Network Virus Propagation Model Based on Variable Propagation Rate

Cong Jin, Qing-Hua Deng, Jun Liu
Department of Computer Science, Central China Normal University, Wuhan, P.R.China
E-mail: jincong@mail.ccnu.edu.cn

Abstract*-In this paper, two different virus models based on different topologies of email network are proposed. By analyzing the means and the characters of email virus spreading, the function of email virus propagation is given, and the maximum time of email virus propagation before the anti-virus software is calculated. The condition in which email virus propagation stops is also proved. The relation between average node degree and power law exponent is discussed later. The models have been testified its rationality through simulation experiments.*

Keywords: Email Network Topology; Propagation Model; Propagation Function; Average Node Degree.

1. Introduction

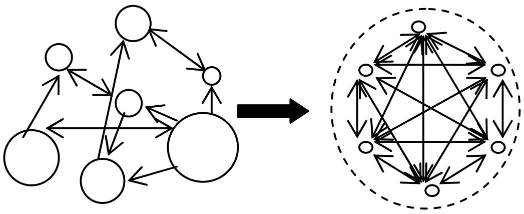
Computer network has become one of the most basic tools for various users. Network virus plays an important role on the security and reliability of the whole network.

Network virus propagation is influenced by various factors, and these factors are regarded as constants in most of the existed models. So, some detail information of network virus propagation is neglected, and the mathematical model [1-3] of network virus propagation is simplified. In fact, many factors are changed during the virus propagation. In this paper, email virus propagation model will be studied, and the email virus propagation rate is designed as a variable for simulating email virus propagation exactly. This research takes the variable factors and the difference of network topologies into account. The variable propagation rate reflects the process that email virus propagate quickly at first and then slowly. To determinate the range of average node degree, the impact of power law exponent on average node degree is concerned. The simulation experiments are performed to demonstrate the validity of our study.

2. Preliminary Knowledge

We describe the logical email network as a directed graph *G*=<*V*, *E*>, where *V* is the set of nodes denotes the email users and *E* is the set of links. If node *A* has the email address of node *B* in its email address book then there is a link from node *A* point to node *B* point and vice versa. If *A* and *B* have the email address of each others then there is an undirected link between *A* and *B*. A remarkable property of email virus propagation is that the email virus must be expanded through email address. *A* must have the email address of *B* before it transfer the email virus to *B*. The directed nature of the email network makes the spread of email viruses qualitatively different from the spread of human diseases.

The in-degree of a user is k_{in} means that there are k_{in} users have the email address of the user. The out-degree of a user is k_{out} means that there are k_{out} email addresses in the user's email address book. Apparently, the bigger the in-degree is, the higher the probability of being infected is. The bigger the out-degree is, the higher the probability of infecting others is. In other words, the highly-connected nodes play a critical role in facilitating virus propagation.



Link loose between the groups Link close in the group

Figure 1. Email network topologies

Cliff Zou etc points out that the nodes degrees of email network satisfied power law distribution by analyze 800,000 email groups in Yahoo[4]. That is $p(k) \propto k^{-\gamma}$, where γ is the power law exponent and the range of most of complex networks is 2≤γ≤3. In fact, the in-degree and the out-degree all satisfied the power law distribution.

The users that have a large of email contacts are fewer. Most of the users have a small email address book. The nature of power law distribution is an important property of email network. Another equal important property is the nature of cluster. It is common that somebody have mutually the email addresses of each others. For example, the members of a department and the workers of an office. They consist of a cluster or a group. The local email network can be regard as a completely connected graph. Actually, the email network is a social network that indicates the relationship of email users. Anybody belong to a group or more and all the big or small groups compose the whole email network. The users in the same group connect closely. Nevertheless, the links between different groups are loose. The email network topology is shown in Figure 1.

3. Email Virus Propagation Model

Email network topology affected heavily email virus propagation. Different topology determine different email virus spreading model. In this paper, we research the email network topology before designing new email virus spreading model. In an email group the nodes consist of a completely connected graph. In the whole network the nodes consist of a loosely connected graph and the nodes degree satisfy power law distribution. Thus, two new email virus spreading models are proposed for adapting different email network topologies respectively.

