Scheduling on All-Optical Broadcast-Star WDM Networks with Restricted Tunable Wavelengths

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Abstract

In this paper we consider the scheduling problem on all-optical broadcast-star WDM networks with W wavelengths and N nodes. Each node is equipped with one tunable transmitter and one fixed receiver and the tuning time of each tunable transmitter is not negligible. Furthermore, we assume that the amount of available tunable wavelengths in each node is restricted to a fixed value γ which is less than W. Given an off-line uniform traffic matrix, the objective is to design a multi-hop scheduling algorithm to minimize the total transmission time slots. We first present an approach to partition the original star architectures into several rings for efficient packet transmission in later stage. Our algorithm is to construct the minimum number of rings to connect all nodes, the problem can be reduced to a special case of the set cover problem. As $\gamma = (W-1)/2$, we can find an optimal set cover if it exists. For the general case, we design a factor of 2 approximation algorithm to choose suitable rings. When the packets are transmitting in logical topology ring obtained by our algorithms, the tuning time can be overlapped with packet transmission time in our scheduling. Thus, Our scheduling strategy has less propagation delay and total scheduling period than other multi-hop scheduling.

1 Introduction

Wavelength division multiplexing schemes have been broadly applied in today's all optical network. And in the WDM network, the star topology is a popular architecture to connect all nodes. This architecture has been used in broadcast and select network. In star network, nodes exchange signals through a broadcast medium, which can be achieved by a passive star couple. [1] [2]

For using bandwidth efficiently, tunability must be provided for at least one end. However, tunability is an expensive option and is typically assumed that only one end is tunable and the other end is fixed. Here we will consider the TT-FR configuration [3].

Since the available bandwidth is divided into multiple channels in WDM network, we considered the channels to be slotted and synchronized. Each slot can accommodate the transmission of one packet per wavelength. Collision avoidance is achieved by sending packets through the star couple. Furthermore, each node is equipped with one full-duplex transceiver; hence it can be source or destination at the same time. Since the TT-FR transceiver allows to tune wavelengths for transmitting data, the time required to tune each node on the transceiver should not be negligible. We assume that the tuning latency needs δ time slots. In fact, with respect to the available optical components, the tuning latency is usually much longer than packet transmission. Given a well-known traffic characteristic, usually presented by matrix, the objective is to transmit packets by distributing them to different wavelength channels such that total transmission period is minimized.

Previous studies basically include two different approaches to solve this problem: single-hop or multiple-hop scheduling [4] [5]. A polynomial-time single-hop algorithm has been proposed to produce the optimal schedule according to a very unrealistic assumption [4]. It shows that, under certain conditions on the traffic matrix, there exits polynomial time algorithms that produce the optimal schedule. On the other hand, the multi-hop scheduling problem can be formally defined as an Integer Linear Programming (ILP) [6] problem. A heuristic algorithm which provides some significant improvements over previous results in terms of running time complexity is presented in [5].

In a WDM broadcast and select network, wavelength channel distribution is an important research topic. In TT-FR assumption, the transmitter is able to tune its channel to all available wavelengths. Hence the single-hop scheduling algorithm can be applied. However, it is obviously that the tuning time for all nodes to tune their transmitter wavelength to another is a very large cost for general traffic characteristic. As we can find a multi-hop scheduling to transmit a part of packets indirectly through other nodes, the ability that each node must tune its transmitter to all available wavelengths is not to be necessary. Furthermore, if each node can use less than W wavelengths to transmit in our scheduling algorithm, it can save economic cost on node transmitter equipments.

See the figure 1, node 1 will send a packet to node 3. Node 1 uses the wavelength λ_1 to send this packet. But the receiver wavelength of node 3 is the wavelength λ_2. In any single-hop scheduling, node 1 can not directly send the packet to node 3 with the wavelength λ_1. The transmitter of node 1 must tune its channel to the wavelength λ_2 to finish this sending task. Now we use the multi-hop scheduling. Let the transmitter wavelength of node 2 is the wavelength λ_2. The receiver of node 2 fixes its channel with wavelength λ_1. So the scheduling strategy might send the packet from node 1 to node 2 with wavelength λ_1, and node 2 sends this packet to node 3 with wavelength λ_2.

