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Title: A new two-dimensional code for optical code division multiple access systems

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Abstract – In this paper, we propose a new family of two-dimensional codes for optical CDMA systems. The new codes are modified from the well-known prime-hop codes. Hence, we refer the new codes as modified prime-hop codes. The modified prime-hop codes preserve the same code correlation properties as original prime-hop codes (cross-correlating constraint equals one and autocorrelation constraint equals zero). Furthermore, the proposed codes can be generated with low cost, low complexity and low power loss encoder/decoder. In addition, more simultaneous users also can be accommodated in our proposed system.

Keywords – Optical CDMA, Prime-Hop code, LAN, Optical Networks

I. Introduction

In recent years, optical communication techniques have been applied to various applications. Local area network (LAN) is one of the popular applications using optical communication techniques. As is known, the traffic model in LAN is a burst and multiple users' one, therefore, asynchronous multiplexing schemes and multiple access techniques are necessary in such an environment. The code division multiple access (CDMA) is such a scheme that is well suited for high speed LAN's. In CDMA systems, the spreading codes play an important role in system performance (in terms of capacity and bit error rate). Therefore, there are many investigations on code construction in order to find codes which are well suited for CDMA systems [1-12].

When we construct codes for a CDMA system, two issues should be considered. One is the correlation property and the other is the code size. The correlation property is relative to the bit error rate and the code size is connected to the capacity of the system which is the major problem for one-dimensional codes, such as OOC's [1-5] and prime codes [6]. In order to lessen this problem, two-dimensional codes are developed to solve such limitation of one-dimensional codes [8-12]. There are number of advantages accruing from using two-dimensional coding. Firstly, both the security and the cardinality of the code family are greatly increased. The large cardinality of the two-dimensional code's family is resulted from the integration of both patterns coded in different domains. Furthermore, since two-dimensional codes are coded in two different domains simultaneously, it is significantly increased the system's security level because the pulses must match in both dimensions. Secondly, to have a correlation peak, in addition to the pulse coincidence, the pulses must also match in their wavelengths. Hence, the peaks of cross-correlation and out-of-phase autocorrelation are reduced.

The representative two-dimensional code coded in time and wavelength is the prime-hop code which is proposed by L. Tancevski *et al* in 1996 [12]. Each pulse of a codeword is transmitted at a different wavelength. Prime-hop codes possess a perfect needle-shaped autocorrelation function with zero sidelobe and a cross-correlation of at most one. Moreover, with a larger prime number p, the prime-hop codes have better performance and larger code size [12]. However, the code generation of prime-hop codes is usually achieved by using parallel architecture (Fig 1). In such architecture, the cost and the complexity of encoder and decoder are increased with larger code weight (p). The large amount of tapped delay lines and filters are required when a large prime number p is selected. For the aforementioned reasons, we want to construct more effective codes by reducing the code weight (removing some pulses) of prime-hop codes, resulting in such a system can be implemented by using low cost and low complexity encoder/decoder. However, we know that if decreasing the code weight with the same code length, there are two factors will influence the system performance. The first is that the threshold will decrease as the code weight reduces. As a result, fewer interfering users can cause an error. The other is that the hit probability will decrease as the code weight decreases. Consequently, more interfering users are needed to motive an error. The former factor will deteriorate the system performance, but the latter factor will improve the system performance. We are interested in which factor dominates the system performance.

The outline of this paper is as follows. In section II, the code construction algorithm is described. The performance of the new codes is analyzed in section III. In section IV, we present the numerical results. At last, in section V, the conclusions of this study are summarized.

