# An Improved Similarity Measure for Image Database Based on 2D C<sup>+</sup>-string

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**Abstract** -In an image database system, the spatial knowledge representation is a technique of abstraction to describe an image. The 2D string and its variants are based on this concept. One of the variants, called 2D  $C^+$ -string, considers sizes of and distances between objects. This method presents three advantages: (1) more accuracy in picture representation and reconstruction; (2) less ambiguity in similarity retrieval; (3) reasoning about relative locations for a symbolic picture is possible. However, the similarity measure based on  $2D C^+$ -string doesn't consider the ratios about sizes of and distances between objects on x- and y-axis together. This neglect may cause distorted result of similarity retrieval. In this paper, we modify the original similarity measure for taking down the variation of ratios between two symbolic pictures and furthermore propose two new types of similarity measure for discriminating pictures more precise. By exploiting the ratios, the pictorial query is more flexible for certain demands.

**Keywords:** Spatial knowledge,2D C<sup>+</sup>-string, Similarity retrieval, Pictorial query.

#### 1. Introduction

An image database system is different from conventional database systems in two respects. First is the raw data of image is a visual information rather than pure numeral or alphabet. Therefore, it is very important to describe some features from the original image for representation and image indexing. Second is the system needs to have the capability of similarity retrieval for adapting human perception.

In the research of image representation, one technique is to abstract an original image as a symbolic picture. A symbolic picture is constituted by some symbols or vocabularies, and these symbols or vocabularies represent iconic objects or real entities in an image. Chang *et al.* [2] proposed the 2D string to represent a symbolic picture based on the knowledge structure of symbolic projections. The basic idea of 2D string is that iconic objects will be pro-



Figure 1. (a) an original image (b) its symbolic picture and 2D string.

jected on the rectangular coordinate plane along xand y-coordinates respectively to build two strings for describing relative positions among objects on xand y-axis. This approach provides an efficient and natural way to construct iconic indexes for symbolic pictures. A symbolic picture and its 2D string are shown in Figure 1.

For the capability of similarity retrieval, three types of 2D subsequence (type-i, i=0, 1, 2) based on 2D string are defined. Therefore, the problem of image retrieval then can be envisioned as a problem of 2D subsequence matching [2].

Subsequently, many spatial knowledge representations for image were proposed, such as 2D G-string, 2D C-string, 2D C<sup>+</sup>-string and so on [1,4,6]. And there are corresponding similarity measures based on these representations.

However, the similarity measure based on 2D C<sup>+</sup>string doesn't consider the ratios about sizes of and distances between objects on x- and y-axis together. This problem may cause distorted pictures of typei1 and type-i2 are retrieved.

In this paper, we improve the similarity measure based on 2D C<sup>+</sup>-string. We modify the equations of type-i1 and type-i2 for taking down the variation of ratios and then explicit the ratios to propose two new types of similarity measure. The improved method provides two major advantages:

1. More precision in similarity retrieval because the additional two new types of similarity measure can discriminate the case of the ratios about sizes on x- and y-axis are equal, so dose the case of distances.

2. More flexibility in the pictorial query because we can assign the specific ratios about sizes of and distances between objects for certain demands.

The remainder of this paper is organized as fol-



Figure 2. The 13 patterns of spatial relations between two objects in one dimension.

lows. Section 2 reviews some variants of 2D string. Section 3 introduces the similarity measure based on 2D C<sup>+</sup>-string and its drawback. The improved similarity measure based on 2D C<sup>+</sup>-string and an example are presented in section 4. Finally, the conclusions are given in the last section.

### 2. Related Works

After 2D string was proposed, Chang *et al.* [1] extended the idea of 2D string by adding a cutting mechanism that called 2D G-string to describe more complex spatial relations among non-zero sized objects. However, more cutting performed in an image will cause more subparts, and then the length of 2D G-string will increase.

To solve the disadvantage of 2D G-string, Lee et al. [6] proposed 2D C-string with a new cutting mechanism based on thirteen patterns of spatial relations between two objects in one dimension. The length of 2D C-string is shorter significantly than 2D G-string because the number of cutting is reduced. The thirteen patterns of spatial relations are shown in Figure 2 and the definitions of spatial operators are shown in Table 1.