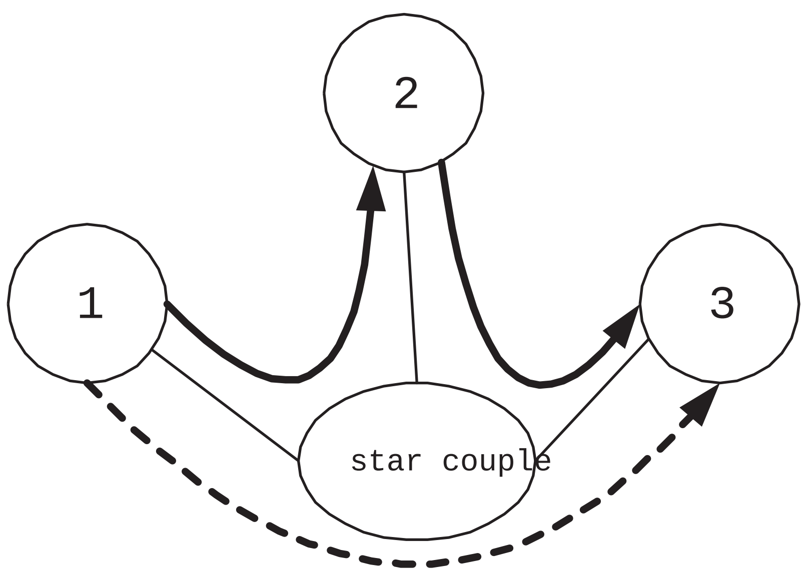

Figure 1: Multi-hop scheduling

We will focus on investigating off-line and uniform traffic characteristics in this problem. With adding the constraints of tunable wavelengths in transceiver, our objective is to minimize the whole transmission period. We will use the pipeline transmission scheduling [7] to increase the efficiency of the transmission in a WDM broadcast and select network. The pipeline transmission scheduling is useful on uniform and ring topology. Thus we first construct an efficient logical topology for the pipeline transmission scheme. The efficient logical topology is said the logical equal-difference rings. Second, design a scheduling strategy to send packets with the pipeline transmission scheme. We show that how to utilize the gaps of tuning time efficiently in the same packet transmission time. Hence we proposed a multi-hop scheduling including with two stages: (1) construct an efficient logical topology and (2) design a scheduling strategy to send packets.

2 Problem Definition

We consider the scheduling problem on All-Optical Broadcast-Star WDM networks with restricted tunable wavelengths. The star network has N nodes and W available wavelength which are denoted by $\lambda_0, \lambda_1, \cdots, \lambda_{W-1}$. Each node is equipped with a tunable transmitter and fixed receiver. Each transmitter only can choose γ fixed tunable wavelengths among W available wavelengths. We call the node with TT-FR transceiver. Transmitting and receiving packets of each node are independent. A packet needs one time slot to be transmitted through the star couple to the destination node. The non-negligible tuning time for each tuning step takes δ time slots.

Mainly, the star network will not have enough available wavelengths as many as the number of nodes. Hence, W is often less than N. And the TT-FR WDM network has at least $\lfloor N/W \rfloor$ nodes which must use the same wavelength as their receiver channel. We decide the receiver wavelength for some nodes, other nodes must transmit packets to them using this fixed wavelength. Since the tuning time can not be ignored, so we need to distribute communication loads uniformly on each wavelength as a receiver channel. In other words we assume nodes which receive the same wavelength are denoted as a "FR-group" and different FR-groups have the same number of nodes. Without loss of generality, assume $N = kW$.

The N Nodes will be partitioned into W "FR-groups"(denoted by $G_0, G_0, \cdots$, and G_{W-1}) and each group has k nodes in it.

We only consider the off-line version of traffic characteristic. Suppose the traffic flow is uniform and known in advance. Assume that each node only needs to send exact one packet to each other in different FR-groups. There are no packets to be sent in the same FR-groups. Since we treat each FR-group as a single node, each node in the uniform traffic matrix does not send packets to itself. We present this characteristic by a $W \times W$ matrix:

$$T = \begin{bmatrix} 0 & k^2 & \cdots & k^2 \\ k^2 & 0 & \cdots & k^2 \\ \vdots & \vdots & \ddots & \vdots \\ k^2 & k^2 & \cdots & 0 \end{bmatrix} = [T_{ij}]; (i, j = 0, 1, 2, \cdots, W-1)$$

where T_{ij} means the number of packets that FR-group G_i must to G_j.

Each transmitter only can choose γ fixed tunable wavelengths among W available wavelengths. So some source nodes may not be able to not tune its transmit wavelength to the receiver wavelength of the destination node. The source node will not directly send one packet to the destination node. In this situation, we must find a multi-hop scheduling strategy for given network traffic. The first step of our algorithm is to design the ring architecture of logical topology to connect all nodes and to decide the γ tunable wavelengths for each node transceiver. The second step is to design an efficient scheduling strategy to send packets such that the total transmission time is minimized on the ring architecture of logical topology obtained in the fist step.

3 Topological Design

Since the number of tunable wavelengths of a node transmitter is a fixed integer γ, $1 \leq \gamma \leq W$, the choice of γ will affect the logical topology. For example, if $\gamma = 1$, it means that each transmitter only uses one fixed wavelength as transmission channel and cannot tune to other wavelengths. Hence it is equivalent to the FT-FR model. To achieve network connectivity, the only logical topology to connect all nodes is a ring architecture. All nodes in the same

FR-group must transmit their packets to others one by one in the logical ring direction.

As $\gamma = 2$, there are no packets needing to be sent in the same FR-group, then we can distribute another tunable wavelength channel to form a logical bi-direction ring.