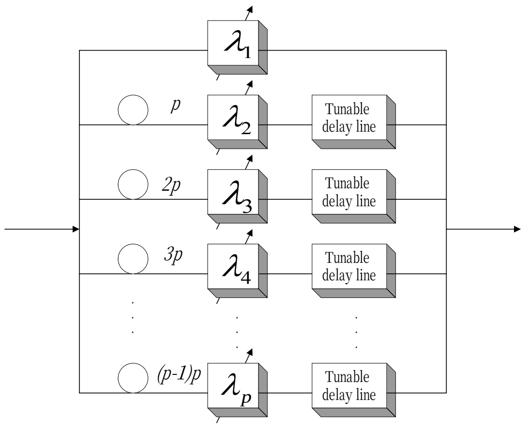

Fig 1. Prime-hop code encoder

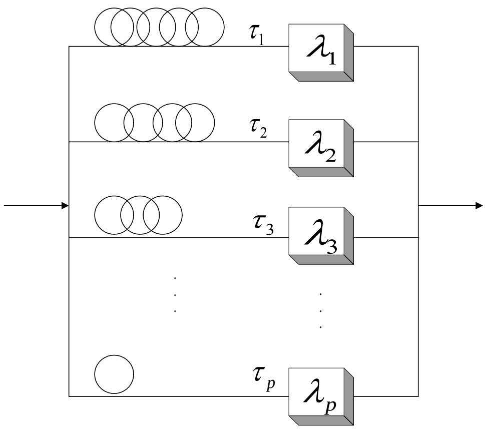

Fig 2. Prime-hop code decoder

II. The algorithm of code construction

In this section, we illustrate how to construct the modified prime-hop codes. The construction algorithm is listed below:

Step1: Select a prime number *p.*

Step2: Write down a set of modified prime sequences $A_i = \{a_{i,0}, a_{i,1}, ..., a_{i,(p-1)/2}\}$, $i = 0,1,.., p-1$. Each element $a_{i,j}$ of A_i is generated by

$$a_{i,j} = [2ij]_p \quad , \qquad (1)$$

where $[\ ]_p$ denotes the modulo p and $j=0,1,\ldots,(p-1)/2$.

Step3: Construct a set of time spreading patterns $S_i = \{s_{i,0}, s_{i,1}, \ldots, s_{i,p^2-1}\}, i = 0,1,\ldots, p-1$. Each element $s_{i,k}$ of S_i is given by

$$s_{i,k} = \begin{cases} 1 & if\ \ k = a_{i,j} + 2jp \\ 0 & otherwise \end{cases}, \qquad (2)$$

where $j = 0,1,\ldots,(p-1)/2$ and $k = 0,1,2,\ldots,p^2-1$.

Step4: Construct a set of hopping patterns $H_i = \{h_{i,0}, h_{i,1}, \ldots, h_{i,(p-1)/2}\}$, $i=0,1,\ldots,p-1$. Each element $h_{i,j}$ is given by

$$h_{i,j} = \begin{cases} p & if\ \ a_{i,j} = 0 \\ a_{i,j} & otherwise \end{cases}, \qquad (3)$$

where $j=0,1,2,\ldots,(p-1)/2$.

The elements of H_i are identical to elements of the modified prime sequences A_i except that the element "*0*" of A_i is replaced by the prime number "*p*". The hopping pattern H_0 is discarded as trivial because its elements have the same value. Hence, there are only *p-1* hopping patterns.

Step 5: Integrate time spreading patterns and wavelength hopping patterns to form the modified prime-hop code $PHC_{l,m} = S_l H_m = \{c_{l,m,0}, c_{l,m,1}, \ldots, c_{l,m,p^2-1}\}$, where $l=0,1,2,\ldots p-1$ *and* $m=1,2,\ldots,p-1$, by the following equation:

$$C_{l,m,k} = \begin{cases} h_{m,j} & \text{if}\ k = a_{m,j} + 2jp \\ 0 & otherwise \end{cases}. \qquad (4)$$

We use the following example to explain the construction algorithm.

Example 1:

Step 1: Let the prime number p equal 5.

