Promoting the Effect of Intuition Concepts in Mathematics with Online Teaching and Web Analogy Materials

Peng-Hen Tsai
Department of Industrial Technology Education, National Kaohsiung Normal University.
mich3860@ms21.hinet.net

Tien-Yu Huang
Department of Computer Science, National Pingtung Teachers College.
tyhuang@mail.npttc.edu.tw

Abstract

This study designed web analogy materials of intuition concepts in eight mathematics conceptions, including angle, discount, ratio, length and area, area and volume, infinite, division, and sequence of number. We sampled thirty-three 6th grade elementary students as experimental group whose ages are between 12 and 13. They used broadband network at home and accepted online teaching and web analogy materials. We sampled other thirty-two students as contrast group and they received normal teaching and materials. The process of the online teaching course was one topic in one and half hour of every week learning at home, and one class of reviewing web analogy materials in computer room at school after four topics. Both groups accepted pretest and posttest to investigate the effect of promoting the intuition concepts in mathematics. After online teaching with web analogy materials, the effect of experimental group was significant difference to contrast group, and the experimental explanation variance was 33.4%.

Keyword：intuition concepts, online teaching, distance learning, analogy, hypermedia

1. Introduction

Online education was growing at a phenomenal rate in many countries, and Taiwan was included. The E-learning market shot up and continued to evolve the learning method. However, according to recent researches, the dropout rate of online students was higher than that in the classroom. The reason lies in that they felt the online courses with little or no personal interaction. These leaded to the following questions: What exactly worked online? Will the course of intuition concepts in mathematics take effect in elementary students?

People facing a problem in an unfamiliar domain use their intuitions rather than general purpose strategies (Newell & Simon 1972). These intuitions came from life experience had carried intrinsic certainty and coercively exclude possible alternative, defined primary intuition usually was wrong notions such as ‘taller thing are bigger than shorter thing’. But the intuitions weren’t unalterable; they could be transformed by learning and formative education to construct accuracy notions, defined secondary intuition. In generally, the primary intuition was existed simultaneously and inconsistent with the secondary intuition.

Stavy (1996) and Tirosh (1999) summarized many research results about students’ conception and proposed four rules of intuition concepts, (a) ‘The more of A, the more of B’, for example, students thought that the longer line contained more points than the shorter one, (b) ‘The same of A, the same of B’, such as the wrong notion that the different rectangles surrounded by two lines of equal length have the same area, (c) ‘The process of successive division is endless’, the process of continuing to divide an object into two equal parts is endless in a geometrical one (for example, a square, a line segment, or a cube), (d) ‘The process of successive division comes to a halt’: if the object is a material one the process is halted when reaches the molecular or atomic level.

Then, how do we transfer students’ intuition concepts that were misconceptions to be accuracy？ Sutula and Krajcik (1988) said the use of an analogy to improve students’ understanding of a concept could be useful, but the analogy itself must be used economically and in a valid and reliable way. Polya (1957) suggests that a useful strategy for solving geometry problems is to search for analogous problems within the domain of geometry.

Most recent researches focused on the use of the Web as a virtual space that either allows people in different locations and at different times to collaborate or enable them to share information and knowledge (Lee 2004). This study tried to transfer students’ intuition concepts in mathematics from primary intuition that was misconception to secondary intuition that was accuracy by web analogy materials designed with hypermedia through online teaching and discussion. To conquer the online problem of little or no personal interaction, the K12 digital school that set up by National Sun Yat-Sen University provided the software ‘JoinNet’ to transmit video and audio with broadband network

between teacher and students. The software 'JoinNet' supported three functions: (a) chartroom: participants communicated through words (Fig. 1), (b) painting board: they could draw what they think to others more clearly (Fig. 2), (c) conference room: they could speak their notions if they could not type (Fig. 3).

Fig. 1 Chartroom

Fig. 2 Painting board

Fig. 3 Conference room

This study designed web analogy materials of intuition concepts in eight mathematics conceptions, including angle, discount, ratio, length and area, area and volume, infinite, division, and sequence of number, and applying online teaching and discussion to transfer students' intuition concepts in mathematics from primary intuition that was misconception to secondary intuition that was accuracy.