As Figure 2 shown, not like the one-direction ring, each FR-group can transmit their packets by choosing proper direction to shorten the node propagation delay. But cost of the tuning time to change the ring direction is still needed to be carefully considered. FR-group G_0 first transmits the packets to $G_1, G_2, \cdots, G_{(w-1)/2}$ one by one using clockwise direction. Other FR-groups also use the same method to transmit packets such as G_0's strategy. Here packets are transmitted using pipeline strategy. After the duration of tuning time to change the logical ring direction, G_0 can send packets passing through $G_{w-1}, G_{w-2}, \cdots, G_{(w-1)/2+1}$ in counterclockwise direction. This is a simple transmission scheduling strategy for $\gamma = 2$, the longest hop distance is reduced by one half. Now we will focus our attention for the value of γ is in the range $1 < \gamma < W$, since then the routing strategies are obvious for $\gamma = 1$ and $\gamma = W$.

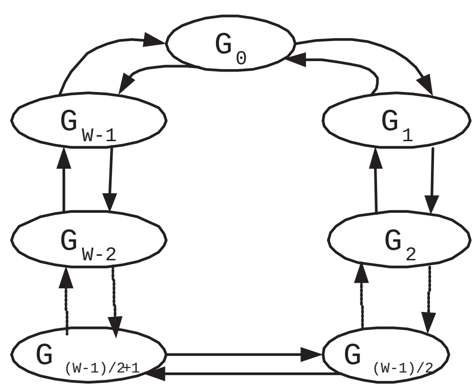

Figure 2: Bi-direction ring topology as $\gamma = 2$

3.1 Equal-Difference Ring

Let a network with N nodes where nodes are named as node $0, 1, 2, \cdots, N-1$. Then an equal-difference ring with difference δ is a ring architecture to connect all nodes such that any connected pair of nodes i and j satisfies $|i - j| = \delta \bmod N$.

For example, if we have 5 nodes names as $0, 1, 2, 3$, and 4. Then an equal-difference ring with difference 2 can be traversed along with $0, 2, 4, 1$, and 3. (See Figure 3)

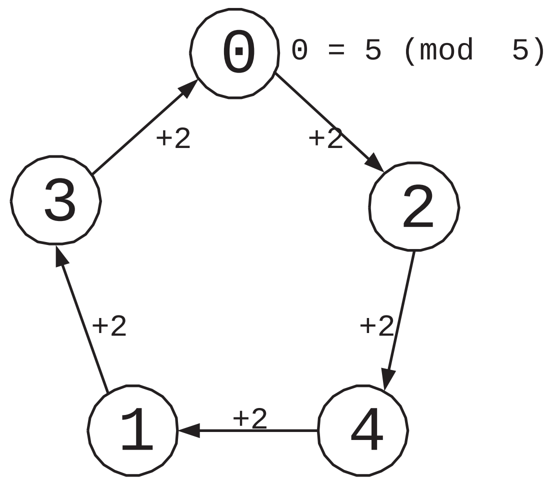

Figure 3: An equal-difference ring with difference 2 of 5 nodes

Hence, we might have W Fr-groups networks with difference as $0, 1, \cdots$, and $W - 1$. The previous logical topology ring can be seen as an equal-difference ring with difference 1, and the counterclockwise ring along with $W - 1, W - 2, \cdots$, to 1 is an equal-difference ring with difference $W - 1$.

3.2 Pipeline Transmission of Ring

Now we introduce the concept of pipeline transmission of ring since we will apply this transmission method on equal-difference rings latter. Consider a connected W nodes ring network and nodes are named as $0, 1, \cdots, W - 1$. The traffic characteristic matrix has been shown in the above section. Each node needs to send exact one packet to each of the other $W - 1$ nodes.

Obviously, each node needs to send $W - 1$ packets with distances of destinations in uni-direction are $1, 2, \cdots, W - 1$ with respect to node number modular W. For example, node 0 needs to send packets to node $1, 2, \cdots, W - 1$ with node difference $1, 2, \cdots, W - 1$; node 1 needs to send packets to node $2, 3, \cdots, W - 1, 0$ with node difference $1, 2, \cdots, W - 1$; and so on. Now we divide the transmission period to $W - 1$ stages, packets with difference i between sources to destinations are sent on the i-th stage. If we assume the transmission time is slotted and each packet through one link needs exact one time slot, the packet with difference i obviously needs i time slots to be sent. In each stage, since packets on the same ring have equivalent transmission distance, we can synchronize the packet transmission starting at the same time to avoid collision on links. Hence the i-th stage only needs exact i time slots, so the total scheduling time is $W(W-1)/2$.

The pipeline transmission scheduling is useful on uniform traffic and ring topology. We only need to focus the processing of packet transmission on one node, and the other nodes are processed by the same scheduling method(only need to change the node number by a proper difference). Since packets are sent by the node number, applying pipeline strategy on logical equal-difference rings in our problem is a trivial problem.

