Maximum-Revenue Multicast Routing and Partial Admission Control for Multimedia Distribution

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Abstract- *In this paper, we intend to solve the problem of maximum-revenue multicast routing with a partial admission control mechanism. Specifically, for a given network topology, a given link capacity, the destinations of a multicast group, and the bandwidth requirement of each multicast group, we attempt to find a feasible routing solution to maximize the revenue of the multicast trees. The partial admission control mechanism means that the admission policy of the multicast group will not be based on a traditional "all or none" strategy. Instead it considers accepting of partial portions of destinations for the requested multicast group. Firstly, we model this problem as an optimization problem. Then, we propose a simple heuristic algorithm and an optimization based heuristic to solve this problem. The methodology taken for solving the problem is Lagrangean relaxation. Computational experiments have been performed on regular networks, random networks, and scale-free networks.*

Keywords: Admission Control, Routing Assignment, Multicast Service, Lagrangean Relaxation.

1. Introduction

With the popularity of the Internet, applications based on network service are growing rapidly. In order to support the advanced applications such as e-learning and video conference, it will be necessary for the service delivery infrastructure to provide multimedia services and multicast data delivery within guaranteed bounds of Quality-of-Service (QoS). Multimedia application environments are characterized by large bandwidth variations due to the heterogeneous access technologies of networks and different receivers' quality requirements, which make it difficult to achieve bandwidth efficiency and service flexibility. There are many challenging issues that need to be addressed in designing architectures and mechanisms for multicast data transmission [1].

In order to meet the requirements for multimedia distribution, network operators invest more and more capital to enlarge their network capacity. In addition to enlarging the network capacity, there is still one way to achieve the goal of revenue maximization, namely: network planning or traffic engineering. Traffic engineering is the process of controlling how traffic flows through a network in order to optimize resource utilization and network performance. At the same time, it can provide QoS. The goal of QoS routing is to select the network routes with sufficient resources for the requested QoS parameters, to satisfy the QoS requirements for every admitted connection, as well as to achieve global efficiency in resource utilization. Admission control is often considered a by-product of QoS routing and resource reservation. If the latter is successfully performed along the route(s) selected by the routing algorithm, the connection request is accepted; otherwise, it is rejected. It is clear from the above introduction to know that in order to consider the QoS assurance issue, the three closely-related mechanisms of admission control, routing and resource reservation should be treated jointly.

In this paper, we jointly considering the above three mechanisms and intend to solve the problem of maximum-revenue multicast routing with a partial admission control mechanism. The partial admission control mechanism means that the admission policy of the multicast group will not be based on a traditional "all or none" strategy. Instead it considers accepting of partial portions of destinations for the requested multicast group. More specifically, for a given network topology, a given link capacity, the destinations of a multicast group, and the bandwidth requirement of each multicast group, we attempt to find a feasible admission decision and routing solution to maximize the revenue of the multicast trees. Firstly, we model this problem as a linear optimization problem. Then, we propose a simple heuristic algorithm and an optimization based heuristic to solve this problem. The methodology taken for solving the problem is Lagrangean relaxation. Computational experiments have been performed on regular networks, random networks, and scale-free networks.

The rest of this paper is organized as follows. In Section 2, we formally define the problem being studied, as well as a mathematical formulation of max-revenue optimization is proposed. Section 3 applies Lagrangean relaxation as a solution approach to the problem. Section 4, illustrates the computational experiments. Finally, in Section 5 we present our conclusions and the direction of future research.

2. Problem Formulation

The network is modeled as a graph where the switches are depicted as nodes and the links are depicted as arcs. A user group, which has one source and one or more destinations, is an application requesting transmission on this network. Given the network topology, the capacity of links and the bandwidth requirement of user groups, we want to jointly determine the following decision variables: (1) the routing assignment (a tree for multicasting, or path for unicasting) of each admitted destination; and (2) the admitted number of destinations of each partially admitted multicast group. We assume that the multicasting is single-rate.

By formulating the problem as a mathematical programming problem, we intend to solve it optimally to obtain a network that fits into our goal, i.e., ensures the network operator can earn maximum revenue from servicing the partially admitted destinations.

This model is based on the following viable assumptions.

- The revenue from each partially admitted group can be fully characterized by two parameters: the entire admitted revenue of the group and the number of admitted destinations.
- The revenue from each partially admitted group is a monotonically increasing function with respect to the number of admitted destinations.
- The revenue function from each partially admitted group is a concave function with respect to the entire admitted revenue of the group and the number of admitted destinations. However, the entire admitted revenue and the number of admitted destinations jointly may not be a concave function.
- The revenue from each partially admitted group is independent.

