

USING ON-LINE TRANSMITTED DOMAIN BLOCKS ORDERED IN VARIANCE FOR NON-ITERATIVE FRACTAL IMAGE CODING

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ABSTRACT

Iteration in the decoding process of conventional fractal image compression not only leads to a high computation complexity but also requires a large size of memory. Moreover, the convergent criterion for the iterated image should be carefully designed. Otherwise, the iterated image will not reach a convergent state. To overcome these problems, we propose a non-iterative method based on a novel domain pool design for fractal image coding. In encoder, the domain pool is built by the use of the mean information in fractal code and is on-line transmitted to the decoder. The domain blocks are selected from the mean image according to the block variance. In computer simulation, we successfully speed up the decoding process when an image is partitioned into 8×8 range blocks. The coding performances of the test images are also well preserved. A better performance than conventional schemes is obtained when an image is partitioned into 4×4 range blocks. However, a competitive computation load is required.

1 INTRODUCTION

The fractal coding scheme is a new technique for image compression and has evolved greatly from the first version proposed by Jacquin [1], [2]. In conventional fractal coding schemes, an image is partitioned into non-overlapping range blocks. The larger domain blocks are selected from the same image and can overlap. A grayscale image is encoded by mapping the domain block D to the range block R with the contractive affine transformation [2]

$$\hat{R} = \iota\{\alpha \cdot (S \circ D) + \Delta g\}, \quad (1)$$

where $S \circ$ represents the contraction operation that maps a domain block to a range block. Then the parameters (called the *fractal code*) describing the contractive affine transformation that has the minimum matching error between the original range block R and

the coded range block \hat{R} are transmitted or stored. The fractal code consists of the contrast scaling α , luminance shift Δg or the block mean (the average pixel value of the range block) μ_R [3], isometry ι , and the position P_D of the best-match domain block in the domain pool. In the decoding stage, an arbitrary image is given as the initial image and the decoded image is repeatedly reconstructed by applying the contractive affine transformation to the iterated image. The iteration process will not stop until a predefined convergence criterion is satisfied. Obviously, the complicated iteration process in the decoding stage is a main drawback in fractal coding techniques.

There are many modified versions proposed to improve the fractal coding techniques. However, only a few fast algorithms [4]~[7] are proposed to speed up the decoding process. Those fast algorithms can only reduce the iteration number or perform non-iterative decoding in special cases. It is thus desired to propose a universal scheme that is non-iterative in the decoding stage for fractal image compression. Therefore, the decoding speed can be greatly improved. Furthermore, the limitations due to the iterations, for example, the contractivity, excessive computation burden, serial block processing, and a large size of memory, will be relaxed.

In this research, we propose a fractal coding scheme that is non-iterative in the decoding stage. Since the block mean can be one of the parameters in the fractal code in a modified contractive affine transformation [3], we can generate the domain pool by using the mean image whose pixel values are the block means of all the range blocks in the encoder. The decoder receives the on-line transmitted mean information and thus an identical domain pool can be reconstructed. We generate the domain blocks those are directly selected from the mean image and ordered in variance. The size of the domain block is the same as that of the range block. Therefore, the encoded and decoded results are exactly identical since we use the same domain blocks in both the encoder and decoder. Simulation results

show that the decoding speed is greatly improved for a large block size. The coding performances of the test images are also well preserved or even better than the conventional fractal coding schemes that require iterations.

The organization of this paper is as follows: We first introduce the conventional fractal coding scheme that uses iterative decoding process in Section 2. Section 3 describes the proposed non-iterative fractal image codec with on-line domain pool transmission. The computer simulation shown in Section 4 verifies the improvement of the proposed non-iterative scheme. Finally, a conclusion is given in Section 5.

2 CONVENTIONAL FRACTAL CODING SCHEME

The proposed non-iterative scheme is compared with the conventional fractal coding schemes that require iterations. In conventional contractive affine transformation, the contrast scaling is used to equalize the dynamic range of the range and the transformed domain blocks. It is usually no more than one to avoid the possible divergence in the iterative decoding process. On the other hand, the luminance shift forces the block mean of range block and the transformed domain block to be identical. The luminance shift can be replaced by the block mean [3] to obtain a good initial image in the decoding stage.

