2. Use a peak detection procedure to locate all points that are local maxima of the squared-error curve. Only those local maxima points with magnitudes larger than a specified threshold are considered.
3. The determined local maxima points that have error magnitudes above certain threshold will be given higher weight values. All the rest points are given a uniform weight such as 1.
4. Apply a grid search procedure to determine the optimal weight value that results in the minimal peak error. This weight value is then utilized as a uniform weight for all points that should be emphasized.

5. PEAK DETECTION

In the weight determination process, a peak location procedure is needed. We describe this procedure briefly in this section. Given the squared-error curve, all the points that have magnitudes larger than their left and right neighbors are treated as peak points. However, not all these points will be more heavily weighted. Only those peak points that have significantly larger error magnitudes will be chosen as candidates that are assigned high weight values.

6. EXPERIMENTAL RESULTS

Due to space limitation, in this section we show only two test examples. These are `bird`, and `big` images. Both of these images have sharp corners, which are very difficult for B-spline curves to synthesize when high compression ratios (CR) are used. In the experiments, the compression ratios for the `bird` and `big` images were set at 40 and 30, respectively.

6.1. Determining the Optimal Weight

Using the procedure described in Section 4, we can determine the optimal weight value. In the implementation, the search grid has a spacing of 0.1. The search range for the `bird` image is from 2 to 11 and that for the `big` image is from 2 to 9. The search results are illustrated in Figs. 1 and 2 for the `bird` image and the `big` image, respectively. The determined optimal weight values for 5.1 and 4.2, respectively.

6.2. Comparison of Approximation Errors

The approximation error e is defined as the sum of squares of approximation errors in the x and y components, i.e.,

$$e \triangleq |e_x|^2 + |e_y|^2 \qquad (17)$$

where e_x is the approximation error of the x component and e_y is the approximation error of the y component. In Tables 1 and 2, we compare the maximum approximation errors of the three tested shapes between LS and WLS designs. Evidently, for these examples, the proposed WLS design gives smaller maximum approximation errors compared to those of the LS design.

Figs. 3(a) and 3(b) compare the approximation errors of all the boundary data points for the `bird` image. These plots exhibit the different approximation characteristics of LS and WLS approaches. Similar plots for the `big` image are shown in Fig. 4. Note that the `big` image stands for a Chinese character which means "big".

6.3. Contour Comparison

Figs. 3(c) and 3(d) compare the original contours, LS-approximated contours, and WLS-approximated contours for the `bird` image, where Fig. 3(d) is an enlarged plot of Fig. 3(c). It is seen clearly from Fig. 3(d) that the proposed WLS approach approximates corners better than conventional LS approach. Similar findings can be obtained from Figs. 4(c)(d) for the `big` image.

7. CONCLUSION

We have proposed a new approach for B-spline shape approximation based on the weighted least-squared error principle. The main difference between the weighted LS and more conventional LS approaches lies in the choice of an appropriate weighting matrix. A simple and effective method for determining this weighting matrix is also clearly presented in this work. Our experiments demonstrate the effectiveness of the proposed technique. The key advantage of using the WLS technique is its ability to greatly reduce large peak approximation errors.

8. REFERENCES

[1] F. S. Cohen and J. Y. Wang, "Part I: Modeling Image Curves Using Invariant 3-D Object Curve Models:A Path to 3-D Recognition and Shape Estimation from Image Contours," *IEEE Tr. on PAMI*, Vol. 16, No. 1, pp.1-12, Jan. 1994.

[2] G. M. Schuster and A. K. Katsaggelos, "An Optimal Polygonal Boundary Encoding Scheme in the Rate Distortion Sense," *IEEE Tr. on Image Proc.*, Vol. 7, No. 1, pp.13-26, Jan. 1998.

[3] R. O. Duda and P. E. Hart, *Pattern Recognition and Scene Analysis*, Johh Wiley, 1973.

[4] A. Kolesnikov and P. Franti, "Polygonal Approximation of Closed Discrete Curves," *Pattern Recognition*, Vol. 40, pp. 1282-1293, 2007.