2. Problems

1. Would the online teaching and web analogy materials work to transfer the intuition concepts in mathematics of elementary students to be accuracy?
2. Would the intuition concepts in mathematics of elementary students not be influenced by the intuitive rule 'The more of A, the more of B' in the concepts of angle, discount, ratio, and infinite?
3. Would the intuition concepts in mathematics of elementary students not be influenced by the intuitive rule 'The same of A, the same of B' in the concepts of ratio, length and area, and area and volume?
4. Would the intuition concepts in mathematics of elementary students be followed by the intuitive rule 'The process of successive division is endless' in the concepts of division, and sequence of number?

3. Method

3.1. Participants

We sampled thirty-three 6th grade elementary students as experimental group whose ages are between 12 and 13. They used broadband network at home and accepted online teaching and web analogy materials. Moreover, we sampled other thirty-two students as contrast group and they received normal teaching and materials.

3.2. Procedure

The process of the online teaching course was one topic in one and half hour of learning every week at home, and one class of reviewing web analogy materials in computer room at school after four topics. Both groups accepted pretest and posttest to investigate the effect of promoting the intuition concepts in mathematics.

3.3. Assessment

This study designed pretest and posttest to investigate the notions and learning effect of intuition concepts in mathematics. In order to calculate the similarity of both tests in the form and the content, we sampled another class to accept both tests and calculated the correlation coefficient between two tests which was .652 as alternate-form reliability. Besides, most content of pretest and posttest were referred to the researches of Stavy (1996), Tirosh (1996), and Tirosh (1999). The results of their survey pointed out students' conceptions and reasoning would be influenced by intuitive rules whose ages were from six to adults.

Each item of the tests includes two parts; (a) chose the answer that students think it is appropriate, (b) wrote down their notions. If the student's answer in part (a) is right, he/she got two points. If his/her notion in part (b) fitted in with the accepted mathematics notions then he/she got five points, or if there was just partial of notions fitted in with the accepted mathematics notions then he/she got three points. If his/her notion in part (b) were not in line with accepted mathematics notions then he/she got zero points.

4. Results and discussion

4.1. Achievement

Table 1 summarized the means, standard deviations, F value of Levene's Test, t value of independent samples t test and R^2 value in terms of the scores from pretest and posttest. The F value of Levene's Test in pretest and posttest were 0.427 and 3.794. They were not significant difference, so the homogeneity tests among experimental group and contrast group was satisfied.

The t value of independent samples test in pretest was -0.267 and there was not significant difference between experimental group and contrast group. But

the t value of independent samples test in posttest was -2.28 and there was significant difference between experimental group and contrast group. This means that after applying the online teaching and web analogy materials, the learning effect of intuition concepts in mathematics of experimental group was significant difference to the learning effect of contrast group. The R^2 value was 0.334, which showed the experimental explanation variance was 33.4%.

Table 1. Summary of independent samples t test: pretest and posttest

Test	Pretest		Posttest	
Group	Exe	Con	Exe	Con
N	33	32	33	32
Mean	29.485	28.406	30.939	22.844
SD	15.926	16.661	16.785	11.202
F	0.427		3.794	
t	-0.267		-2.28*	
R^2	0.334			

Exe = Experimental Group, Con = Contrast Group, *p<0.05

4.2. Notions of intuition concepts

The answers of intuition concepts written by both groups of students in pretest and posttest were gathered statistically and summarized as follows: (Exe = Experimental group, Con = Contrast group)

4.2.1. Angle

Which angle is the bigger one?			
Question 1		Question 2	
Pretest	Posttest	Pretest	Posttest
			

According to the answers in question 1 and question 2 listed in Table 2, we found that the notion of the intuition concept in angle of both groups of students was 'the longer of arm, the larger of angle'. Nearly two thirds of students chose angle B as the answer (wrong answer) in posttest, even though the experimental group understood the internal angle of two different proportion triangles was equal on the web analogy materials.