Two design methods of the domain pool in conventional fractal coding schemes are described as follows: Basically, the domain blocks are selected from the original image and the block size is four-time size of the range block. First, the domain pool consists of the domain blocks subsampled from the original image and it is denoted as the *subsampling* method. For an image of size $M \times M$, the sampling period T in both the horizontal and vertical directions is determined by

$$T = \left\lceil \frac{M - 2B}{\sqrt{N_D} - 1} \right\rceil, \quad T \geq 1, \quad (2)$$

where $2B \times 2B$ is the domain block size and N_D is the number of the domain blocks in the domain pool, and $\lceil \cdot \rceil$ denotes choosing a smaller and the closest integer of the real number in the bracket. Second, we choose the N_D domain blocks that are neighboring to the range block and it is denoted as the 'neighboring' method. Then the contractive affine transformation is used to find the fractal code for each range block.

In decoding stage, the initial image consists of the range blocks whose pixel values are equal to the corresponding block means. The decoded image is iteratively reconstructed with the contractive affine transformations denoted in the fractal codes. Since the initial image in the decoder is different from the original

image in the encoder, the domain blocks in the encoder are different from that found in the decoder. There exists a distortion between the coded image in the encoder and the decoded image in the decoder. To reduce this distortion, it is desirable to generate the same domain pool in both the encoder and decoder. Here the proposed non-iterative scheme can solve this problem.

The criterion for the decoded image to achieve a convergent state is determined as follows: Let the n th iterated image be denoted as $f^{(n)}$. The average error $e(n)$ between the n th and $(n-1)$ th decoded images is calculated by

$$e(n) = \frac{1}{512^2} \sum_{i=1}^{512} \sum_{j=1}^{512} (f_{i,j}^{(n)} - f_{i,j}^{(n-1)})^2, \quad (3)$$

where $f_{i,j}^{(n)}$ denotes the (i, j) th pixel in n th decoded image. If the ratio,

$$\gamma = \frac{|e(n) - e(n-1)|}{e(n-1)}, \quad (4)$$

is less than a threshold value γ_{th} , the decoded image converges and the iteration process stops. Otherwise, the iteration process will not stop until the convergence criterion

$$\gamma \leq \gamma_{th}, \quad (5)$$

is satisfied. The determination of the convergent criterion is an important issue in conventional fractal coding schemes. It should be carefully chosen such that we can obtain a good decoded image. If we choose a big γ_{th} , the decoded image does not reach stable yet when the iteration process stops. On the other hand, if the γ_{th} is very small, the iterated images can not satisfy the convergent criterion and the iteration process continues and will not be easy to stop.

3 NON-ITERATIVE FRACTAL IMAGE CODEC DESIGN

3.1 Encoder

The flow chart of the encoder in the proposed non-iterative scheme is shown in Figure 1. The input $M \times M$ image is partitioned into the non-overlapping range blocks of size $B \times B$. First of all, we sequentially measure the mean and variance of each range block. After all the means of the range blocks are obtained, we can generate a mean image of size $M/B \times M/B$ with each pixel corresponding to the block mean. If the variance of the range block,

$$\text{Var}\{R\} = \sum_{0 \leq i, j < B} (r_{i,j} - \mu_R)^2 \quad (6)$$

(where $r_{i,j}$ denotes the (i,j) th pixel in the range block) is smaller than the threshold value E_{th} , then the range block is coded by the block mean. Otherwise, the range block will be coded with the affine transformation. Note that in this case, the size of the mean image should be much larger than the size of the domain block, i.e., $M/B \times M/B \gg B \times B$. Otherwise, it will not be easy to find a good mapping between the domain and range blocks because only a few domain blocks can be taken from the mean image. The size of the domain block is the same as that of the range block and thus the contraction procedure in conventional fractal coding schemes is eliminated. We therefore proceed with a new contractive affine transformation between the range block and the domain block generated from the mean image.