[5] H. Freeman, "On the Encoding of Arbitrary Geometric Configurations," *IEEE Tr. Electron. Comput.*, Vol. EC-10, No. 2, pp.260-268. 1961.

[6] C. T. Zahn and R. S. Roskies, "Fourier Descriptors for Plane Closed Curves," *IEEE Tr. Computers*, pp.269-281, Mar. 1972.

[7] D. Paglieroni and A. K. Jain, "A Control Point Theory for Boundary Representation and Matching," *Proc. ICASSP*, Vol. 4, pp.1851-1854, Tampa, Fla., 1985.

[8] G. C. Chung and C.-C. J. Kuo, "Wavelet Descriptor of Planar Curves: Theory and Applications," *IEEE Tr. on Image Proc.*, Vol. 5, No. 1, pp.56-70, Jan. 1996.

[9] F. S. Cohen, Z. Huang, and Z. Yang, "Invariant Matching and Identification of Curves Using B-Splines Curve Representation," *IEEE Tr. on Image Proc.*, Vol. 4, No. 1, pp. 1-10, Jan 1995.

[10] H.-G. Kang and D. Kim, “Real-Time Multiple People Tracking Using Competitive Condensation,” *Pattern Recognition*, Vol. 38, pp. 1045-1058, 2005.

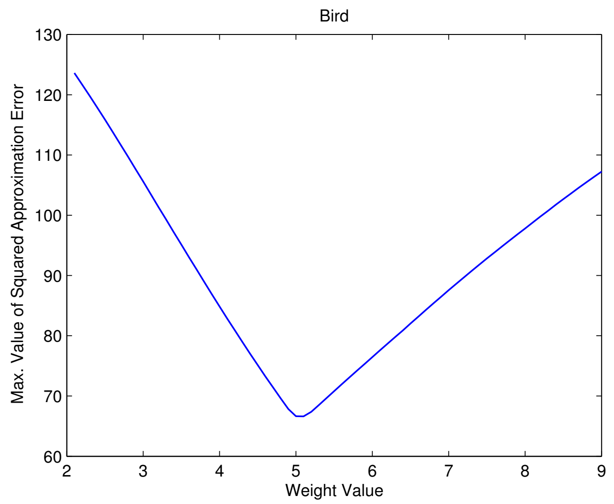

Fig. 1. The peak squared-error curve for determining the optimal weight in the `bird` image.

Table 1. Comparison of maximum approximation errors of the `bird` image between LS and WLS approaches.

	LS	WLS
$e_{\max}$	140.1435	66.6146

Table 2. Comparison of maximum approximation errors of the `big` image between LS and WLS approaches.

	LS	WLS
$e_{\max}$	58.9560	29.4962

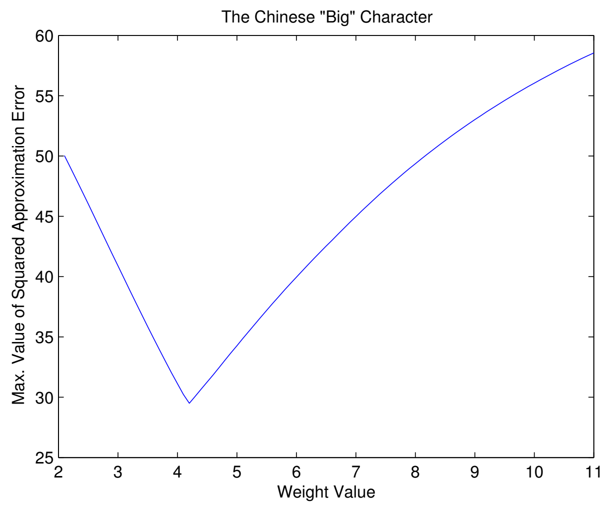

Fig. 2. The peak squared-error curve for determining the optimal weight in the `big` image.