Although the length of 2D C-string is shortened, the neglect about the sizes of and distances between objects will cause ambiguity in picture representation, reconstruction, and similarity retrieval. Thus, Huang *et al.* [4] proposed 2D C<sup>+</sup>-string, which extends the work of Lee *et al.* [6,7] by attaching metric information to the strings. Such information can be easily counted by pixels in an image. In the 2D C<sup>+</sup>string, each object in a symbolic picture is given two pairs of begin-bounds and end-bounds, one for x-axis and the other for y-axis to calculate the sizes and the distances for objects.

Table 1. The definitions of spatial operators.

Notations Conditions				
A < B	end(A) < begin(B)			
A B	end(A) = begin(B)			
A%B	begin(A) < begin(B), end(A) > end(B)			
A[B	begin(A) = begin(B), end(A) > end(B)			
A]B	begin(A) < begin(B), end(A) = end(B)			
A/B	begin(A) > begin(B), end(A) > end(B)			
A = B	begin(A) = begin(B), end(A) = end(B)			

The symbolic pictures and their corresponding 2D C<sup>+</sup>-string extended from 2D C-string with metric information in one dimension are represented as follows:

1.  $A_s$  denote object A with size s where s=end(A)-begin(A). Following object is represented by  $A_6$ .

2. Operator "<" with distance d between object A and B, denoted as A< dB, d=begin(B)-end(A). Following pattern is represented by A<  $_3$ B.



3.Operator "%" with distance d between object A and B, denoted as A%dB, d=begin(B)-end(A). Following pattern is represented by A%<sub>5</sub>B.



4. Other operators: no metric information is needed.

However, the similarity measure based on 2D C<sup>+</sup>string may cause distorted result of similarity retrieval.

## 3. Similarity Measure Based on 2D C<sup>+</sup>-string and Its Drawback

The similarity retrieval is very important since pictures are represented slightly different in image



 $\begin{array}{ll} f_1: u: A_2]C_1|C_1 <_1 D_4\%_1(B_2 = E_2) & v: B_1 <_3 A_2](C_1 = D_1)|C_1 = D_1 <_1 E_2 \\ f_2: u: A_8]C_1|C_7 <_1 E_7](D_4]B_3)|D_{14}[B_1 & v: B_1 <_3 A_2](C_1 = D_1)|C_1 = D_1 <_1 E_2 \\ \\ & \text{Figure 3. An example of the original similarity measure between } f_1 \text{ and } f_2. \end{array}$ 

database may regard as the same by a user. Therefore, the system needs a similarity measure for evaluating the similarity of images.

The following notations will be used in defining the similarity measure based on 2D  $C^+$ -string: [4]

- (AB), a pair objects A and B, it is regarded as the type-*ij* similar pair.
- $C_{AB}$ , the spatial relation for (AB) corresponds to the category that classified by Lee *et al* [7].
- $r_{AB}^{u}$ , the spatial relation for (AB) on x-axis.
- $r_{AB}^{v}$ , the spatial relation for (AB) on y-axis.
- $S_{Au}$ , the size of object A on x-axis.
- $S_{Av}$ , the size of object A on y-axis.
- $D_{AB}^{u}$ , the ditance between object A and B on x-axis.
- $D_{AB}^{v}$ , the ditance between object A and B on y-axis.

The definition of similar type between two symbolic pictures is the picture f' is a type-ij similar unit picture of a picture f. The picture f and f' both contain (AB), represented as

1. In 
$$f$$
,  
 $u: A r_{AB}^{u} B v: A r_{AB}^{v} B$   
 $S_{Au} D_{AB}^{u} S_{Bu} S_{Av} D_{AB}^{v} S_{Bv}$   
2. In  $f'$ ,  
 $u: A r_{AB}^{u'} B v: A r_{AB}^{v'} B$   
 $S_{Au'} D_{AB}^{u'} S_{Bu'} S_{Av'} D_{AB}^{v'} S_{Bv'}$ 

The nine types of similarity (type-ij, i = 0, 1, 2, and j = 0, 1, 2) that the pair objects (AB) both in picture f and f' are defined as follows:

- 1. (type-00):  $C(r_{AB}^{u}, r_{AB}^{v}) = C(r_{AB}^{u'}, r_{AB}^{v'})$
- 2. (type-10):  $C(r_{AB}^{u}, r_{AB}^{v}) = C(r_{AB}^{u'}, r_{AB}^{v'})$  and  $(r_{AB}^{u} = r_{AB}^{u'} \text{ or } r_{AB}^{v} = r_{AB}^{v'})$
- 3. (type-20):  $C(r^{u}_{AB}, r^{v}_{AB}) = C(r^{u'}_{AB}, r^{v'}_{AB})$  and  $(r^{u}_{AB} = r^{u'}_{AB} \text{ and } r^{v}_{AB} = r^{v'}_{AB})$
- 4. (type-01): type-00 and  $(S_{Au}/S_{Bu} = S_{Au'}/S_{Bu'}$ and  $S_{Av}/S_{Bv} = S_{Av'}/S_{Bv'}$ )

- 5. (type-11): type-10 and  $(S_{Au}/S_{Bu} = S_{Au'}/S_{Bu'}$ and  $S_{Av}/S_{Bv} = S_{Av'}/S_{Bv'}$ )
- 6. (type-21): type-20 and  $(S_{Au}/S_{Bu} = S_{Au'}/S_{Bu'}$ and  $S_{Av}/S_{Bv} = S_{Av'}/S_{Bv'}$ )
- 7. (type-02): type-01 and  $(S_{Au}/D^{u}_{AB} = S_{Au'}/D^{u'}_{AB}$ and  $S_{Av}/D^{v}_{AB} = S_{Av'}/D^{v'}_{AB}$ )
- 8. (type-12): type-11 and  $(S_{Au}/D^{u}_{AB} = S_{Au'}/D^{u'}_{AB})$ and  $S_{Av}/D^{v}_{AB} = S_{Av'}/D^{v'}_{AB}$
- 9. (type-22): type-21 and  $(S_{Au}/D^{u}_{AB} = S_{Au'}/D^{u'}_{AB}$ and  $S_{Av}/D^{v}_{AB} = S_{Av'}/D^{v'}_{AB}$ )

According to the definition of most similar picture between two pictures was proposed by Lee et al. [7] and the similarity retrieval algorithm based on 2D  $C^+$ -string [4]. We illustrate the drawback of similarity measure based on 2D C<sup>+</sup>-string with the example shown in Figure 3. There are five objects (A, B, C, D, and E) between two pictures  $f_1$  and  $f_2$  respectively. The type-20 similar subjicture of  $f_1$  and  $f_2$  contains objects A, B, C, and D. Then type-21 similar subpicture contains objects A, B, and C. The finest granularity, type-22 similar subpicture contains objects A and B. Obviously, type-21 and type-22 similar subpictures between  $f_1$  and  $f_2$  are distorted critically because the ratios about sizes and distances of objects on x- and y-axis aren't taken into consideration together.

## 4. An Improved Similarity Measure Based on 2D C<sup>+</sup>-string

improved In the similarity measure, we change the original equation  $(S_{Au}/S_{Bu})$ =  $S_{Au'}/S_{Bu'}$  and  $S_{Av}/S_{Bv}$  $S_{Av'}/S_{Bv'}$ in =type-*i*1 into a new equation  $(S_{Au}/S_{Au'})$ =  $S_{Bu}/S_{Bu'}$  and  $S_{Av}/S_{Av'} = S_{Bv}/S_{Bv'}$ . The original equation  $(S_{Au}/D_{AB}^{u'} = S_{Au'}/D_{AB}^{u'})$  and  $S_{Av}/D_{AB}^{v} =$  $S_{Av'}/D_{AB}^{v'}$  in type-*i*2 is modified as a new equation  $(S_{Au}/S_{Au'} = D^u_{AB}/D^{u'}_{AB}$  and  $S_{Av}/S_{Av'} =$  $D_{AB}^{v}/D_{AB}^{v'}$ ). The new equations can compute the ratios about sizes of and distances between

objects. Then we add a parameter k for representing the variation of ratios between two symbolic pictures. By exploiting the parameter k, we can retrieve pictures more precise if the ratios on xand y-axis are equal. Notice that the notations of parameter k have different meanings in size and distance.