3.3 Topological Design by Choosing Equal-Difference Rings

Now we need to consider how to assign these γ wavelengths, where $1 \leq \gamma \leq W$ to each node and the assignment will affect the logical topology for each tuning-connected graph. For $\gamma = 2$, the tunable two wavelengths of FG-group G_i are λ_{i+1}, and λ_{i-1} . When all G_i tunes their transmitter to λ_{i+1}, the logical topology is an equal-difference ring with difference 1. Hence packets with difference $1, 2, \cdots, (W-1)/2$ between node group number can be transmitted in this tuning graph. On the other hand, as G_i tunes their transmitter to λ_{i-1}, the logical topology is an equal-difference ring with difference $W-1$ (respect to modular W). And packets with difference $W-1, W-2, \cdots, (W-1)/2+1$ between node group number can be transmitted in this counterclockwise tuning-connected graph.

In this example, we partition packets into two parts and find two suitable equal-difference rings to transmit them. It is expected to shorten half packet propagation delay and pay extra cost of the tuning time between different equal-difference rings then one-direction ring. We will present a logical topology design strategy by generalize this method. That is, given known tuning latency δ, we select proper γ equal-difference rings and transmit packets on them efficiently.

We can see that using more equal-difference rings will shorten more packets propagation delay, and need to pay more tuning time costs. Hence our goal is to construct the minimum number of equal-difference rings to cover all nodes to obtain the minimization of scheduling period.

3.4 The Existence of Equal-Difference Ring

Now we are interested with how many equal-difference rings can be chosen in W nodes network. We first apply some important algebra knowledge to construct the equal-difference rings.

Definition 3.1 (Cyclic group and generator). Let G be a group and a be any element of G. The set $< a >= \{x \in G | x = a^n$ for some $n \in Z\}$ is called the cyclic subgroup generated by a. The group G is called a cyclic group if there exists an element $a \in G$ such that $G =< a >$. In this case a is called a generator of G.

Definition 3.2 (Congruence class). Let a and $n > 0$ be integers. The set of all integers which have the same remainder as a when divided by n is called the congruence class of a modulo n, and is denoted by $[a]_n$, where $[a]_n = \{x \in Z | x \equiv a (\mod n)\}$. The collection of all congruence classes modulo n is called the set of integers modulo n, denoted by Z_n.

Theorem 3.1. *Let $G = (Z_w, +)$ be a group of congruence classes modulo W. If W is prime , then G is a cyclic group and for any element $a \in G - \{0\}$, $< a >= G$.*

Corollary 3.2. *Let $G = (Z_w, +)$ be a group of congruence classes modulo W. Then the W nodes network exists an equal-difference ring with difference a if and only if $< a >= G$.*

Corollary 3.3. *Let $G = (Z_w, +)$ be a group of congruence classes modulo W. If W is prime, then there are $W - 1$ equal-difference ring with difference a ,where $a = 1, 2, \cdots, W - 1$.*

This is the reason why we assume "W is prime", since we will have the most equal-difference rings to choose. If W is not a prime number, we can choose the largest prime number less than W to apply our algorithm.

Now our purpose is to choose suitable γ rings from known $W - 1$ equal-difference rings, we will show how to reduce it to a set cover problem.

3.5 Notations and Estimating Function

Let W FR-groups be represented as $Z_w = \{0, 1, \cdots, w-1\}$ and w be prime. Define the set of equal-difference rings $R_w = \{r_1, r_2, \cdots, r_{w-1}\}$, where r_i is an equal-difference ring with difference i. Note that $r_i = (0, i(\mod w), 2i(\mod w), \cdots, (w-1)i(\mod w)) = (\sigma_0^{(i)}, \sigma_1^{(i)}, \cdots, \sigma_{w-1}^{(i)})$ which is also a permutation of Z_w.

Then we want to find a subset $R^{'} = \{r_1^{'}, r_2^{'}, \cdots, r_\gamma^{'}\} \subseteq R_w, |R^{'}| = \gamma$, and a partition of $w-1$ kinds of packets $P = \{S_1, S_2, \cdots, S_\gamma\}$ where

- $\bigcup_{i=1}^{\gamma} S_i = 1, 2, \cdots, w-1$
- $S_i \cap S_j = \phi$ if $i \neq j, i, j = 1, 2, \cdots, \gamma$
- $\rho_i = |S_i|$ and $\rho = \lceil (\sum_{i=1}^{\gamma} \rho_i)/\gamma \rceil = \lceil (w-1)/\gamma \rceil$
- $S_i = s_{i_1}, s_{i_2}, \cdots, s_{i\rho_i}$, it means the packets with difference S_{ij} will transmit on $\gamma_i^{'}$ $(i = 1, 2, \cdots, \gamma$ and $j = 1, 2, \cdots, \rho_i)$