The notations used to model the problem are listed in Table 1.

Table 1. Description of notations

Given Parameters	
Notation	**Descriptions**
F_g	Revenue generated from admitting partial users of multicast group g, which is a function of f_g and a_g
a_g	Revenue generated from admitting multicast group g
α_g	Traffic requirement of multicast group g
G	The set of all multicast groups
V	The set of nodes in the network
L	The set of links in the network
D_g	The set of destinations of multicast group g
C_l	Capacity of link l
I_v	The incoming links to node v
r_g	The multicast root of multicast group g
I_{r_g}	The incoming links to node r_g
P_{gd}	The set of paths user d of multicast group g may use
δ_{pl}	The indicator function which is 1 if link l is on path p and 0 otherwise
Decision Variables	
Notation	**Descriptions**
x_{gpd}	1 if path p is selected for group g destined for destination d and 0 otherwise
y_{gl}	1 if link l is on the subtree adopted by multicast group g and 0 otherwise
f_g	The number of admitted destinations in multicast group g

According to the description in previous, the max-revenue problem is formulated as a combinatorial optimization problem in which the objective function is to maximize revenue from servicing the partially admitted destinations. Of course a number of constraints must be satisfied.

Optimization Problem:

Objective function:

$$\min \; -\sum_{g\in G} F_g(a_g, f_g) \qquad \text{(IP)}$$

subject to:

$$\sum_{g\in G}\alpha_g y_{gl} \le C_l \qquad \forall l\in L \qquad (1)$$

$$\sum_{l\in L} y_{gl} \ge \sum_{d\in D_g}\sum_{p\in P_{gd}} x_{gpd} \qquad \forall g\in G \qquad (2)$$

$$\sum_{d\in D_g}\sum_{p\in P_{gd}} x_{gpd}\delta_{pl} \le |D_g| y_{gl} \qquad \forall g\in G, l\in L \qquad (3)$$

$$\sum_{l\in I_v} y_{gl} \le 1 \qquad \forall g\in G, v\in V-\{r_g\} \qquad (4)$$

$$\sum_{l\in I_{r_g}} y_{gl} = 0 \qquad \forall g\in G \qquad (5)$$

$$y_{gl} = 0 \text{ or } 1 \qquad \forall l\in L, g\in G \qquad (6)$$

$$x_{gpd} = 0 \text{ or } 1 \qquad \forall g\in G, p\in P_{gd}, d\in D_g \qquad (7)$$

$$\sum_{p\in P_{gd}} x_{gpd} \le 1 \qquad \forall d\in D_g, g\in G \qquad (8)$$

$$\sum_{d\in D_g}\sum_{p\in p_{gd}} x_{gpd} = f_g \qquad \forall g\in G \qquad (9)$$

$$f_g \in \{0,1,2,......,|D_g|\} \qquad \forall g\in G \qquad (10)$$

The objective function of (IP) is to maximize the total revenue of servicing the partially admitted destinations in multicast groups g, where $g\in G$ and G is the set of user groups requesting transmission. F_g reflects the priority of partial users belonging to group g, while different choices of F_g may provide different physical meanings of the objective function. For example, if F_g is chosen to be the mean traffic requirement of partial users belonging to group g, then the objective function is to maximize the total system throughput. On the other hand, if F_g is chosen to be the earnings of servicing partial users belonging to group g, then the objective function is to maximize the total system revenue. In general, if user group g is to be given a higher priority, then the corresponding F_g may be assigned a larger value.

Constraint (1) is the capacity constraint, which requires that the aggregate flow on each link l does not exceed its physical capacity C_l. Constraint (2) requires that if one path is selected for group g destined for destination d, it must also be on the subtree adopted by multicast group g. Constraint (3) is the tree constraint, which requires that the union of the selected paths for the destinations of user group g forms a tree. Constraints (4) and (6) require that the number of selected incoming links y_{gl} is 1 or 0 and each node, excepting the root, has only one incoming link. Constraint (5) requires that there is no selected incoming link y_{gl} that is the root of multicast group g. As a result, the links we select can form a tree. Constraints (7) and (8) require that at most one path is selected for each admitted multicast source-destination pair, while Constraint (9) relates the routing decision variables x_{gpd} to the auxiliary variables f_g. The introduction of the auxiliary variables f_g may facilitate the decomposition in the Lagrangean relaxation problem to be discussed later. Constraint (10) requires that the number of admitted destinations in multicast group g is the set of integers.