There are $(M/B - B + 1) \times (M/B - B + 1)$ possible domain blocks in the mean image. The measurement of block variance for all possible domain blocks is performed at first. For each possible domain block, we measure its block variance,

$$\text{Var}\{D\} = \sum_{0 \leq i,j < B} (d_{i,j} - \mu_D)^2, \quad (7)$$

where $d_{i,j}$ denotes the (i,j) th pixel in the domain block and μ_D is the mean of domain block. Then all the domain blocks are ordered from the maximal block variance to the minimal one. We can arbitrary choose the size of the domain pool from the ordered domain blocks. The domain pool can be constructed by choosing the domain blocks with larger variance. For example, we construct a domain pool of size K by choosing the first K blocks whose variance is larger than other blocks. The range of K can be from 16 to 1024 (or bigger) in our design.

The parameters used in our new contractive affine transformation are specified as follows: The luminance shift is replaced by the block mean [3] which is coded by six bits. The contrast scaling is usually smaller than one to avoid the divergence caused from iterations in conventional fractal coding schemes. Although it is possible to use an extended range for the contrast scaling since no divergence problem is necessarily concerned with, we choose the range of contrast scaling to be adaptive because the variance of the selected domain blocks changes gradually. In our design, the contrast scaling is determined from the values in the set $\{n \times L/8, n=1, 2, 3, \dots, 8\}$, where L denotes the range of contrast scaling and

$$L = \begin{cases} 0.5, & \text{if } K=16, 32 \\ 1.0, & \text{if } K=64, 128, 256 \\ 1.25, & \text{if } K=512, 1024 \end{cases} \quad (8)$$

We thus need three bits to code the contrast scaling. On the other hand, the eight isometries for shuffling

the pixels in the block are the same as those in [2] and are coded with three bits.

The new contractive affine transformation can be expressed by

$$\hat{R} = \iota\{\alpha \cdot D + \mu_R - \alpha \cdot \mu_D\} = \iota\{\alpha \cdot (D - \mu_D) + \mu_R\}. \quad (9)$$

Note that in Equation 9 the contraction procedure is eliminated and the term $\mu_R - \alpha \cdot \mu_D$ is equivalent to the luminance shift shown in Equation 1. After testing all the combinations of the parameters in Equation 9, the fractal code is determined such that the coded block \hat{R} has the minimum distortion from the original range block R . The distortion between the original and coded range blocks is represented by the mean-squared-error (MSE) measurement defined as

$$\text{MSE}(R, \hat{R}) = \frac{1}{B^2} \sum_{0 < i,j \leq B} (r_{i,j} - \hat{r}_{i,j})^2, \quad (10)$$

where $\hat{r}_{i,j}$ denotes the (i,j) th pixel in the coded range block. We finally attach a header for each range block to denote its coding status (either coded by the mean or affine transformation). Therefore, the decoder can correctly reconstruct each range block according to the header.

3.2 Decoder

Figure 2 shows the flow chart of the decoder in the proposed non-iterative fractal coding scheme. We first receive all the fractal codes and determine whether or not the range block is coded by the mean from its header. At the same time, the mean image is reconstructed with the on-line transmitted mean information in the fractal codes. Note that this mean image is exactly identical to the mean image used in the encoder since both are constructed by the same block means. Therefore, the domain blocks generated from two identical mean images are also the same and thus the decoded image is the same as the coded image in the encoder. Here the same process is performed to obtain the same domain blocks. It requires some extra computation to measure the block variance and to order them by block variance.

If the block is coded by the mean, the value of each pixel in the decoded block is equal to the mean value. Otherwise, we perform the new contractive affine transformation to reconstruct the coded range block. The decoding process ends when the new contractive transformation is applied once only. At this point, no iterations are required and thus no convergence criterion and divergence problem for the decoded image to be concerned with. The decoding of the conventional fractal coding scheme requires two iteratively refreshed images in the iteration process. On

the contrary, only the fixed mean image that can be reconstructed from the received fractal codes is required in our non-iterative scheme.