Table 2. Results of intuition in angle

Ans.	Question 1				Question 2			
	Pretest		Posttest		Pretest		Posttest	
	Exe	Con	Exe	Con	Exe	Con	Exe	Con
A	3	5	0	1	1	1	0	2
B	12	10	1	1	20	15	19	17
Equal	18	15	32	28	12	15	14	13
Incomparable	0	2	0	2	0	1	0	0

We designed the animations (Fig. 4) fabricated by 'Macromedia Flash MX' to display two lines expand to be an angle and the pictures (Fig.5) to show the degree of angle is irrelevant to the length of arm.

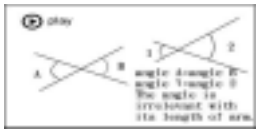


Fig. 4 Animation of definition of angle

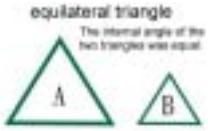


Fig. 5 The relation between arm and angle

4.2.2. Discount

Question 3

Pretest	Posttest
John wants to buy a toy, the price of store A is 20% discount and the price of store B is 40% discount, which price of store is cheaper than the other with the same original price?	A buys a ball with \$150 and B buys the same one with \$180 at store U, who gets more allowance?

Most of students chose the right answer in pretest and posttest, but six students of experiment group and three students of contrast group thought that the answer is incomparable in posttest because they didn't know the original price.

Table 3. Results of intuition in question 3

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
A	8	5	23	27
B	25	22	3	2
Equal	0	2	1	0
Incomparable	0	3	6	3

4.2.3. Ratio

Question 4

Pretest	Posttest
A girl saves quarter of her pocket money and a boy saves half of his pocket money, whose money is more after six months?	A girl's father saves one sixth of his salary and a boy's father saves one twelfth of his salary, whose father saves more in one month?

Table 4. Results of intuition in question 4

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
Girl	5	5	14	18
Boy	18	20	2	1
Equal	3	2	5	3
Incomparable	7	5	12	10

In question 4, about one third of students understood that they couldn't only make decision just

according to the ratio but also consider the base, so they thought that it was incomparable. However, many students thought 'the larger ratio means the more quantity'.

Question 5

Pretest
The price of a toy decreased 10% in February according to the price in January, and then increased 10% in March according to the price in February. A says 'The price in March is equal to the price in January.' B says 'The price in March is more expensive than the price in January.' C says 'The price in March is cheaper than the price in January.' Who is right?
Posttest
The price of a toy increased 10% in February according to the price in January, and then decreased 10% in March according to the price in February. A says 'The price in March is equal to the price in January.' B says 'The price in March is more expensive than the price in January.' C says 'The price in March is cheaper than the price in January.' Who is right?

In question 5, most of students considered the price being not different after the process of decreasing and increasing. Some students chose answer c as the right answer, but they didn't use valid solution strategy. Only one or two students used algebra or segment strategy to solve the question.

Table 5. Results of intuition in question 5

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
A	24	18	24	17
B	3	2	3	2
C	5	7	5	10
None	1	5	1	3

We use segment (Fig. 6) to compare the quantity of different ratio of two objects in different situation.

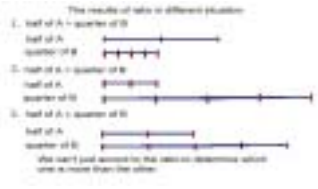


Fig. 6 The results compared with different ratio

4.2.4. Length and area

Question 6

The polygon B is cut some small rectangles of rectangle A, which has the longer perimeter?

Pretest	Posttest
	

In question 6, some students considered the perimeter of rectangle A was longer because they thought the polygon B is smaller; some students considered the perimeter of polygon B was longer than rectangle A because they thought the number of sides is more; and most of students considered their perimeter is equal because the perimeter didn't change although the area was changed.

Table 6. Results of intuition in question 6

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
Rectangle A	11	9	7	5
Polygon B	4	7	6	6
Equal	17	16	20	19
Incomparable	1	0	0	2

Question 7

Which figure constructed between two parallel lines has the bigger area?