3.1. Email virus propagation in the group

We assume that the email virus propagation is a discrete time process, *i.e.*, $t = 0, 1, 2, 3, \ldots\ldots$. The unit of time is day (24 hours). The size of the group is M. I_t is the number of infected users at time t in the group. δ is the probability of cleanup virus in the group. Users open the unsafe email with the probability α and the interval of checking email is μ. Therefore, the opening probability in unit time is $\frac{\alpha}{\mu}$. At time $t+1$ the number of infected users I_{t+1} is composed of two parts. One is the users that have been infected at time t but have not been clean at time $t+1$. The other is the newly infected users, *i.e.*, the users who are healthy at time t but infected at time $t+1$. Because of having the email addresses of each other within a group, all the other users receive the email virus copies as long as one of them has been infected. Here, we did not consider the restriction of network bandwidth. That is to say, there are $(M - I_t)$ users may be infected newly at any time. We name them suspicious nodes like the epidemiological model. Whether the suspicious users would be infected or not is determined by whether they would open the email. Some hackers are very tricky. They embed virus in the email text but not the attachment. Email users are infected after checking the email in despite of not opening the attachment. Email virus like this is more covert than others. So we assume that the users are infected once they open the email. The model applied to email group is given as follows:

$$I_{t+1} = (1-\delta)\, I_t + \frac{\alpha}{\mu}\,(M - I_t). \qquad (1)$$

Where $I_0 = 1$, from the Equation (1), we get

$$I_t = \left(1 - \frac{\alpha M}{\delta\mu + \alpha}\right) e^{-(\delta + \frac{\alpha}{\mu})t} + \frac{\alpha M}{\delta\mu + \alpha}. \qquad (2)$$

Equation (2) shows that the maximum number of infected users is depended on the proportion of opening probability and cleanup probability. Smaller value of opening probability and bigger value of cleanup probability imply a smaller number of maximal infected users. In this paper, let the size of group be 20, *i.e.*, $M = 20$. Cleanup probability $\delta = 0.2$ and opening probability $\alpha = 0.7$. According to the habits of email users, the interval of checking email is $\mu = 1$.

Experiment shows that the infected virus number increases greatly within a short time and then tends to a steady state in general. Instead of spreading continually, email virus propagation terminates at an equilibrium point result in some users remain healthy at the end of the propagation. Email virus outbreak quickly and also terminate quickly in the group.

3.2. Email virus propagation in the Internet

It is often the case that the anti-virus software is updated only after a virus has spread for some time. In the beginning, email users know so little about the new virus that none of strategy can be use to stop the spreading of virus. The new virus propagates unrestrictedly until the malicious activities caught the attention of people. Once the anti-virus software appearing, it can be used to throttle the further propagation of the virus from the infected users. Therefore the propagation of virus can be classified into two phases.

3.2.1. The initial phase

Suppose that the anti-virus software starts to be available at the time T_0. Before the time T_0, *i.e.*,

$t < T_0$, the spreading of email virus is modeled as follows

$$I_{t+1} = I_t + \frac{\alpha}{\mu}\ \beta(t)\ I_t\,. \qquad (3)$$

Where, $\beta(t)$ is the function of email virus propagation. Rather than all the email users are infected with the same probability, the users are infected by the infected contacts in the email addresses. The pervasion of email virus is implemented by spreading the virus copy to the contacts in the email address. The spreading of email virus is active but not passive. Exactly, the users who may be infected at time $t+1$ are the users that link with the user who have been infected at time t. This model takes the initiative of email virus propagation into account and believes that the number of email virus copies is $\beta(t)I_t$. Thus the number of newly infected users is $\frac{\alpha}{\mu}\beta(t)I_t$.

The function of email virus propagation $\beta(t)$ is varied with time and related with the average node degree of email users. The average node degree is greater, the $\beta(t)$ is bigger. Because of the feature of cluster email virus likely transfers the email virus copies to the infected users. The number of infected user increase sharply when it infects a healthy group in the first time. If most of the users in a group have been infected, email virus propagates mildly. Only the healthy users are favor of the spreading of email virus. Thus $\beta(t)$ is also related with the proportion of healthy users. We design the definition of $\beta(t)$ based on the two factors analyzed above. $\beta(t) = \bar{k}\frac{N - I_t}{N}$, where $\bar{k}$ is the average node degree of email users, and $\frac{N - I_t}{N}$ is the proportion of healthy users to total email users.

Replace $\beta(t)$ in Equation (3) with $\bar{k}\ \frac{N - I_t}{N}$, and we obtain $I_{t+1} = I_t + \frac{\alpha}{\mu}\ \bar{k}\ \frac{N - I_t}{N}\ I_t$. Furthermore, the differential of I_t indicates the increasing rate of email virus and we can obtain the differential of I_t described by $\frac{dI_t}{dt} = -\frac{\alpha\bar{k}}{\mu N}(I_t - \frac{N}{2})^2 + \frac{N\alpha\bar{k}}{4\mu}$. Where the infected users is 5 at the initial time, namely $I_0 = 5$. While $I_t = \frac{N}{2}$, *i.e.*, $t = \frac{\mu}{\alpha\bar{k}}\ln\frac{N-5}{5}$, $\frac{dI_t}{dt}$ takes maximum value $\frac{N\alpha\bar{k}}{4\mu}$. In other words, email virus propagates most quickly when half of the email users are infected before the anti-virus program is available. In order to restrain the large-scale outbreak of email virus we should try our best to run the anti-virus software before the time $t = \frac{\mu}{\alpha\bar{k}}\ln\frac{N-5}{5}$. That is to say, the bigger the value of t is, there are more time for the anti-virus experts to research the anti-virus software. So email users should open the email with long interval and small probability to delay the time t. To store as small email addresses as possible in the email address book is also helpful to delay t.