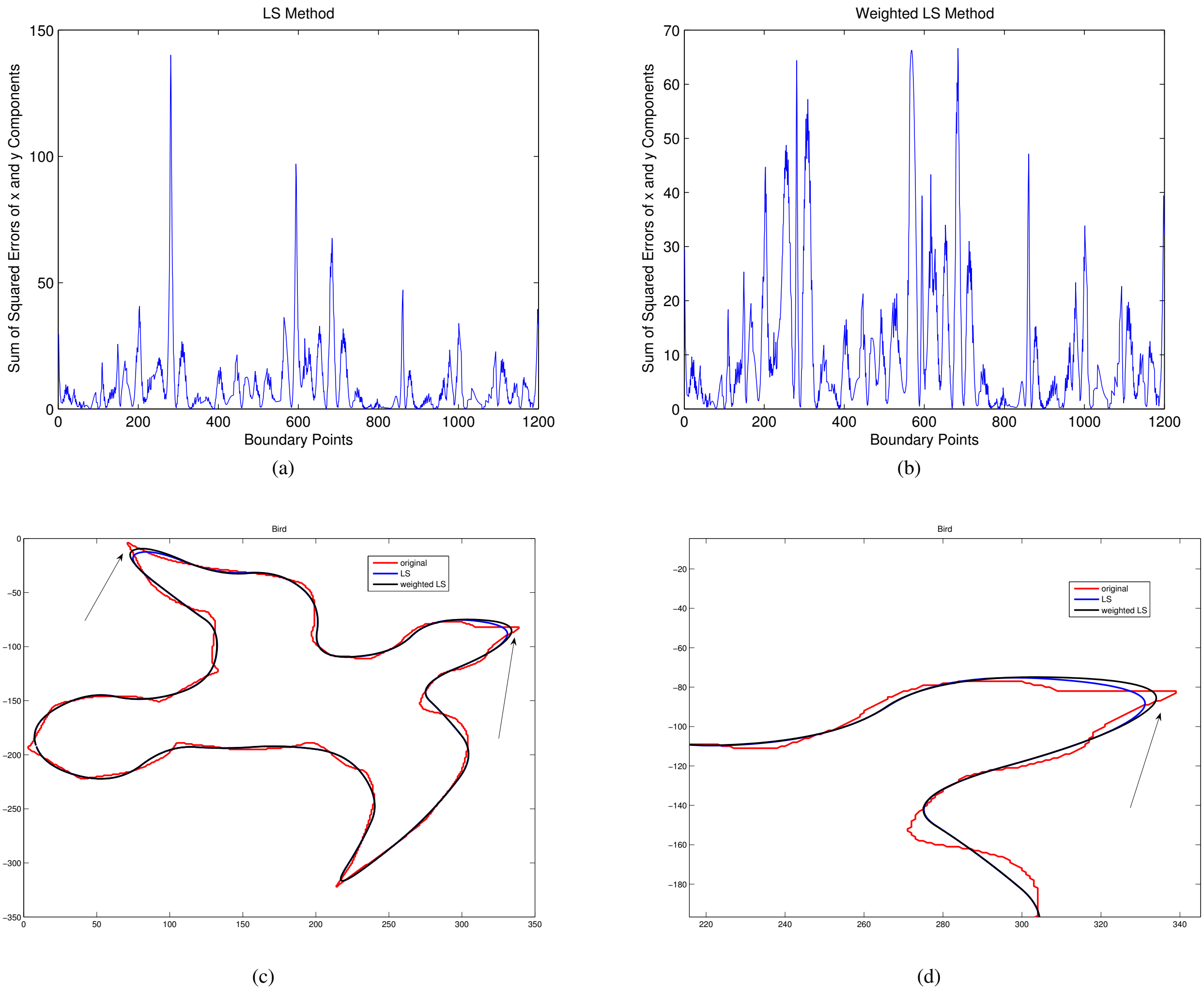

Fig. 3. Approximation errors of all the boundary data points for the `bird` image. (a) LS, (b) WLS, (c) original and approximated contours, (d) enlarged plot.