4 COMPUTER SIMULATION

In computer simulation, two 512×512 images (Lena and Jetplane, shown in Figure 3(a) and (b)) with eight-bit grayscale resolution is used to test the proposed non-iterative fractal coding scheme. The performance of the decoded image quality is evaluated by the peak signal-to-noise-ratio (PSNR) and the bit rate (the required bits per pixel). In our simulation, an image is partitioned into range blocks with a single size, either 8×8 or 4×4 . Therefore, a general form for the PSNR of the decoded image is defined as

$$\text{PSNR} = 10 \log_{10} \frac{255^2}{\sum_{i=1}^{N_8} \text{MSE}(R_{8_i}, \hat{R}_{8_i}) + \sum_{i=1}^{N_4} \text{MSE}(R_{4_i}, \hat{R}_{4_i})} \text{ dB} \quad (11)$$

where N_8 and N_4 are the total numbers of the 8×8 range block R_8 and the 4×4 range block R_4 , respectively. On the other hand, the calculation of the bit rate will be given in the following contents.

For all the schemes used in our simulation, we set the threshold values E_{th} for the variance of 8×8 and 4×4 range blocks to be 1600 and 400, respectively. The size of the domain pool is represented by the number of domain blocks in it. There are seven sizes for the domain pool: 16, 32, 64, 128, 256, 511, and 1024 domain blocks used in our simulation. On the other hand, the sampling periods T in the conventional domain pool design is determined according to Equation 2.

In our simulation, the range block of size 8×8 or 4×4 is considered. The length of the attached header I_h to the fractal code for each range block is only one bit (*i.e.*, $I_h=1$) because it only denotes whether or not the range block is coded by the block mean. Therefore, the bit rate can be calculated by

$$B = \frac{(N_\mu + N_{af})(I_h + I_\mu) + N_{af}(I_\alpha + I_t + I_{P_D})}{512^2} \text{ bit/pixel}, \quad (12)$$

where I_μ, I_α, I_t , and I_{P_D} denote the required bits for the block mean, contrast scaling, isometry, and the position of the domain pool, respectively. In addition, N_μ and N_{af} denote the number of the block coded by the block mean and affine transform, respectively.

For an image partitioned into 8×8 range blocks, we measure every block mean and obtain a 64×64 mean image. Figure 4(a) shows that the mean image is very similar to the original Lena image. There are 57×57 possible domain blocks in this mean image. We calculate the block variance for each possible domain block and order them from the maximum to minimum. Therefore the domain pools of different sizes can be obtained from the ordered domain blocks. The image is

coded by using the contractive affine transformation shown in Equation 9. Tables 1 and 2 show the simulation results for Lena and Jetplane images. The numbers shown in the table represent the different sizes of the domain pool. The bit rates of all schemes while using the same size of the domain pool are the same. A smaller size for the domain pool leads to a lower bit rate and PSNR, and vice versa.

For the image partitioned into 4×4 range blocks, the 128×128 mean image for Lena is shown in Figure 4(b). Similarly, we construct the domain pools of different sizes from the ordered domain blocks. The simulation results for Lena and Jetplane images based on the domain pool of different sizes are also shown in Tables 1 and 2. The proposed non-iterative scheme has a competitive performance with the conventional fractal coding schemes. The PSNR of the decoded image partitioned by the 4×4 block size is much higher than that partitioned by the 8×8 block size since a smaller block size leads to a smaller matching error for the affine transformation. However, the bit rate increases significantly because the number of the 4×4 range blocks is four times the number of the 8×8 range blocks.

In Figure 5(a) and 5(b), we demonstrate two domain pools constructed from the domain blocks ordered in block variance. The size of the domain pool is 256. Obviously, no uniform blocks are included and thus it is an efficiently design. Figure 6(a)~(d) shows some graphic results of the decoded images in Table 1. These four images are based on the proposed non-iterative scheme which uses the block size 8×8 . As the size of the domain pool increases, the fidelity of the decoded image increases very much and can be better than that of the conventional fractal scheme.