- $k_u^s$ , the ratio about size on x-axis for (AB) between two symbolic pictures.
- $k_v^s$ , the ratio about size on y-axis for (AB) between two symbolic pictures.
- $k_u^{sd}$ , the ratio about size and distance on x-axis for (AB) between two symbolic pictures.
- $k_v^{sd}$ , the ratio about size and distance on y-axis for (AB) between two symbolic pictures.

The improved similarity measure between the pair objects (AB) in a picture f and the same pair objects (AB) in another pictures f' are defined as follows:

- 1. (type-00):  $C(r_{AB}^{u}, r_{AB}^{v}) = C(r_{AB}^{u'}, r_{AB}^{v'})$
- 2. (type-10):  $C(r^{u}_{AB}, r^{v}_{AB}) = C(r^{u'}_{AB}, r^{v'}_{AB})$  and  $(r^{u}_{AB} = r^{u'}_{AB} \text{ or } r^{v}_{AB} = r^{v'}_{AB})$
- 3. (type-20):  $C(r^{u}_{AB}, r^{v}_{AB}) = C(r^{u'}_{AB}, r^{v'}_{AB})$  and  $(r^{u}_{AB} = r^{u'}_{AB}$  and  $r^{v}_{AB} = r^{v'}_{AB})$
- 4. (type-01): type-00 and  $(S_{Au}/S_{Au'} = S_{Bu}/S_{Bu'} = 1/k_u^s$  and  $S_{Av}/S_{Av'} = S_{Bv}/S_{Bv'} = 1/k_v^s$ )
- 5. (type-11): type-10 and  $(S_{Au}/S_{Au'} = S_{Bu}/S_{Bu'} = 1/k_u^s$  and  $S_{Av}/S_{Av'} = S_{Bv}/S_{Bv'} = 1/k_v^s$ )
- 6. (type-21): type-20 and  $(S_{Au}/S_{Au'} = S_{Bu}/S_{Bu'} = 1/k_u^s$  and  $S_{Av}/S_{Av'} = S_{Bv}/S_{Bv'} = 1/k_v^s$ )
- 7. (type-02): type-01 and  $(S_{Au}/S_{Au'} = D^u_{AB}/D^{u'}_{AB} = 1/k_u^{sd}$  and  $S_{Av}/S_{Av'} = D^v_{AB}/D^{v'}_{AB} = 1/k_v^{sd}$
- 8. (type-12): type-11 and  $(S_{Au}/S_{Au'} = D^u_{AB}/D^{u'}_{AB} = 1/k^{sd}_u$  and  $S_{Av}/S_{Av'} = D^v_{AB}/D^{v'}_{AB} = 1/k^{sd}_v$
- 9. (type-22): type-21 and  $(S_{Au}/S_{Au'} = D^u_{AB}/D^{u'}_{AB} = 1/k^{sd}_u$  and  $S_{Av}/S_{Av'} = D^v_{AB}/D^{v'}_{AB} = 1/k^{sd}_v$ )
- 10. (type-31): type-21 and  $k_u^s = k_v^s$
- 11. (type-32): type-22 and  $k_u^{sd} = k_v^{sd}$