Step 2: By following equation (1), the modified prime sequences are listed as follows:

$$
\begin{aligned}
A_0 &= \{0, 0, 0\}, \\
A_1 &= \{0, 2, 4\}, \\
A_2 &= \{0, 4, 3\}, \\
A_3 &= \{0, 1, 2\}, \\
A_4 &= \{0, 3, 1\}.
\end{aligned}
$$

Step 3: Derive the p time spreading patters from equation (2).

$$
\begin{aligned}
S_0 &= \{10000\ 00000\ 10000\ 00000\ 10000\}, \\
S_1 &= \{10000\ 00000\ 00100\ 00000\ 00001\}, \\
S_2 &= \{10000\ 00000\ 00001\ 00000\ 00010\}, \\
S_3 &= \{10000\ 00000\ 01000\ 00000\ 00100\}, \\
S_4 &= \{10000\ 00000\ 00010\ 00000\ 01000\},
\end{aligned}
$$

Step 4: The hopping patterns are derived from equation (3).

$$
\begin{aligned}
H_0 &= \{5, 5, 5\}, \\
H_1 &= \{5, 2, 4\}, \\
H_2 &= \{5, 4, 3\}, \\
H_3 &= \{5, 1, 2\}, \\
H_4 &= \{5, 3, 1\}.
\end{aligned}
$$

The hopping pattern H_0 is discarded as trivial because elements of H_0 have identical values.

Step5: Finally, we generate $p(p-1)$ distinct code sequences, called as modified prime-hop codes, by integrating $p-1$ wavelength hopping patterns and p time spreading patterns. The modified prime-hop codes for $p = 5$ are listed in Table 1. From Table 1, it is clearly shown that the modified prime-hop codes have pulses only in even-number blocks. As a result, the modified prime-hop codes have the same code length p^2 but less code weight $(p+1)/2$ in compared with the original prime-hop codes. Moreover, through theorems proposed in [12], it is shown that the autocorrelation constrain λ_a of prime-hop codes is zero and cross-correlation constraint λ_c of prime-hop codes is 1. Due to the process of removing some pulses doesn't increase the values of autocorrelation and cross-correlation, consequently, the modified prime-hop codes have the same code correlation properties with prime-hop codes.

Table 1 Modified prime-hop codes for $p=5$

S_0H_1	*10000*	*00000*	*30000*	*00000*	*50000*	S_1H_1	*10000*	*00000*	*00300*	*00000*	*00005*
S_2H_1	*10000*	*00000*	*00003*	*00000*	*00050*	S_3H_1	*10000*	*00000*	*03000*	*00000*	*00500*
S_4H_1	*10000*	*00000*	*00030*	*00000*	*05000*	S_0H_2	*10000*	*00000*	*50000*	*00000*	*40000*
S_1H_2	*10000*	*00000*	*00500*	*00000*	*00004*	S_2H_2	*10000*	*00000*	*50000*	*00000*	*40000*
S_3H_2	*10000*	*00000*	*05000*	*00000*	*00400*	S_4H_2	*10000*	*00000*	*00050*	*00000*	*04000*
S_0H_3	*10000*	*00000*	*20000*	*00000*	*30000*	S_1H_3	*10000*	*00000*	*00200*	*00000*	*00003*
S_2H_3	*10000*	*00000*	*00002*	*00000*	*00030*	S_3H_3	*10000*	*00000*	*02000*	*00000*	*00300*
S_4H_3	*10000*	*00000*	*00020*	*00000*	*03000*	S_0H_4	*10000*	*00000*	*40000*	*00000*	*20000*
S_1H_4	*10000*	*00000*	*00400*	*00000*	*00002*	S_2H_4	*10000*	*00000*	*00004*	*00000*	*00020*
S_3H_4	*10000*	*00000*	*04000*	*00000*	*00200*	S_4H_4	*10000*	*00000*	*00040*	*00000*	*02000*

III. Performance analysis

In this section, we analyze the performance of the system with modified prime-hop codes. Here, we assume that the performance deterioration is only due to the multiple access interference (MAI) without the negative effects of thermal noise and shot noise in the photodetection process. Moreover, for simplification, the Poisson characteristic of the optical direct detection process is not considered. We also assume that the receivers are already synchronized with the transmitters. In addition, chip synchronization is also assumed here for the sake of mathematical convenience.