Here we also list the computation time based on a PC with a Pentium 200 MMX CPU and 64M SDRAM for the proposed and conventional methods. The threshold value γ_{th} for the convergence criterion in the conventional fractal coding scheme is set by 0.005. Tables 3 and 4 show their CPU time (in seconds, the decoding program is not optimized) for decoding Lena and Jetplane images. As shown in both tables, the decoding time of the proposed non-iterative scheme is very close under different sizes of domain pool. However, the decoding time of conventional scheme varies very much. In some cases, the iteratively decoded image does not reach convergent since it can not satisfy the criterion shown in Equation 5. The proposed domain pool design for the non-iterative fractal coding scheme has no need to satisfy the convergent criterion. It greatly speeds up the decoding procedure for the image partitioned into 8×8 block size and the fidelity of the decoded image is well preserved. On the other hand, the decoding time for the image partitioned into 4×4 block size might be longer than that required in

conventional schemes. The additional decoding time causes from the variance measurement and ordering process of a large number of domain blocks. However, the PSNR increases as the size of domain pool increases.

The conventional fractal coding scheme takes more than eight iterations to achieve a convergent state for decoding the Lena image [2]. Usually, the iteration number is image dependent and so is the decoding time. Although there are some methods proposed to speed up the iteration process for the conventional fractal coding schemes, at least three or four iterations are required [5], [6] to reach the convergence criterion in the decoding of the Lena image. A fractal coding scheme proposed in Ref. [4] is non-iterative in some special cases. The proposed non-iterative scheme exactly accomplishes decoding process by applying contractive affine transformation only once.

The proposed non-iterative scheme can be applied to any block-based fractal coding scheme that contains mean information in the fractal codes. A higher compression ratio can be achieved by using a variable-size partition for the image. For example, by using the quadtree partition [9], the source image is partitioned into the range blocks of maximum size 32×32 and minimum size 4×4 , according their complexity. We can only encode the range blocks whose sizes are larger than 8×8 by the mean. The bit rate can be reduced although the image quality might be slightly degraded.

5 CONCLUSION

In this paper, we propose a novel fractal image coding scheme that is non-iterative in the decoding stage. The decoder receives the on-line transmitted domain pool and thus the non-contractive affine transformation can be applied only once. Without iterations, the decoding process becomes deterministic and the divergence problem does not exit. Simulation results show that we make an improvement on the decoding speed for a large block size. The coding performances of the test images are comparable with or even better than those in the conventional fractal coding schemes. We will try to find a more efficient method for the domain pool design such that we can reduce the extra computation required for the measurement of block variance and the ordering process to speed up the decoding process in our future research.

6 ACKNOWLEDGEMENT

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Block size	DP size		16	32	64	128	256	512	1024
8×8	PSNR (dB)	CS	26.93	28.57	28.97	29.69	30.18	30.72	30.85
		CN	28.76	29.03	29.37	29.56	29.86	30.19	30.36
		PV	28.10	28.55	28.82	29.24	29.78	30.49	30.91
	Bit rate (bpp)		0.190	0.199	0.207	0.215	0.223	0.231	0.239
4×4	PSNR (dB)	CS	32.31	33.18	33.86	34.24	34.59	34.91	35.51
		CN	33.31	33.80	34.30	34.62	34.98	35.37	35.68
		PV	33.01	33.41	34.11	34.71	35.29	35.88	36.29
	Bit rate (bpp)		0.661	0.684	0.706	0.729	0.751	0.773	0.796

Table 1: Coding performance of the Lena image by the use of the proposed and conventional methods. (DP denotes domain pool, CS denotes conventional subsampling scheme, CN denotes conventional neighboring scheme, and PV denotes the proposed non-iterative scheme with domain blocks ordered in variance.)