Pretest	Posttest
	

In question 9, some students chose the area of two figures being equal according to the number of sides. Some students considered it is incomparable because they didn't know the base length so that they couldn't calculate it.

Table 7. Results of intuition in question 7

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
A	4	8	11	10
B	10	5	5	7
Equal	18	18	10	6
Incomparable	1	1	7	9

Question 8

The length of sides in figure B and figure A is changed, which figure has longer perimeter and bigger area (or volume)?

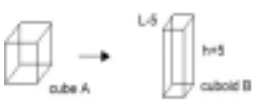
Pretest	Posttest
	

Table 8. Results of intuition in question 8

Ans.		Pretest		Posttest	
		Exe	Con	Exe	Con
Perimeter	A	3	2	5	3
	B	5	5	6	7
	Equal	23	23	22	19
	Incomparable	2	2	0	3
Area or Volume	A	11	6	13	11
	B	3	1	2	0
	Equal	18	20	16	20
	Incomparable	1	5	2	1

Question 9

The length of sides in figure B and figure A is changed, which figure has longer perimeter and bigger area (or volume)?

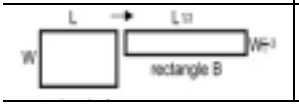
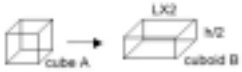
Pretest	Posttest
	

Table 9. Results of intuition in question 9

Ans.		Pretest		Posttest	
		Exe	Con	Exe	Con
Perimeter	A	5	5	7	2
	B	6	5	6	8
	Equal	22	18	18	20
	Incomparable	0	4	2	2
Area or Volume	A	7	6	10	10
	B	3	2	7	3
	Equal	23	20	14	18
	Incomparable	0	4	2	1

In question 8 and question 9, the notions of those students choosing equal were ‘the same increase and decrease of length, the same perimeter’ and ‘the same perimeter, the same area or volume’. The notions of perimeter, area, or volume were not equal just according to their intuitions or guess and the visional difference of figures, few students used algebra strategy.

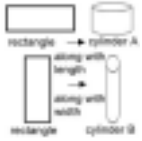
We try to give an example with animation (Fig. 7) to show the process and result of changing.

Fig. 7 Animation of changing process and result

4.2.5. Area and volume

Question 10

We roll up a rectangle paper in two ways. One rolled along with length and the other is rolled along with width, which one has bigger volume?

Pretest	Posttest
	

Those students chose the answer being equal with the notions of intuition concepts ‘The same area, the same volume’ or ‘They are just placed in different ways’. The notions of those students choosing A were ‘It is wider’ or ‘It has larger base area’. The notions of those students choosing B were ‘It is the longer one’ or ‘It looks larger’. The notion of choosing incomparable was ‘I don’t know its length, I can’t calculate’.

Table 10. Results of intuition in question 10

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
A	8	8	12	8
B	5	4	3	1
Equal	17	15	16	20
Incomparable	3	5	2	3

We gave an example and applied animation (Fig. 8) to show the volumes of the two different cylinders are not equal.

Fig. 8 Animation of volume with different cylinders

4.2.6. Infinite

Question 11

Which figure has much more points?

Pretest	Posttest
	

According to Cantorian set theory, any two line segments contain the same number of points. The notion of intuition concept in infinite of those students choosing A was ‘The longer line contains the more points’. The notions of those students choosing equal were ‘They both contain countless points’ or ‘They both contain two end points’.

Table 11. Results of intuition in question 11

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
A	23	23	19	26
B	1	3	0	0
Equal	3	4	5	3
Incomparable	6	2	9	3

The web material presented infinite decimal including recurring decimal and nonrecurring decimal that they had learned, and the ratio of the circumference of the circle to the diameter.

4.2.7. Division

Question 12

Can the process of continuing to divide a figure into two equal parts be endless?

Pretest	Posttest
	

The notions of intuition concepts in division of those students choosing ‘Yes’ were ‘It will be smaller and smaller’, ‘The midpoint is always exist’ or ‘It can be divide into two equal parts in mathematics objects’. These notions presented the intuitive rule ‘The process of successive division is endless’.