In an optical CDMA system using on-off keying modulation only bit “1” is encoded by the signature code sequence. Therefore, an error occurs only when the accumulative MAI at a particular user whose transmitting data bit is “0” reaches over the decision threshold. In a system with modified prime-hop code whose prime number is p, the pulses in a codeword are transmitted at different wavelengths. Hit (or the MAI) happens only when the pulse belonging to a particular user overlaps with the pulse of the desired user and their wavelengths are also the same. Since the cross-correlation constraint of modified prime-hop codes is equal to one, two codewords cannot hit more than once during a data bit interval. However, the code patterns of modified prime-hop codes use only a portion of available wavelengths. The exact value of hit probability depends on the code patterns used by the desired user and interfering users. Consequently, it is difficult to compute the exact distribution of hit. In order to simplify the computational complexity, we use the average hit instead of the exact hit to analyze the system performance. The average hit is defined as the mean value of the exact distribution of hits in a full load environment and can be obtained through the computer program with the exhausted method. Table 2 lists the average values of hit for prime number $p \le 41$. Through this section, we use these values to approximate the performance of modified prime-hop codes. Let $\bar{p}$ denote the average hit. Then, the probability that a pulse of a particular user hitting with one of the pulses of the desired user is given by:

$$q = \frac{\bar{p}}{2L}, \qquad (5)$$

where the factor 1/2 accounts for the probability that the interferer is transmitting a “1” and L is the code length equal to p^2. The number of users that interfere with the desired user has a binomial distribution with parameters *N-1* and *q*, where *N* is the total number of users. Let $P_I(I=i)$ denote the probability of *i* interfering users, we have

$$\mathrm{P}_I(I=i)=\binom{N-1}{i}q^i(1-q)^{N-1-i},\ i=0,1,\ldots,N\text{ -1} \quad (6)$$

The approximation bit error rate is given by

$$P_e=\frac{1}{2}\sum_{i=Th}^{\infty}P_I(I=i) \quad (7)$$

Substituting eqn.(6) into (7), we derive the P_e in form of

$$P_e=\frac{1}{2}\sum_{(p+1)/2}^{N-1}\binom{N-1}{i}\left(\frac{\overline{p}}{2L}\right)^i\left(1-\frac{\overline{p}}{2L}\right)^{N-1-i} \quad (8)$$

Table 2 Average hit of modified prime-hop codes

The average hit					
Prime number	Average value	Prime number	Average value	Prime number	Average value
p=5	1.947	*p=17*	4.966	*p=31*	8.491
p=7	2.46	*p=19*	5.486	*p=37*	9.985
p=11	3.477	*p=23*	6.489	*p=41*	10.987
p=13	3.98	*p=29*	7.991		

IV. Numerical Analysis and Simulation Results

In this section, the numerical analysis and simulation results of the proposed schemes are given. Firstly, we compare the simulation probability of bit error of the proposed scheme with the bit error probability derived from eqn.(8). The simulation curve is shown in Fig.3. It is shown that the curve of the approximation analysis is very close to the simulation one. Hence, the validity of our theoretical analysis is verified.