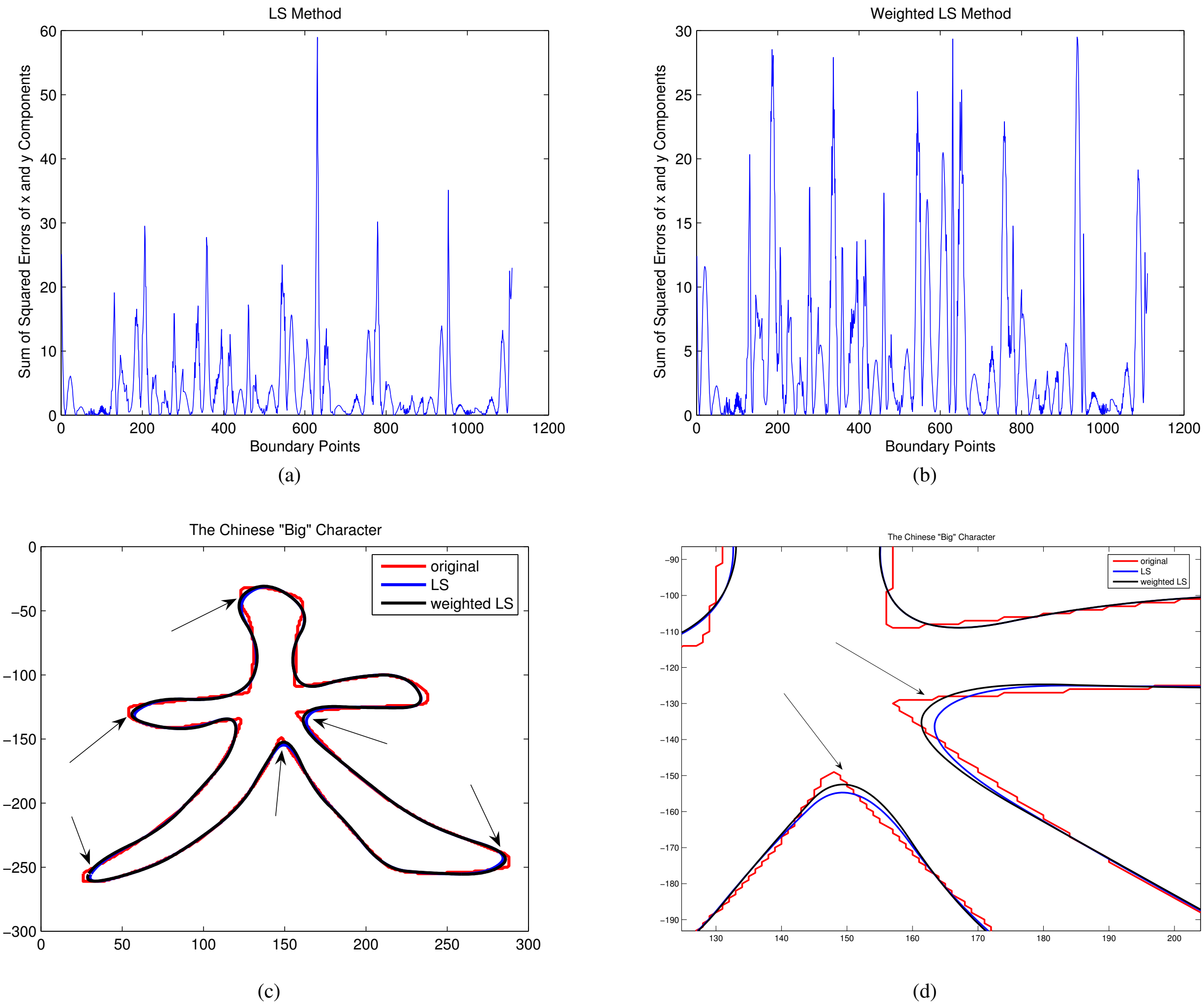

Fig. 4. Approximation errors of all the boundary data points for the `big` image. (a) LS, (b) WLS, (c) original and approximated contours, (d) enlarged plot.