Choosing each set $R^{'}$ and partitioning packets to be sent on each node, we need estimate the scheduling period. The function TS to estimate the scheduling time slots is defined in the following way:

$$TS(R^{'}, P) = TS(R^{'}, \{S_1, S_2, \cdots, S_\gamma\}) = \sum_{i=1}^{\gamma} \tau(r_i, S_i) + (\gamma - 1)\delta$$

where $\tau(r_i, S_i) = \tau(r_i, \{s_{i_1}, s_{i_2}, \cdots, s_{i_{\rho_i}}\}) = \sum_{k=1}^{\rho_i} \varphi(r_i, s_{i_k})$ and the function $\varphi(r_i, x)$ is defined in the following. Let $r_i = (\sigma_0^{(i)}, \sigma_1^{(i)}, \cdots, \sigma_{w-1}^{(i)})$. We define $\varphi(r_i, x) = j$, which j satisfies $\sigma_j^{(i)} = x, x \in \{1, 2, \cdots, w-1\}$. It means the packets with difference x need j time slots to transmit on r_i.

3.6 Choose Equal-Difference Rings

Given a tuning latency δ, we want to find a set of equal-difference rings $R^{'}$ and partition packets P to minimize $TS(R^{'}, P)$. We will present an algorithm $GAR(r)$(Greedy Algorithm to choose Rings), where r is the average number of packets transmitting on each ring. We can obtain the minimization of scheduling period by $\min_{\rho=1,2,\cdots,w-1}\{TS(R', P)$ returned from $GAR(r)\}$.

We will first reduce the choice of equal-difference ring to a set cover problem. Consider the W FR-groups as $Z_w = \{0, 1, \cdots, w-1\}$, equal-difference rings $R_w = \{r_1, r_2, \cdots, w-1\}$, $r_i = (\sigma_0^{(i)}, \sigma_1^{(i)}, \cdots, \sigma_{w-1}^{(i)})$. Define $r_i'(\rho) = \{\sigma_1^{(i)}, \sigma_2^{(i)}, \cdots, \sigma_\rho^{(i)}, \}$; $R'(\rho) = \{r_1'(\rho), r_2'(\rho), \cdots, r_r'(\rho)\}$. Then we want to find a set cover $R'(\rho)$ containing $\{1, 2, \cdots, w-1\}$. We use the greedy algorithm of the set cover to solve this problem.

3.6.1 Special case for $\rho = 2$

Consider $\rho = 2$, and we want to choose a set cover of $\{1, 2, \cdots, w-1\}$. If each $r_i(2)$ is an edge, and $\{1, 2, \cdots, w-1\}$ is a node set, then it can be reduced to a maximum matching problem. Define $e = 2$, $e^n = 2n(\bmod w-1)$. And we can easily find the matching edges as $\{1, e\}, \{e^2, e^3\}, \{e^4, e^5\}, \cdots, \{e^{w-3}, e^{w-2}\}$. This is because nodes in the reduced graph are connected by edges numbered by multiplying a factor of 2, and $(Z_{w-1} - \{0\}, *)$ is a cyclic group which can be generated by 2. We present the following theorem to conclude the above observation.

Theorem 3.4. *Let w be a prime. If $w-1$ is the smallest number such that $2^{w-1} \equiv_w 1$, then $Z_w - \{0\} = \{1, e, e^2, \cdots, e^{w-2}\}$ and we can choose the set cover as $\{1, e\}, \{e^2, e^3\}, \{e^4, e^5\}, \cdots,$ $\{e^{w-3}, e^{w-2}\}$.*

Theorem 3.5. *If t is the smallest number such that $2^t \equiv_w 1$, then $\exists y \ni (w-1) = y\dot{t}$ and we will find y subgroups with t elements from $Z_w - \{0\}$;*

- As t is even, we can find a maximal matching from original problem.

- As t is odd, there will be leaving one element in each subgroup. Hence we need to use greedy methods to choose other better equal-difference rings.

3.6.2 General case for $\rho > 2$

For $\rho = 2$, we can find a maximal matching to solve the choice of equal-difference rings problem as above theorems. As $\rho > 2$, the problem can be reduced to a set cover problem. But the set cover problem is an NP-complete problem. Thus it cannot solve the problem in polynomial time, and we will show an approximation algorithm for the choice of equal-difference rings problem. First we give some definition and notations for our algorithm.

Definition 3.3 (Independent numbers). Let p be prime and the network have $p-1$ nodes. Each ring will be only selected ρ numbers, let the selected ρ rings contain the number x, $1 \leq x \leq p-1$. If these ρ rings do not include the number y, then we say that the numbers x and y are independent each other.

The greedy algorithm is shown as follows:

Step 1: Choose all independent numbers from the first number of each ring. Select those rings that the independent numbers locate on the first number for each ring.

Step 2: Select the ring, if it contains the unselected numbers more than other rings. Repeat until all numbers are selected.

The concept of the greedy algorithm has the property that the numbers of selected rings are different each other is better. And we hope that selected rings of step 1 satisfy the above property. The following lemma shows that the numbers of m selected rings of step 1 are different each other.