3.2.2. The latter phase

After the anti-virus software is available, *i.e.*, $t > T_0$, the cleanup probability is not zero anymore. The case of email virus propagation is modeled as follows

$$I_{t+1} = (1 - \delta\,)\,I_t + \frac{\alpha}{\mu}\ \bar{k}\ \frac{N - I_t}{N}\ I_t \qquad (4)$$

Furthermore,

$$\frac{dI_t}{dt} = -\frac{\alpha\bar{k}}{\mu N}[I_t - \frac{(\alpha\bar{k} - \delta\mu)N}{2\alpha\bar{k}}]^2 + \frac{N(\delta\mu - \alpha\bar{k})^2}{4\mu\alpha\bar{k}} \qquad (5)$$

$$\frac{1}{I_t} = [\frac{1}{I_0} - \frac{\alpha\bar{k}}{N(\alpha\bar{k} - \delta\mu)}]e^{(\delta - \frac{\alpha}{\mu}\bar{k})t} + \frac{\alpha\bar{k}}{N(\alpha\bar{k} - \delta\mu)} \qquad (6)$$

There are 5000 infected email users in the Internet when the anti-virus software appears, *i.e.*, $I_0 = 5000$, and $\frac{dI_t}{dt}$ is the increasing rate of email virus in unit time. While $\frac{dI_t}{dt} < 0$, the number of infected users lessen and the email virus no longer spreads. From Equation (5), we know that when $I_t > \frac{(\alpha\bar{k} - \delta\mu)N}{\alpha\bar{k}}$, $\frac{dI_t}{dt} < 0$. Combining Equation (5) and (6), we get the condition of convergence of email virus propagation which is depicted as

$$\delta > (1 - \frac{I_0}{N})\frac{\alpha\bar{k}}{\mu}\,. \qquad (7)$$

Inequality (7) points out the restriction among various factors. The users who have large email address book should cleanup virus frequently to control virus propagation. Some users are accustomed to check email with short interval. These users should also cleanup virus with a high frequency. If users open email with low probability, a low cleanup probability is also useful to control propagation. During the process of email virus propagation, if the cleanup probability δ, the opening probability in unit time $\frac{\alpha}{\mu}$ and the average degree $\bar{k}$ satisfy the inequality (7), $\frac{dI_t}{dt} < 0$, *i.e.*, email virus will disappear gradually.

4. Discussion of Average Node Degree

The average node degree is a crucial factor of email virus propagation. To a great extent, the speed of email virus spreading depends on the average node degree. However, it is really difficult to decide the value of average node degree by statistic data due to the hugeness of email network. Thus, we discuss the relativity of average node degree and the power law exponent for ascertaining the value. The average node degree can be expressed as $\bar{k} = \sum kp(k)$, where $p(k)$ is the probability of any given node with degree *k*. The degree of email network satisfied the power law distribution, thus $p(k) = \frac{k^{-\gamma}}{\zeta(\gamma)}$, where γ is the power law exponent and $\zeta(\gamma)$ is the Riemann zeta function, and $\zeta(\gamma) = \sum_{1}^{\infty} k^{-\gamma}$ [5]. Power law exponent of many actual complex networks are different from each other and the range is $2 \le \gamma \le 3$. From the analysis above, we get $\bar{k} = \frac{1}{\zeta(\gamma)} \sum_{k=1}^{\infty} k^{1-\gamma}$. By integral operation we can obtain

$$\bar{k} = \frac{\gamma - 1}{\gamma - 2}. \tag{8}$$

Most users have a small-scale email address book, so the value of $\bar{k}$ is impossible to be infinite and γ is not equal to 2, *i.e.*, γ is greater than 2. When the value of γ increases, the value of $\bar{k}$ decreases. The value of $\bar{k}$ gets the minimum 2 while γ reaches the maximum 3. If we know the value of exponent power law exactly, the value of average node degree $\bar{k}$ can be figured out from Equation (8). We established the basis for selecting the value of $\bar{k}$. It is helpful for designing the function of propagation and then further developing the propagation model.