Block size	DP size		16	32	64	128	256	512	1024
8×8	PSNR (dB)	CS	25.36	26.11	26.59	27.28	27.87	28.57	29.03
		CN	25.68	25.98	26.26	26.50	26.81	27.21	27.64
		PV	25.20	25.71	26.52	27.25	27.53	28.11	29.49
	Bit rate (bpp)		0.192	0.199	0.207	0.214	0.222	0.229	0.236
4×4	PSNR (dB)	CS	29.90	30.60	30.94	31.28	31.45	31.86	32.31
		CN	28.88	29.73	30.98	31.43	32.62	33.49	34.25
		PN	30.48	31.56	32.84	34.05	34.63	35.21	35.72
	Bit rate (bpp)		0.661	0.684	0.706	0.729	0.751	0.773	0.796

Table 2: Coding performance of the Jetplane image by the use of the proposed and conventional methods.

Block size	DP size		16	32	64	128	256	512	1024
8×8	Time (seconds)	CS	7.15	7.94	8.08	7.24	N/A	N/A	N/A
		CN	13.25	38.86	13.41	15.89	7.03	10.88	14.64
		PV	4.68	4.68	4.67	4.71	4.66	4.64	4.66
4×4	Time (seconds)	CS	5.66	N/A	5.70	N/A	N/A	N/A	N/A
		CN	6.9	10.10	9.57	9.64	N/A	16.24	11.28
		PV	14.56	14.45	14.77	14.92	15.57	15.18	15.44

Table 3: Decoding time of the conventional and proposed methods for the Lena image. (N/A means not available. The decoding process can not reach the defined convergent state.)

Block size	DP size		16	32	64	128	256	512	1024
8×8	Time (seconds)	CS	6.81	12.80	N/A	10.64	12.89	19.81	N/A
		CN	6.55	10.11	8.98	11.92	7.80	34.01	38.11
		PV	4.04	4.04	4.02	4.06	4.01	4.05	4.06
4×4	Time (seconds)	CS	4.32	N/A	7.02	11.23	23.42	N/A	11.89
		CN	9.48	6.3	7.40	17.09	19.77	13.07	23.13
		PV	15.96	16.60	16.32	15.95	16.10	16.19	15.99

Table 4: Decoding time of the conventional and proposed methods for the Jetplane image.

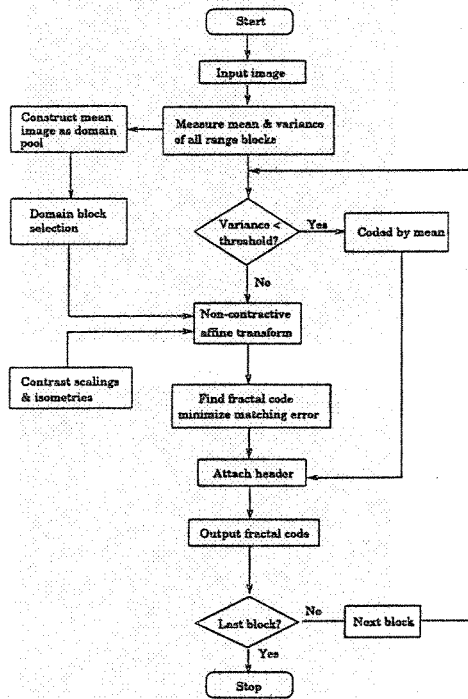


Figure 1: The flow chart of the encoder in the proposed non-iterative fractal coding scheme.



(a)



(b)

Figure 3: The original images (512×512, 8 bit/pixel) used to test the proposed non-iterative fractal coding scheme: (a) Lena, and (b) Jetplane.