3. Solution Procedure

3.1 Lagrangean relaxation

By using the Lagrangean Relaxation method [2][3], we can transform the primal problem (IP) into the following Lagrangean Relaxation problem (LR) where constraints (1), (2), (3), and (9) are relaxed.

For a vector of Lagrangean multipliers, a Lagrangean Relaxation problem of (IP) is given by

Optimization problem (LR):

$$Z_D(\beta,\lambda,\theta,\varepsilon)=$$

$$\begin{aligned}\min\ & -\sum_{g\in G} F_g(a_g,f_g)+\sum_{g\in G}\sum_{l\in L}\beta_l\alpha_g y_{gl} \\ & -\sum_{l\in L}\beta_l C_l+\sum_{g\in G}\sum_{l\in L}\lambda_g y_{gl}-\sum_{g\in G}\sum_{d\in D_g}\sum_{p\in P_{gd}}\lambda_g x_{gpd} \\ & +\sum_{g\in G}\sum_{l\in L}\sum_{d\in D_g}\sum_{p\in P_{gd}}\theta_{gl}x_{gpd}\delta_{pl} \\ & -\sum_{g\in G}\sum_{l\in L}\theta_{gl}\left|D_g\right|y_{gl} \\ & +\sum_{g\in G}\sum_{d\in D_g}\sum_{p\in P_{gd}}\varepsilon_g x_{gpd}-\sum_{g\in G}\varepsilon_g f_g\end{aligned}$$

subject to: (4)(5)(6)(7)(8)(10).

Where β_l, λ_g, θ_{gl} and ε_g are Lagrangean multipliers and $\beta_l,\theta_{gl}\geq 0$. To solve (LR), we can decompose (LR) into the following five independent and easily solvable optimization subproblems.

Subproblem 1: (related to decision variable x_{gpd})

$$Z_{Sub1}(\lambda,\theta,\varepsilon)=$$
$$\min\ \sum_{g\in G}\sum_{d\in D_g}\sum_{p\in P_{gd}}(\varepsilon_g-\lambda_g+\sum_{l\in L}\theta_{gl}\delta_{pl})x_{gpd}$$

subject to: (7)(8).

The Subproblem 1 is to determine x_{gpd} and it can be further decomposed into $|G||D_g|$ independent shortest path problems with nonnegative arc weights θ_{gl}. Each shortest path problem can be easily solved by Dijkstra's algorithm.

Subproblem 2: (related to decision variable y_{gl})

$$Z_{Sub2}(\beta,\lambda,\theta)=$$
$$\min\ \sum_{g\in G}\sum_{l\in L}(\beta_l\alpha_g+\lambda_g-\theta_{gl}\left|D_g\right|)y_{gl}$$

subject to: (4)(5)(6).

The Subproblem 2 can be decomposed into |G| independent problems. For each multicast group $g\in G$:

$$Z_{Sub2.1}(\beta,\lambda,\theta)=$$
$$\min\ \sum_{l\in L}(\beta_l\alpha_g+\lambda_g-\theta_{gl}\left|D_g\right|)y_{gl}$$

subject to: (4)(5)(6).

The algorithm to solve to Subproblem 2.1 is stated as follows:

1. Compute the coefficient $\beta_l\alpha_g+\lambda_g-\theta_{gl}|D_g|$ for all links in the multicast group g.
2. Sort the links in descending order according to the coefficient.
3. According to the order and complying with constraints (4) and (5), if the coefficient is less than zero, assigns the corresponding negative coefficient of y_{gl} to 1; otherwise 0.

Subproblem 3: (related to decision variable f_g)

$$Z_{Sub3}(\varepsilon)=\min\ -\sum_{g\in G}(F_g(a_g,f_g)+\varepsilon_g f_g)$$

subject to: (10).

We can easily solve Subproblem 3 optimally by exhaustively searching from the known set of f_g.

According to the weak Lagrangean duality theorem [4], for any $\beta_l,\theta_{gl}\geq 0$, $Z_D(\beta_l, \lambda_g, \theta_{gl}, \varepsilon_g)$ is a lower bound on Z_{IP}. The following dual problem (D) is then constructed to calculate the tightest lower bound.