5. Simulation Experiment

In order to demonstrate that the proposed models are reasonable, some simulation experiments are given.

To simplify the problem we study email virus propagation on discrete time, *i.e.*, $t = 0, 1, 2, 3, \ldots\ldots$. The unit of time is day (24 hours). In the experiment, the parameters are set as, the size of email users is 10000, *i.e.*, $N = 10000$, the interval of checking email μ =1, and the average of contacts $\bar{k}$ is 6. The following figures suggest that the experiment result is consistent with the above analyses.

Figure 2 shows that email virus spread freely before anti-virus software appearing and the speed is fast. Email virus would infect all the email users without anti-virus software. The larger the opening probability is, the higher the speed of spreading is. The time at which email virus propagates fastest is pointed out through the dashed line in Figure 2.

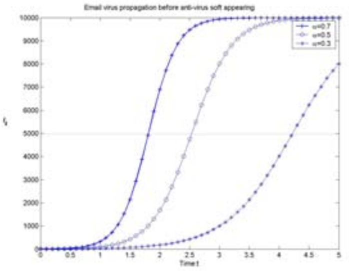


Figure 2. Email virus propagation on different α and δ

Figure 3 clearly shows that email virus propagation has two cases after anti-virus software is used. Either it increase sharply and tend to a stable state or decrease and tend to zero. α is smaller and δ is greater, email virus propagation is slower. When the inequality (7) is tenable, email virus propagation goes down and the number of infected users reduce gradually. When the inverse case is tenable, email virus propagation goes up and the number of infected users adds.

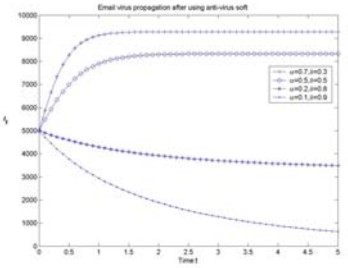


Figure 3. Different α and δ

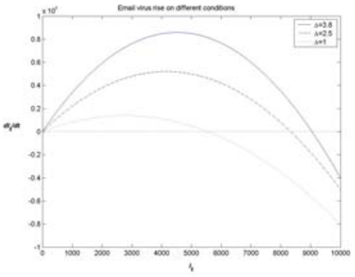


Figure 4. Effect of Δ

Let $\Delta = |\delta\mu - \alpha\bar{k}|$. Figure 4 shows that the increasing rate is increased initially and then decreased. The dashed line shows the time when email virus propagation start to fade away. While more than half of the email users are infected, the number of newly infected users is negative with $\Delta = 1$. In other word, if $\Delta = 1$, about half of the users are safe during the process of email virus propagation.

6. Conclusions

In this paper, two models of email virus propagation based on dissimilar topologies and variable propagation rate are proposed. The two models develop previous work by giving concrete mathematical models to describe email virus propagation. Email virus propagation is controlled by several parameters. The models are helpful to control email virus spreading. Analysis of the model reveals that the time of anti-virus software appearing is a vital factor in controlling virus propagation.

The terminative condition of email virus propagation plays a significant role on control. Equation (7) reveals that highly-connected users request large cleanup probability. Low opening probability and large checking interval request a comparatively small cleanup probability. Instead of a fixed value, δ is different for different users to stop spreading.

Equation (8) shows that average node degree is inversely proportional to power law exponent. Considering the relation between $\bar{k}$ and γ bring the model to be self-adaptive. By adjusting the power law exponent automatically, the model is suitable for different topologies. According to Equation (8), the email network is less likely to be BA scale-free network. The equation can be used to evaluate the email network model.

References

[1] M. M. Williamson and J. Léveillé. “An epidemiological model of virus spread and cleanup”. Proceedings of Virus Bulletin, Toronto, Canada, Sept. 25-26, 2003.

[2] A.Ramani, A.S.Carstea, R.Willox, and B. Grammaticos. “Oscillating epidemics: a discrete-time model”. *Physica A*, *vol*.333: 278-292, 2004.

[3] B.K.Mishra and D.Saini. “Mathematical models on computer viruses”. *Applied Mathematics and Computation*, *vol*.187, *no*.2, 929-936, 2007.

[4] C.C.Zou, D.Towsley, and W.B.Gong. “Email virus propagation modeling and analysis”. Technical Report: TR-CSE-03-04, University of Massachusetts, Amherst 2003.

[5] J.T.Xiong. “ACT: attachment china tracing scheme for email virus detection and control”. Proceedings of the 2004 ACM Workshop on Rapid Malcode, October 29-29, Washington DC, USA, 11-22, 2004.