For clear understanding, we divide the improved similarity measure into four parts for explaining. First, type-*i*0 similarity is the same as proposed by Huang *et al.* [4]. Second, although the original equation  $(S_{Au}/S_{Bu}=S_{Au'}/S_{Bu'})$  and the new equation  $(S_{Au}/S_{Au'}=S_{Bu}/S_{Bu'})$  in type-*i*1 have the same result in the form of  $(S_{Au}*S_{Bu'}=S_{Au'}*S_{Bu})$ , they mean two different implications. The original equation compares the ratios about sizes for a pair objects (AB) on x-axis in one picture to another that also contains the same pair objects (AB). The improved equation compares the ratios about sizes for objects A between two pictures to the ratios about sizes for objects B between the same two pictures on x-axis. The new one can help us acquire the ratios of pair objects (AB) on x-axis between two pictures. Then, we add a notation  $1/k_u^s$  for representing the ratio about sizes for (AB) on x-axis. The same modifications are applied to y-axis, such as  $(S_{Av}/S_{Av'} = S_{Bv}/S_{Bv'} = 1/k_v^s)$ . Notice that the notations  $1/k_u^s$  and  $1/k_v^s$  directly add after original equation can't acquire the ratios about sizes. An example is shown in Figure 3. Based on the improved type-21 similarity, we can realize the ratios about sizes on x- and y-axis for (AC) are different since  $(S_{Au}/S_{Au'} = S_{Cu}/S_{Cu'}) \Rightarrow 2/8 = 2/8$ and  $(S_{Av}/S_{Av'}) = S_{Cv}/S_{Cv'}$  $\Rightarrow 2/2 = 2/2.$ However, based on the original type-21 simwe can't realize the ratios ilarity. about sizes on x- and y-axis for (AC) are differsince  $(S_{Au}/S_{Bu}=S_{Au'}/S_{Bu'})$  $\Rightarrow 2/2 = 8/8$  $\operatorname{ent}$ and  $(S_{Av}/S_{Bv}=S_{Av'}/S_{Bv'}) \Rightarrow 2/2=2/2.$ The reason is the ratios about sizes of objects on x- and y-axis aren't considered together. Third, similar modifications also perform on the original type-i2. The new equation of type-*i*2 is  $(S_{Au}/S_{Au'} = D_{AB}^u/D_{AB}^{u'} = 1/k_u^{sd}$  and  $S_{Av}/S_{Av'} = D_{AB}^v/D_{AB}^{v'} = 1/k_v^{sd}$ . Finally, we can propose two new types of similarity measure for retrieving pictures more precise because we have the ratios about sizes and distances for each pair objects on x- and y-axis. The definition of type-31 is (AB) satisfies type-21 and the ratios about sizes on x- and y-axis are equal. The type-32 satisfies type-22 and the ratios about sizes and distances on x- and y-axis are equal. There is a clear summary is shown in Table 2. Notice that the sign of " $\times$ " in Table 2 means no corresponding type because type-i0 doesn't take any metric information into consideration.

Table 2. The improved similarity measure based on 2D C<sup>+</sup>-string.

metric $info(j)$	spatial relations $(i)$			
	category	either	both	the rations
	relation	x- or	x- and	on x- and
		y-	y-axis	y-axis are
		axis		equal
	0	1	2	3
0	00	10	20	×
size 1	01	11	21	31
size and 2	02	12	22	32
distance				

In this paper, we also propose a type-ij\_Similarity\_Retrieval algorithm based on the concept of "query-by-example" [3]. This algorithm is similar to the one's proposed by Huang *et al.* [5]. We will introduce some notations that will be used in defining the similarity retrieval.



 $f_4$ 

Figure 4. An example of type-ij\_Similarity\_Retrival based on improved similarity measure.

- $f_q$ , a query picture in pictorial query.
- $f_d$ , pictures in image database.
- $O_{fq}$ , the set of all objects in a query picture  $f_q$ .
- $O_{fd}$ , the set of all objects in a database picture  $f_d$ .
- $c_{AB}$ , category relation for (AB) in a picture,  $c_{AB} \in \{\text{DISJOINT}, \text{JOINT}, \text{PART_OVLP}, \text{CONTAIN}, \text{BELONG}\}, \text{they can be abbreviated as } d, j, p, c, b [7].$
- $C_{fq}$ , the set of all category relations for each pair objects in  $f_q$ .
- $C_{fd}$ , the set of all category relations for each pair objects in  $f_d$ .
- $r_{AB}^u$  and  $r_{AB}^v$ , the spatial relations based on 2D C<sup>+</sup>-string for (AB) on x- and y-axis,  $r_{AB} \in \{<,$

 $|, \%, [, ], /, = \}.$ 

- $R_{fq}^u$  and  $R_{fq}^v$ , the set of all spatial relations for each pair objects on x- and y-axis respectively in  $f_q$ .
- $R^u_{fd}$  and  $R^v_{fd}$ , the set of all spatial relations for each pair objects on x- and y-axis respectively in  $f_d$ .