Dual Problem (D):

$$Z_D=\max Z_D(\beta_l,\lambda_g,\theta_{gl},\varepsilon_g)$$

subject to: $\beta_l,\theta_{gl}\geq 0$.

There are several methods for solving the dual problem (D). The most popular is the subgradient method [5], which is employed here. Let a vector s be a subgradient of $Z_D(\beta_l, \lambda_g, \theta_{gl}, \varepsilon_g)$. Then, in iteration k of the subgradient optimization procedure, the multiplier vector is updated by $\omega^{k+1}=\omega^k+t^k s^k$. The step size t^k is determined by $t^k=\delta(Z^h_{IP} - Z_D(\omega^k))/||s^k||^2$. Z^h_{IP} is the primal objective function value for a heuristic solution (an upper bound on Z_{IP}). δ is a constant and $0 < \delta \leq 2$.

3.2 Getting primal feasible solutions

After optimally solving the Lagrangean dual problem, we get a set of decision variables. However, this solution would not be a feasible one for the primal problem since some of constraints are not satisfied. Thus, minor modification of decision variables, or the hints of multipliers must be taken, to obtain the primal feasible solution of problem (IP). Generally speaking, the best primal feasible solution is an upper bound (UB) of the problem (IP), while the Lagrangean dual problem solution guarantees the lower bound (LB) of problem (IP). Iteratively, by solving the Lagrangean dual problem and getting the primal feasible solution, we get the LB and UB, respectively. So, the gap between UB and LB, computed by (UB-LB)/LB*100%, illustrates the optimality of problem solution. The smaller gap computed, the better the optimality.

To calculate the primal feasible solution of the maximum revenue tree, the solutions to the Lagrangean relaxation problems are considered. The set of $\{x_{gpd}\}$ obtained by solving Subproblem 1 may not be a valid solution to problem (IP) because the capacity constraint is relaxed. However, the capacity constraint may be a valid solution for some links. The set of $\{y_{gl}\}$obtained by solving Subproblem 2 may not be a valid solution because of the link capacity constraint and the union of $\{y_{gl}\}$ may not be a tree. Also, because the constraint (9) is released, the set of $\{f_g\}$ obtained by solving Subproblem 3 may not be a valid solution.

Here we propose a comprehensive, two-part method to obtain a primal feasible solution. It utilized a Lagrangean multipliers based heuristic, followed by adjustment procedures. While solving the Lagrangean relaxation dual problem, we may get some multipliers related to each OD pair and links. According to the information, we can make our routing more efficient. We describe the Lagrangean based heuristic below.

[Lagrangean Multipliers based heuristic]

Step 1 Use $\beta_l\alpha_g+\lambda_g-\theta_{gl}|D_g|$ as link l's arc weight and run the T-M heuristic [6] to get a spanning tree for each multicast group.

Step 2 **Drop procedures**:

2.1 Check the capacity constraint of each link. If there is a link violate the capacity constraint, go to Step 2.2, otherwise Step 3.

2.2 Sort the links in descending order according to $\{C_l$ – the aggregate flow on the link$\}$. Choose the maximal overflow link and drop the group with the maximal subgradient $(-F_g(a_g,f_g)-\varepsilon_g f_g)$. Go to Step 2.1.

Step 3 **Add procedures:**

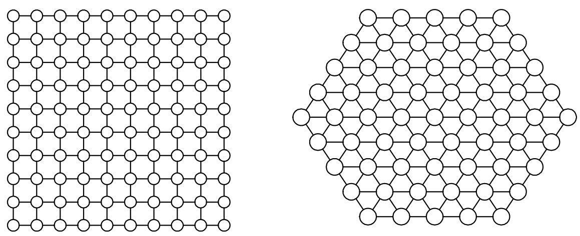
3.1 Sort the dropped group in ascending order according to the subgradient $(-F_g(a_g,f_g)-\varepsilon_g f_g)$.

3.2 In accordance with the order, re-add the groups to the network. Use $\beta_l\alpha_g+\lambda_g-\theta_{gl}|D_g|$ as link l's arc weight, removes the overflow links from the graph and run the T-M heuristic. If it can not find a route for the destinations, drop the destinations.

4. Computational Experiments

In this section, computational experiments on the Lagrangean relaxation based heuristic and other primal heuristics are reported. The heuristics are tested on three kinds of networks - regular networks, random networks, and scale-free networks. Regular networks are characterized by low clustering and high network diameter, and random networks are characterized by low clustering and low diameter. The scale-free networks, which are power-law networks, are characterized by high clustering and low diameter. Reference [7] shows that the topology of the Internet is characterized by power laws distribution. The power laws describe concisely skewed distributions of graph properties such as the node degree.