Lemma 3.6. *Let the number i locate on the first number of the ith ring. The first number of the jth ring is j. If the numbers i and j are independent each other, then all number of the ith ring and the jth ring are different each other.*

Proof. The details of proof are shown in the Appendix. □

Let a network with $n = p-1$ nodes. Assume that there are m independent numbers from the first number of each ring. By above lemma, step 1 can select $m\rho$ numbers, and remains $n - m\rho$ numbers unselected. The remained $n - m\rho$ numbers can be divided into A and B. A is the set that x is the member of A if its location of rings only contain an unselected number x. One of the location of rings of x that is the member of B has at least 2 unselected numbers. Clearly, the unselected numbers on A will use $|A|$ rings to cover. How many rings will cover the unselected numbers on B, it will decide the number of equal-difference rings for the algorithm. We can see the following discussion.

Lemma 3.7. *The greedy algorithm uses at most $m + n - m\rho$ rings for the choosing equal-difference rings problem.*

Proof. The details of proof are shown in the Appendix. □

Theorem 3.8. *The greedy algorithm achieves an approximation guarantee of* 2 *for the choice of equal-difference rings problem.*

Proof. By above lemma, the algorithm will use at most $m + n - m\rho$ rings for all numbers. The worst case is created by as $|A| = n - m\rho$ and $|B| = 0$. We only consider from the first number to the ρth number of each ring. Each number must only locate on ρ equal-difference rings.

Step 1 selects $m\rho$ different numbers, these numbers only present once time on that m choice rings, and they must present $\rho - 1$ times on other $n - m$ rings. Now we see the other $(n-m)\rho$ spaces of $n-m$ rings. Each number of A must locate on ρ rings of the $n-m$ rings. So

$$(n - m) = (n - m\rho)\rho.$$

The other $(n - m)(\rho - 1)$ spaces will be hold by the $m\rho$ numbers of the above m rings. So

$$m\rho(\rho - 1) = (n - m)(\rho - 1).$$

Hence $n - m\rho = m$ and $m + n - m\rho = 2m$. The algorithm is based on the lower bound $\frac{n}{\rho}$ on the choice equal-difference rings problem. Clearly,$m \leq \frac{n}{\rho}$.Hence $m + n - m\rho \leq 2\frac{n}{\rho}$. □

4 Packets scheduling

In topological design step, we define the function TS to estimate the scheduling time slots. It is only a function to decide which equal-difference rings $R^{'}$ and which packets partition is better, since we treat each FR-group as a single node. In the section, we will compute the actually scheduling time slots with uniform traffic packets and given $R^{'}$ and P.

Consider the traffic matrix in assumption.

$$T = \begin{bmatrix} 0 & k^2 & \cdots & k^2 \\ k^2 & 0 & \cdots & k^2 \\ \vdots & \vdots & \ddots & \vdots \\ k^2 & k^2 & \cdots & 0 \end{bmatrix} = [T_{ij}]; (i, j = 0, 1, 2, \cdots, W-1)$$

where T_{ij} means the number of packets that FR-group G_i must send to G_j and let $R' = \{r'_1, r'_2, \cdots, r'_\gamma\}$ and $P = \{S_1, S_2, \cdots, S_\gamma\}$ which are obtained in topological design step.

Although we need δ tuning time slots to tune to different equal-difference rings r'_i, we have k nodes in each FR-groups and nodes in the same group can tune their transmitter when other nodes are transmitting and using the transmitting wavelength channel. Hence it doesn't need to reserve a tuning time slots and stop all nodes transmission to enter tuning steps. Thus the gap of tuning time may be overlapped with packet transmission in scheduling period.

The details of our approach are shown as follows. Use the pipeline transmission scheduling in the above logical rings. We only consider the processing of packet transmission on one group, and the other groups are processed by the same scheduling method. The group G_0 has k nodes noted by $N_{0,1}, N_{0,2}, \cdots, N_{0,k}$. In logical step, the greedy algorithm can obtain the equal-difference rings of $R'(\rho) = \{r'_1(\rho), r'_2(\rho), \cdots, r'_r(\rho)\}$. The tunable γ wavelengths of all nodes' transmitter of G_0 are $\lambda_{\sigma_1^{(1)}}, \lambda_{\sigma_1^{(2)}}, \cdots, \lambda_{\sigma_1^{(\gamma)}}$. First $N_{0,1}$ transmits one packet to $G_{\sigma_1^{(1)}}, G_{\sigma_2^{(1)}}, \cdots, G_{\sigma_\rho^{(1)}}$ with wavelength $\lambda_{\sigma_1^{(1)}}$. The other nodes can use the same method and wavelength to transmit one packet to $G_{\sigma_1^{(1)}}, G_{\sigma_2^{(1)}}, \cdots, G_{\sigma_\rho^{(1)}}$, and $N_{0,1}$ can tune the wavelength to $\lambda_{\sigma_1^{(2)}}$. The k nodes of G_0 must do again $k-1$ times for the above processing.