Then we review a notation that a pair objects (AB) is regarded as the type-*ij* similar pair. Definition 4.1

A picture  $f_d$  in image database is type-ij similar to a query picture  $f_q$  if for every two objects A, B  $\in f_q$ , there exist a type-ij similar pair  $(AB)\in f_d$ . Algorithm:type-ij\_Similarity\_Retrieval

**Input:**A query picture  $f_q$  and a sequence of 2D C<sup>+</sup>string representing picture in image database



Figure 5. An example of flexible query by using parameter k.

**Output:**type-*ij*\_Similar\_List (pictures in image database similar to  $f_q$ )

- 1. Generate  $f_q[C_{fq}, R^u_{fq}, R^v_{fq}]$ . /\*  $f_q[C_{fq}, R^u_{fq}, R^v_{fq}]$ denotes  $C_{fq}, R^u_{fq}, R^v_{fq}$  for  $f_q$  \*/ 2. /\* search pictures in image database \*/
- For each picture  $f_d$  in image database
  - if  $O_{fq} \subseteq O_{fd}$  then (a) Derive  $f_d[C_{fd}, R^u_{fd}, R^v_{fd}]$ . /\*  $f_d[C_{fd},$  $[R_{fd}^u, R_{fd}^v]$  denotes  $C_{fd}, R_{fd}^u, R_{fd}^v$  for  $f_d */$ (b) Apply type-ij similarity measure to  $f_q[C_{fq}, R^u_{fq}, R^v_{fq}]$  and  $f_d[C_{fd}, R^u_{fd}, R^v_{fd}]$ .
    - if  $f_d$  is type-ij similar to  $f_q$ add  $f_d$  to type-ij\_Similar\_List
- 3. Return type-*ij*\_Similar\_List

To illustrate the power of the improved method, let's look at an example shown in Figure 4. Suppose that we have a  $f_q$  and four  $f_d$ . By applying the  $f_4$ , type-21 \_Similar\_List of  $f_q$  is  $\{f_2, f_3, f_4\}$ , and type-22 \_Similar\_List of  $f_q$  is  $\{f_3, f_4\}$ . Obviously, we can look at  $f_3$  is significantly different form  $f_q$ . Thus, we can further eliminate  $f_3$  by applying type-32 and precisely discriminate  $f_4$  is more similar to  $f_q$ .

Beside the power of retrieving pictures more precise, we will illustrate the parameter k can provide flexible query. An example is shown in Figure 5. We recognize the  $f_q$  as a cake with three candles of different color (R, G, B) and the base is white (W). Then we want to retrieve  $f_d$  containing candles and base that is twice as taller as  $f_q$ . Therefore, we apply type-21 to  $f_q$  and  $f_d$ , and we assign  $k_u^s = 1$ ,  $k_v^s = 2$ . Take pair objects (RG) for example,  $(S_{Ru}/S_{Ru'}=S_{Gu}/S_{Gu'}=1/k_u^s \Rightarrow 1/S_{Ru'} =$  $1/S_{Gu'}$ , drives  $S_{Ru'} = 1$  and  $S_{Gu'} = 1$ ). The size of object R on y-axis is 6 and object G is 4 because  $(S_{Rv}/S_{Rv'}=S_{Gv}/S_{Gv'}=1/k_v^s \Rightarrow 3/S_{Rv'}=$  $2/S_{Gv'}$ , drives  $S_{Rv'} = 6$  and  $S_{G'} = 4$ ). The calculation also conducts other pairs of objects. Finally, we retrieve  $f_1$  from database.

#### 5. Conclusions

In this paper, we propose two new types of similarity measure based on 2D C<sup>+</sup>-string to retrieve images more precise than the previous method. The improved method modifies the equations of original type-i1 and type-i2 simialrity measure and adds the parameter k to take down the ratios about relative sizes and distances for objects on x- and y-axis. The problem of distortion between two symbolic pictures can be aware if the ratios on x- and y-axis are quite different. Additional, the parameter k let user can assign the specific value about ratios for certain flexibility of pictorial query.

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