In Fig.4, we show the variation in the system performance with modified prime-hop codes for different prime numbers. It is shown that the systems with larger prime number have better performance. This property is similar to that of the prime-hop codes [12]. Finally, we compare the performance of our proposed scheme with that of prime-hop codes. As illustrated in Fig.5 and Fig.6, we find that modified prime-hop codes perform better than original prime-hop codes when the traffic load is heavy. We have mentioned in the beginning of this paper that there are two factors influence the system performance resulting from reducing the code weight. The first is that the threshold will decrease as the code weight reduces. As a result, fewer interfering users can cause an error. The other is that the hit probability will decrease as the code weight decreases. Consequently, more interfering users are needed to motive an error. The numerical results in Fig.5 and Fig.6 show that the latter dominates the system performance when the traffic load is heavy. This result satisfies our expectancy that the system can be implemented with low complexity encoder/decoder and, in the meanwhile, maintains good system performance. From Fig.5 and Fig.6, we find that the cross point is under the maximum BER required in optical CDMA system (10^{-9}) when $p>=37$. Table 3 shows the system hardware requirement and maximum number of simultaneous users for systems with prime-hop codes and modified prime-hop codes. From Table 3, it is shown that the proposed scheme can accommodate more users than the system with prime-hop codes when selecting proper prime number.

V. Conclusions

In this paper, we have proposed a new two-dimensional code, which we term as modified prime-hop codes. The modified prime-hop codes are constructed by adjusting the code weight of prime-hop codes to a reasonable value. From our results, it is shown that the modified prime-hop codes are more efficient than original prime-hop codes because of many advantages, including low complexity of hardware implementation, low power loss and better performance. Moreover, more simultaneous users can be accommodated in the system with modified prime-hop codes.

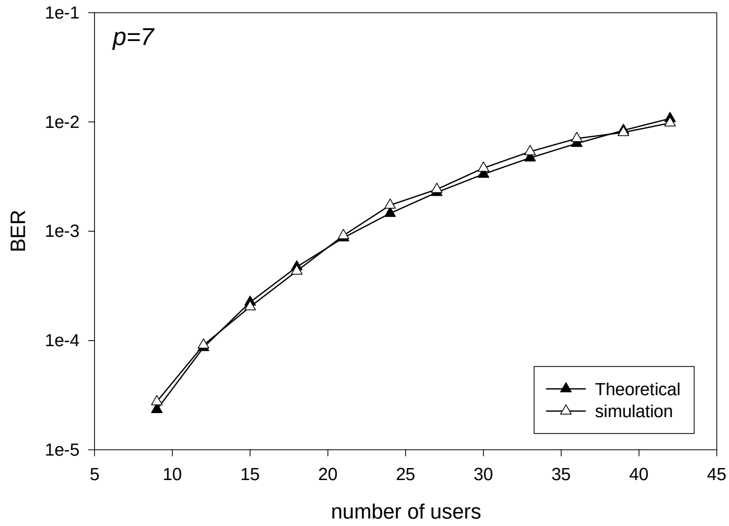

Fig.3 Simulation results of modified prime-hop codes for *p* = *7* compared to the analytical approximation.

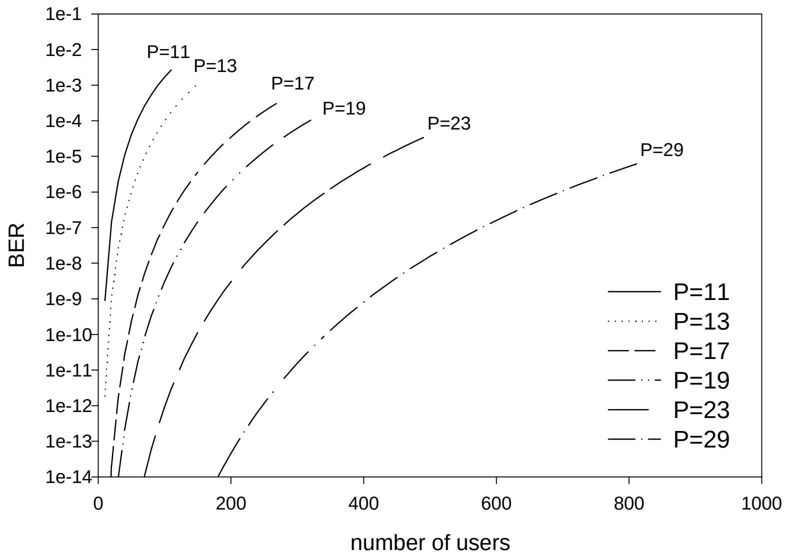

Fig.4 BER calculations for modified prime-hop codes for various prime numbers.