Note that $\tau(r_i, S_i)$ means the time slots for each partition S_i needing to be transmitted on equal-difference r_i as FR-group is a single node. Hence each FR-group with k nodes needs $k\tau(r_i, S_i)$ time slots as tuning the ring to r_i. And in the same FR-group, one node can tune to next wavelength as other $w-1$ nodes are transmitting, hence each node will obtain $(k-1)k\tau(r_i, S_i)$ tuning time slots between r'_i to r'_{i+1}. As k is large, the tuning time is possible overlapped with packet transmission in the scheduling.

Consider $k = 2$ in figure 4 , there are two nodes $N_{i,1}$ and $N_{i,2}$ in each FR-groups G_i.

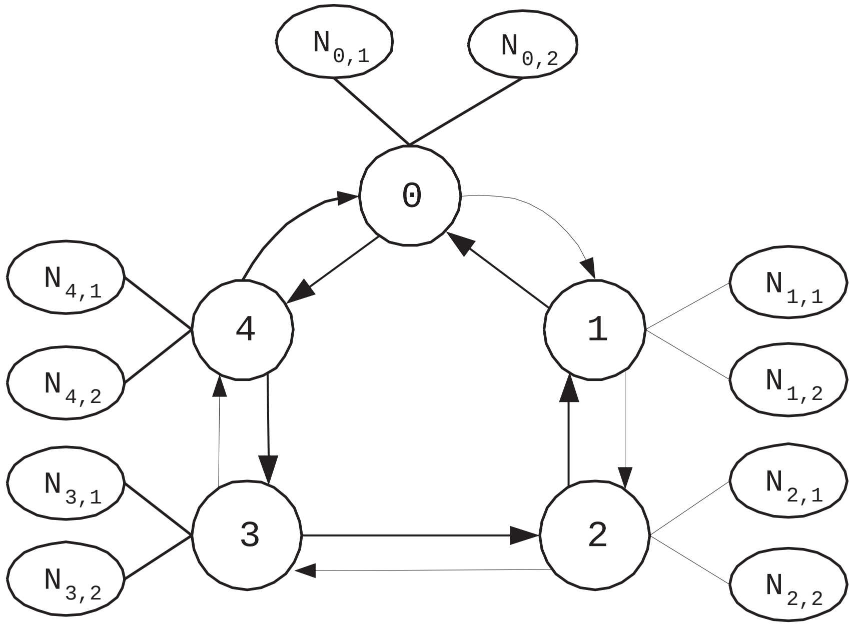

Figure 4: Example as W=5 and k=2

Observing G_0, the transmitting channel of $N_{0,1}$ and $N_{0,2}$ is λ_1 in first equal-difference ring r_1. First, we let $N_{0,1}$ transmit the packets to G_1 and G_2, it costs $2\times\varphi(r_1,1)+2\times\varphi(r_1,2) = 2\times 1+2\times 2 = 6$ time slots . Then $N_{0,2}$ used the same method and wavelength λ_1 to transmit packets to G_1 and G_2, and $N_{0,1}$ entered the tuning step to tune to λ_4 in this time slots. Hence we need $\delta - 6$ slots of tuning gap to wait $N_{0,1}$ tune to λ_4. As $\delta < 6$, then we will mask the tuning time gap in scheduling period.It says that although we use tuning technique to reduce packets propagation delay, but will not increase total scheduling period. The advantage of our scheduling strategy shorten packets propagation delay and total scheduling period.

When the tuning latency is less than $(k-1)k\frac{W(W-1)}{2}$.The time slots of tuning time will be overlapped with packet transmission in scheduling. Assume that we do not consider the high cost of tunability transmitter. We can delete the term $(\gamma-1)\delta$ for tuning latency, and each transmitter node will tune to all wavelengths. Thus the transmission time is at most $k^2\frac{W(W-1)}{2}$ slots.

5 Conclusion

In this paper, we proposed a multi-hop scheduling algorithm in WDM broadcast star network with TT-FR transceiver system. We partition the problem into two subproblems:(1)construct logical topology configuration for efficient communication, and(2) design pipeline transmission scheduling.

In topology design step, we observed that reducing the tuning times of wavelengths for each node transmitted was helpful to shorten the tuning gap in whole transmission period. Hence given a known tuning latency δ, and a bound $\gamma(\gamma < W)$ of the number of tunable wavelengths for each node "FR-group" , our algorithm constructs some number of equal-difference rings in tuning stages. We reduced our problem to the set cover problem. For the special case as $\gamma = \frac{W-1}{2}$, we can reduce it to the maximum matching problem. For the other cases, we find a factor of 2 approximation algorithm with respect to the number of rings generated.

In packet scheduling step, we observe that more nodes in the same FR-groups, more extra tuning time will be needed for each node. And the gap of tuning time may be overlapped with packet transmission in scheduling to add the throughput for each wavelength channel. Our results show that the relation between N, W, and δ affect the tunable wavelengths bound γ. Actually, as the amount of nodes in each FR-groups is large enough to mask the tuning gap, our scheduling strategy shorten packets propagation delay and total scheduling period. Hence our algorithm can be applied generally in WDM broadcast and selected star network.