The notions of intuition concepts in division of those students choosing ‘No’ were ‘The material is limited’, ‘It will be none at last’ or ‘It can’t be seen in

the end', these notions presented the intuitive rule 'The process of successive division comes to a halt'.

Table 12. Results of intuition in question 12

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
Yes, it can.	10	13	10	10
No, it can't.	23	19	23	22

Because elementary students weren't easy to recognize the difference of the figures with real objects or geometrical shapes drawn on paper, we designed animations like cartoons to tell the basic elements of real objects are molecular or atomic, but the basic element of geometrical shapes is point. According to the definition of point in Euclidean geometry, the point only occupied the location and there is no length, so the process of successive division is endless.

4.2.8 Sequence of number

Question 13

There is a sequence of number as follow:

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128} \ldots\ldots\ldots$$

Pretest	Posttest
Would the sequence be endless?	Is zero the last element of the sequence?

The notions of intuition concept in sequence of number of those students choosing the infinite answers were 'The fraction can be infinitesimal', 'The denominator can be infinity' or 'The fraction can't be divided to be zero'. The notions of those students choosing the limit answer were 'The number is limit', 'At last, the fraction will be zero' or 'The fraction can't be continue divided'.

Table 13. Results of intuition in question 13

Ans.	Pretest		Posttest	
	Exe	Con	Exe	Con
Yes	29	26	5	6
No	4	6	28	26

We designed the animation (Fig.9) to display the number can be infinity and infinitesimal.

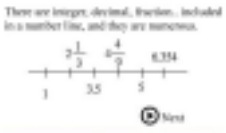


Fig. 9 Animation of number line

5. Conclusions

This study designed web analogy material of intuition concepts in eight mathematics conceptions and applied online teaching in the K12 digital school that set up by National Sun Yat-Sen University. The learning effect of the experimental group was significant difference to contrast group and the experimental explanation variance was 33.4%.

Although most of the misconceptions can have been converted after the online teaching and web analogy materials, there were still several misconception difficult to be converted due to the self-evident, perseverance, certainty, and coerciveness of intuitions concept. But we could base on the relationship to predict and improve the misconceptions of students.

6. Acknowledgements

This research was supported by the National Science Council（NSC）of Taiwan, the Republic of China （R.O.C.） under Grant No. NSC92-2520-S-153-002.

7. References

(1) A. Newell, & H. Simon, *Human problem-solving*. Englewood Cliffs, NJ: Prentice-Hall., 1972.

(2) C.-I. Lee, & F.-Y. Tsai, "Internet project-based learning environment: the effects of thinking styles on learning transfer.", *Journal of Computer Assisted Learning*, 20, 31-39, 2004.

(3) D. Tirosh, & R. Stavy, "Intuitive Rules in Science and Mathematics : The Case of "Everything can be Divided by Two"."., *International Journal of Science Education*, 1996, Vol. 18, No. 6, 669-683, 1996.

(4) D. Tirosh, & R. Stavy, "The Intuitive Rules Theory and Inservice Teacher Education. In Fou-Lai Lin（ED）", *Proceedings of the 1999 International Conference on Mathematics Teacher Education.* Department of Mathematics, National Taiwan Normal University : Taipei, Taiwan, 205-225, 1999.

(5) G. Polya, *How to solve it.* Princeton, NJ: Princeton Univ. Press, 1957.

(6) R. Stavy, & D. Tirosh, "Intuitive Rules in Science and Mathematics : The Case of "More of A -- More of B"."., *International Journal of Science Education*, Vol. 18, No. 6, 653-667, 1996.

(7) R. Stavy, D. Tirosh, P. Tsamir, & H. Ronen, *The Role of Intuitive Rules in Science and Mathematics Education.* Unpublished thesis. School of Education, Tel-Aviv University, Israle, 1996.

(8) V. Sutula, & J.S. Krajcik, "*The effective use of analogies for solving mole problems in high school chemistry.*", Annual Meeting of the National Association of Research in Science Teaching, Lake Ozark, MO, 1988.