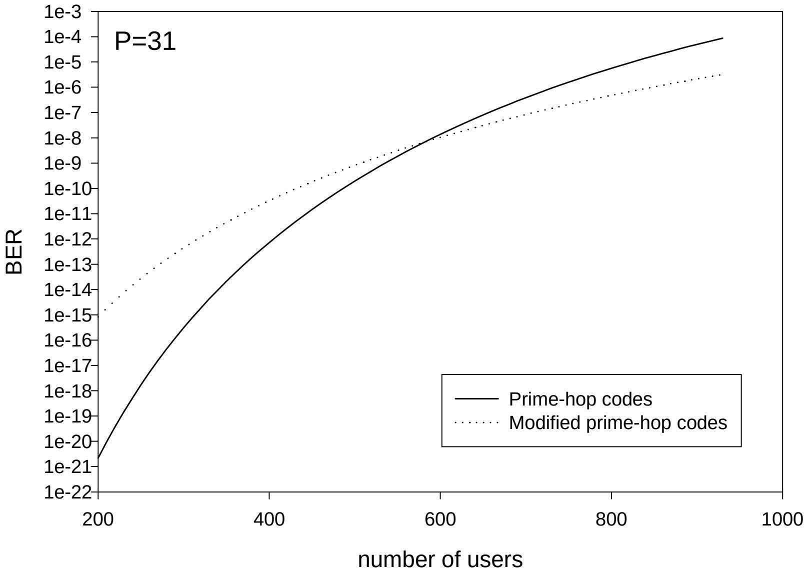

Fig.5 BER versus the number of users for *p = 31*

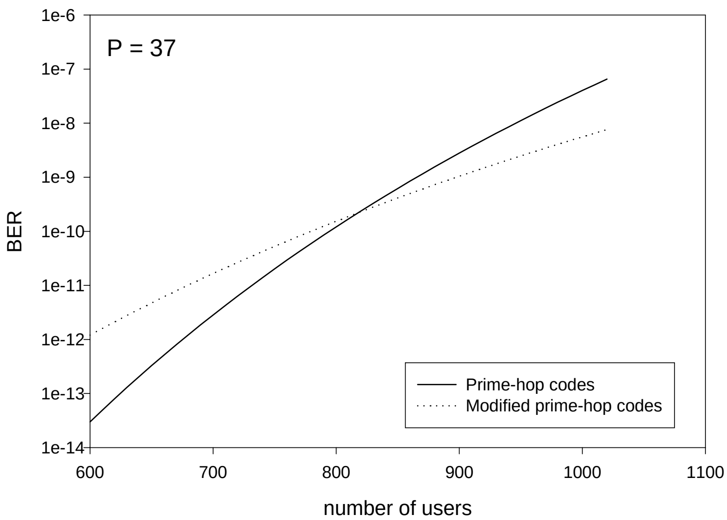

Fig.6 BER versus the number of users for *p = 37*

Table 3 Given BER<=10^{-9}, maximum number of users can be accommodated in systems employing prime-hop codes and modified prime-hop codes

	Modified prime-hop codes			**Prime-hop codes**		
Prime number	Max number of users (BER<=10^{-9})	Hardware requirement		Max number of users (BER<=10^{-9})	Hardware requirement	
		filters	tapped delay lines		filters	tapped delay lines
P=23	182	24	24	231	46	46
P=29	407	30	30	446	58	58
P=31	508	32	32	536	62	62
P=37	898	38	38	866	74	74

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