6 Appendix

Lemma 3.5 Let the number i locate on the first number of the ith ring. The first number of the jth ring is j.If the numbers i and j are independent each other, then all number of the ith ring and the jth ring are different each other.

Proof. The kth number of the ith ring is $ki \bmod p$. The lth number of the jth ring is $lj \bmod p$. Let $i < j$, and $1 \leq k, l \leq p-1$, $k \neq l$ Assume that two numbers ki and lj are equal for modulo p. Foe general case, let ki mod p= lj mod p, and p is prime.

$$ki - rp = lj - sp$$

$$\Rightarrow ki = lj - (s - r)p, t = s - r$$

$$\Rightarrow ki = lj - tp \cdots\cdots (1)$$

By the equation (1)

$$j = \frac{ki + tp}{l} \cdots\cdots (2)$$

$$i = \frac{lj - tp}{k} \cdots\cdots (3)$$

$$i = j + ((l - 1)j - (k - 1)i) - tp \cdots\cdots (4)$$

Now we see the following term:

$$(l - 1)j - (k - 1)i$$

$$= (i - j) + (lj - ki)$$

$$= (i - j) + tp$$

So the equation (4) can be represented as follows:

$$i = j + ((l - 1)j - (k - 1)i) - tp$$

$$\Rightarrow i = j + (p + i - j) - p$$

Hence the numbers i and j are on the ring with difference $p + i - j$. So it produces contradiction. $\square$

Lemma 3.6 The greedy algorithm uses at most $m + n - m\rho$ rings for the choosing equal-difference rings problem.

Proof. The lemma say that the worst case is $m + n - m\rho$ rings for this greedy algorithm, as $|A| = n - m\rho$ and $|B| = 0$. To prove the lemma, we use Mathematical Induction.First let it use at most R rings to cover all numbers.

Starting the Induction: If $|A| = n - m\rho$, $|B| = 0$ or $|A| = n - m\rho - 1$, $|B| = 1$, then clearly $R \leq m + n - m\rho$. This starts the induction.

The Inductive step: We assume that $|A| = n - m\rho - k$, $|B| = k$, and $R \leq m + n - m\rho$ to hold. Let it use at most R_k rings to cover all member of B. And clearly it must use $n - m\rho - k$ rings to cover all numbers of A. So,

$$R \leq m + (n - m\rho - k) + R_k \leq m + n - m\rho$$

It implies that $R_k \leq k$.

Now we discuss that $|A| = n - m\rho - (k + 1)$, $|B| = k + 1$. All numbers of A will be cover by $n - m\rho - (k + 1)$ rings. All member of B will be cover by at most $R_k + 1$ rings. So we obtain

$$R \leq m + (n - m\rho - (k + 1)) + (R_k + 1) = m + n - m\rho - (k - R_k)$$

$$R \leq m + n - m\rho$$

Thus by Induction, we prove it. □

References

[1] P. W. Dowd, "Optical Bus and Star-Couple Parallel interconnection," *in Proc 4th International Parallel Processing Symp.* (Los Anaeles, CA) , pp. 824-838, Apr. 1990.

[2] K. Bogineni and P. W. Dowd, "A Collisionless Multiple Access protocol for a Wavelength Division Multiplexed Star-Couple Configuration: Architecture and Performance Analysis," *in IEEE, Journal of Lightware Technology*, vol. 10, no. 11, November 1992.

[3] G. I. Papadimitriou, A. S. Pomportsis, and M. S. Obaidat, "Adaptive bandwidrh allocation schemes for lightwave LANs with asymmetric traffic," *Performance, Computing, and Communications , 2001. IEEE International Conference on.*, pp. 45-50, 2001.

[4] A. Dasylva and R. Srikant, "Optimal WDM schedules for optical star networks," *in IEEE/ACM transcations on Networking*, vol. 7, no. 3, June 1999.

[5] M. A. MArsan, A. Bianco, E. Leonardi, F. Neri, and A. Nucci, "Multi-hop packet scheduling in WDM/TDM broadcast-and-select optical networks with arbitrary transceivers tuning latencies," *Global Telecommunications Conference, 1998. GLOBECOM 1998. The Brige to Global Integration., IEEE*, vol. 2, pp. 1105-1111, 1998.

[6] M. A. MArsan, A. Bianco, E. Leonardi, F. Neri, and A. Nucci, "Multi-hop Scheduling Algorithm for All-Optical WDM/TDM broadcast and select networks with arbitrary transceivers tuning latencies," *IFIP TC-6 Conference on Optical Network Design and Modeling, Rome, Italy*, Feb. 1998.

[7] X. J. Zhang and C. M. Qiao, "Pipelined Transmission scheduling in all -optical TDM/WDM rings Compter Communications and Neiworks," *1997. Proceedings., Sixth International Conference on*, pp. 144-149, 1997.