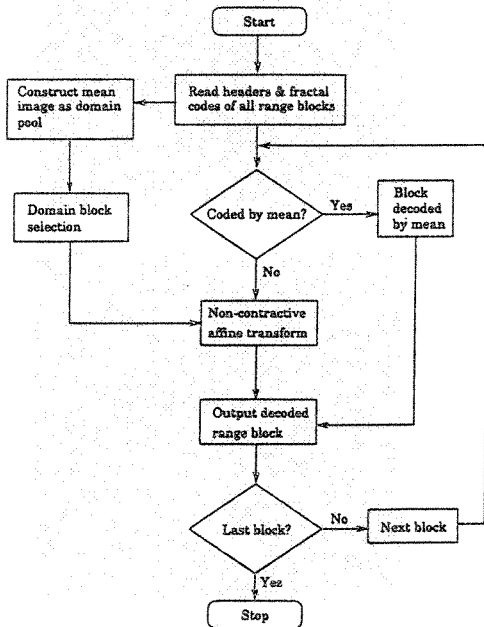


Figure 2: The flow chart of the decoder in the proposed non-iterative fractal coding scheme.



(a)



(b)

Figure 4: Two mean images of size (a) 64×64 and (b) 128×128 for the Lena image.

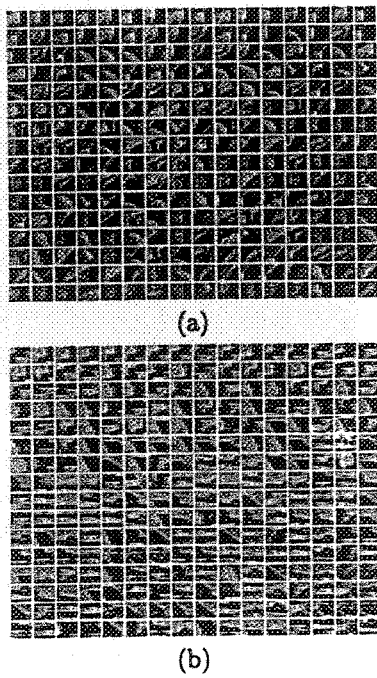


Figure 5: The domain pools (consist of 256 8×8 domain blocks) built from the mean image for (a) Lena and (b) Jetplane by the use of the proposed ordered-in-variance method.

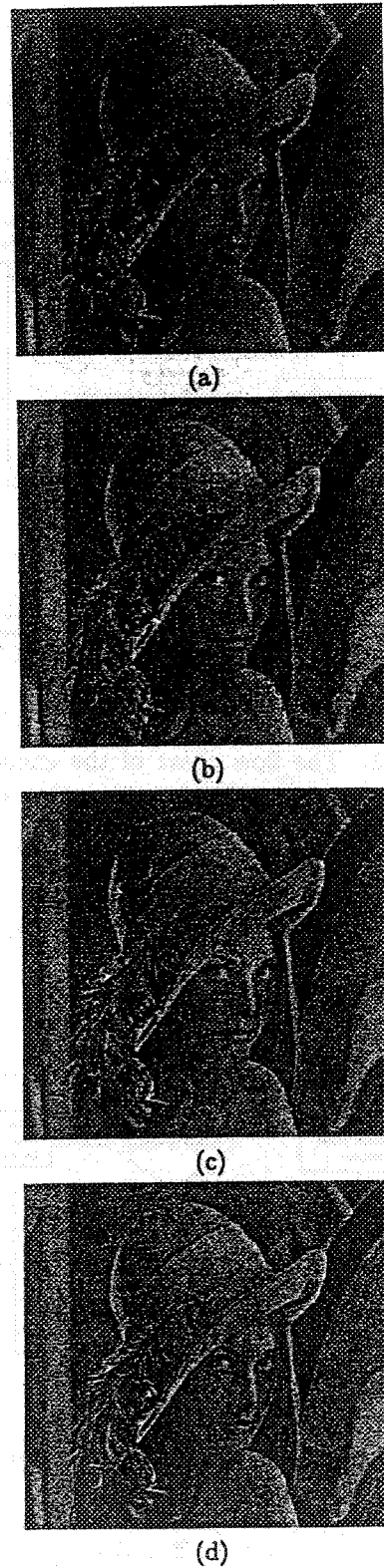


Figure 6: The decoded Lena images based on the proposed non-iterative scheme with different domain pool sizes: (a) 16, (b) 64, (c) 256, and (d) 1024. (Range block size: 8×8)