Two regular networks shown in Figure 1 are tested in our experiment. The first one is a grid network that contains 100 nodes and 180 links, and the second is a cellular network containing 61 nodes and 156 links.



(a) Grid Network (b) Cellular Network

Figure 1: Regular Networks

Random networks tested in this paper are generated randomly, each having 100 nodes. The candidate links between all node pairs are given a probability following the uniform distribution. In the experiments, we link the node pair with a probability smaller than 2%. If the generated network is not a connected network, we generate a new network.

Reference [8] shows that the scale-free networks can arise from a simple dynamic model that combines incremental growth with a preference for new nodes to connect to existing ones that are already well connected. In our experiments, we applied this *preferential attachment method* to generate the scale-free networks. The corresponding preferential variable (m_0, m) is (2, 2). The number of nodes in the testing networks is 100.

In order to prove that our heuristics are good enough, we also implement a simple algorithm to compare with our heuristic.

[Simple Algorithm]

Step 1 Set link l's arc weight to 1 and run the T-M heuristic to get a spanning tree for each multicast group.

Step 2 **Drop procedures**:

2.1 Check the capacity constraint of each link. If there is a link violate the capacity constraint, go to Step 2.2, otherwise Step 3.

2.2 Sort the links in descending order according to {C_l – the aggregate flow on the link}. Choose the maximal overflow link and drop the group with the minimal revenue. Go to Step 2.1.

Step 3 **Add procedures:**

3.1 Sort the dropped group in descending order according to the unit revenue {Group revenue/number of destination of the group}.

3.2 In accordance with the order, re-add the groups to the network. Remove the overflow links from the graph, set each link's arc weight to the aggregate flow of the link and run the T-M heuristic. If it can not find a feasible route for the destinations, drop the destinations.

For each testing network, several distinct cases, which have different pre-determined parameters such as the link capacity, the number of multicast group and the number of nodes in a group, are considered. The traffic demands for each multicast group are drawn from a random variable uniformly distributed in pre-specified categories {1, 2, 5, 10, 15, 20}. We conducted 120 experiments for each kind of network. For each experiment, the result was determined by the group source and destinations generated randomly. Table 3 summaries the selected results of the computational experiments.

For each testing network, the maximum improvement ratio between the simple heuristic and the Lagrangean based heuristic is 186.46 %, 93.37%, 137.08 %, and 139.17%, respectively. In general, the Lagrangean based heuristic performs well compared to the simple heuristic. We also find that in more congested network, either with more destinations or with less link capacity, the Lagrangean based heuristic outperforms the simple heuristic such as the case D of grid network and case F of scale-free network.

There are two main reasons of which the Lagrangean based heuristic works better than the simple algorithm. First, the Lagrangean based heuristic makes use of the related Lagrangean multipliers which include the potential cost for routing on each link in the topology. Second, the Lagrangean based heuristic is iteration-based and is guaranteed to improve the solution quality iteration by iteration. Therefore, in a more complicated testing environment, the improvement ratio is higher.

To claim optimality, we also depict the percentile of gap in Table 3. The results show that most of the cases have a gap of less than 40%. We also found that the simple heuristic performs well in many cases, such as the case I of grid network and case G of random network.

5. Conclusions

In this paper, we attempt to solve the problem of capacitated max-revenue multicast routing and partial admission control for multimedia distribution. Our achievement of this paper can be expressed in terms of mathematical formulation and experiment performance. In terms of formulation, we propose a precise mathematical expression to model this problem well. In terms of performance, the proposed Lagrangean based heuristic outperforms the simple heuristics. Our model can be easily extended to deal with the constrained multicast routing and admission control problem for multi-layered multimedia distribution by modifying some constraints. These issues will be addressed in future works.

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Table 3. Selected results of computational experiments

CASE	Cap.	G #	N #	SA	UB	LB	GAP	Imp.
Grid Network								**Max Imp. Ratio: 186.46 %**
A	20	20	20	-1777.01	-1998.12	-2400	16.75%	12.44%
B	20	20	50	-2010.87	-3536.48	-5274.67	32.95%	75.87%
C	20	50	20	-3052.47	-3731.75	-5918.61	36.95%	22.25%
D	20	50	50	-1998.79	-5725.72	-8123.47	29.52%	186.46%
E	20	100	20	-3744.71	-5859.17	-9232.08	36.53%	56.47%
F	20	100	50	-5844.29	-9574.34	-14114.3	32.17%	63.82%
G	40	20	20	-2170.69	-2208.75	-2322.6	4.90%	1.75%
H	40	20	50	-3854.31	-4105.33	-5450	24.67%	6.51%
I	40	50	20	-3613.02	-3636.07	-5086.36	28.51%	0.64%
J	40	50	50	-5382.09	-6862.85	-10767.9	36.27%	27.51%
K	40	100	20	-6118.06	-6506.32	-11033.3	41.03%	6.35%
L	40	100	50	-10594.3	-14500.4	-20074.9	27.77%	36.87%
Cellular Network								**Max Imp. Ratio: 93.37 %**
A	20	20	20	-1531.19	-1748.98	-2340	25.26%	14.22%
B	20	20	50	-4686.17	-5394.88	-5600.02	3.66%	15.12%
C	20	50	20	-4212.02	-4407.76	-5813.74	24.18%	4.65%
D	20	50	50	-4262.02	-8241.3	-9765.03	15.60%	93.37%
E	20	100	20	-4620.5	-6083.93	-8604.21	29.29%	31.67%
F	20	100	50	-7117.66	-12337.8	-14587.2	15.42%	73.34%
G	40	20	20	-2031.8	-2040	-2044.53	0.22%	0.40%
H	40	20	50	-4329.65	-4529.65	-4900	7.56%	4.62%
I	40	50	20	-5244.04	-5352.35	-5840	8.35%	2.07%
J	40	50	50	-8684.42	-9577.18	-11413.8	16.09%	10.28%
K	40	100	20	-7301.44	-7538.01	-11184.8	32.61%	3.24%
L	40	100	50	-13701.3	-17705.2	-20706.6	14.49%	29.22%
Random Networks								**Max Imp. Ratio: 137.08 %**
A	20	20	20	-1799.28	-1945.48	-2060	5.56%	8.13%
B	20	20	50	-4161.85	-4609.93	-4750	2.95%	10.77%
C	20	50	20	-4204.04	-4541.83	-5460	16.82%	8.03%
D	20	50	50	-5168.37	-7950.94	-11279.4	29.51%	53.84%
E	20	100	20	-4323.71	-4979.78	-9704.55	48.69%	15.17%
F	20	100	50	-5050.87	-11974.8	-18540.9	35.41%	137.08%
G	40	20	20	-2033.63	-2044.82	-2123.08	3.69%	0.55%
H	40	20	50	-5153.08	-5239.75	-5450	3.86%	1.68%
I	40	50	20	-6155.9	-6160	-6175.29	0.25%	0.07%
J	40	50	50	-12539.6	-12676.2	-16000	20.77%	1.09%
K	40	100	20	-5811.08	-5962.94	-10734.4	44.45%	2.61%
L	40	100	50	-11940.5	-15569	-23335.7	33.28%	30.39%
Scalefree Network								**Max Imp. Ratio: 139.17 %**
A	20	20	20	-1969.75	-2117.51	-2580	17.93%	7.50%
B	20	20	50	-2997.46	-3343.65	-4892.02	31.65%	11.55%
C	20	50	20	-2933.91	-3426.09	-5429.09	36.89%	16.78%
D	20	50	50	-4588.44	-7384.51	-10542.8	29.96%	60.94%
E	20	100	20	-2908.92	-4809.64	-8109.17	40.69%	65.34%
F	20	100	50	-4146.94	-9918.12	-14771.3	32.86%	139.17%
G	40	20	20	-2184.54	-2216.01	-2237.26	0.95%	1.44%
H	40	20	50	-3980.47	-4096.46	-4857.93	15.67%	2.91%
I	40	50	20	-4062.27	-4171.08	-5440	23.33%	2.68%
J	40	50	50	-7237.48	-9053.87	-12152.2	25.50%	25.10%
K	40	100	20	-5421.27	-6266.76	-10723.7	41.56%	15.60%
L	40	100	50	-9482.01	-14139.5	-19914.6	29.00%	49.12%

Cap.: The capacity of each link
G#: The number of multicast group
N#: The number of destinations in each multicast group
SA: The result of the simple algorithm
UB: Upper bounds of the Lagrangean based heuristic
LB: Lower bounds of the Lagrangean based heuristic
GAP: The error gap of the Lagrangean relaxation
Imp.: The improvement ratio of the Lagrangean based heuristic