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On the Complexity of Power-Aware Multiple Routing for Mobile Ad Hoc Networks*

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Abstract*-Most of the existing routing protocols for mobile ad hoc networks are designed primarily to carry best-effort traffic and only concerned with shortest-path routing. Little attention is paid to the issues related to the quality-of-service (QoS) requirement of a route. In this paper, we will consider the problem of searching for multiple paths between multiple pairs of sources and destinations satisfying the power requirement in a mobile ad hoc network. We will show that the problem is NP-complete.*

Keywords: Ad Hoc Network, Multiple Routing, NP-completeness, Power-Aware, Quality-of-Service, Wireless Mobile Network.

1. Introduction

A mobile ad hoc network (MANET) is formed by a group of mobile hosts (or called mobile nodes) without an infrastructure consisting of a set of fixed base stations. A mobile host in a MANET can act as both a general host and a router, i.e., it can generate as well as forward packets. Two mobile hosts in such a network can communicate directly with each other through a single-hop route in the shared wireless media if their positions are close enough. Otherwise, they need a multi-hop route to finish their communications. In a multi-hop route, the packets sent by a source are relayed by several intermediate hosts before reaching their destination. MANETs are found in applications such as short-term activities, battlefield communications, disaster relief situations, and so on. Undoubtedly, MANETs play a critical role in environment where a wired infrastructure is neither available nor easy to establish.

The research of MANETs has attracted a lot of attentions recently. In particular, since MANETs are characterized by their fast changing topologies, extensive research efforts have been devoted to the design of routing protocols for MANETs. At present, a number of efficient routing protocols have been developed. The Ad-hoc On-Demand Distance Vector routing protocol (AODV) [1], the Dynamic Source Routing (DSR) [2], and the Temporary-Ordered Routing Algorithm (TORA) [3] are among them. Most routing protocols have demonstrated their abilities to transmit connectionless data packets efficiently through the MANET. To be more specific, most of the existing routing protocols are designed primarily to carry best-effort traffic and only concerned with shortest-path routing. On the other hand, since the operations of most mobile hosts in a MANET today mainly rely on battery power, power consumption becomes an important issue. Many power-aware routing protocols have been designed to evenly distribute packet-relaying loads to each node to prevent the battery power of any node from being overused or abused [4-7]. However, most of the existing power-aware routing protocols are concerned with primarily minimizing the power consumption of the entire MANET (implying maximizing the lifetime of MANETs). Little attention is paid to the issues related to the QoS requirement of a route, i.e., to provide guaranteed battery power for the transmission of packets along a path from a source to a destination such that any node in the path does not run out of its power during the transmission of packets. Only recently have people turned their attentions to establishing routes with QoS given in terms of power [8].

In [8], the authors have proposed a routing algorithm that provides guaranteed power for the transmission of packets along a path from a source to a destination. They observed that each node along the path must have enough power for the transmission and reception of the required packets. In their routing algorithm, all those links along

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which no transmission can take place are removed. In other words, firstly, the routing request is infeasible (no transmission can take place) when the current residual power capacity of destination is less than the required reception power. Secondly, each outgoing link from the source is removed when its transmission power consumption is greater than the current residual power capacity of source. Thirdly, if one of the remaining nodes connected to the source is selected then it must relay packets (it must receive them and transmit them). Thus, the node is removed from the network if its current residual power capacity is less than the required reception power. Finally, for any relay node left in the network, all its outgoing links are removed whose transmission power consumption is larger than the residual power capacity of the relay node minus the required reception power. As a result, all the paths in the reduced network can now be used for packet transmission. In summary, the presented routing algorithm in [8] is indeed a unicast routing algorithm with power considerations for MANETs. It can guarantee the transmission of a given amount of packets along a single path in the MANET that can be supported by the residual power capacity of each node along the path.

In a realistic MANET, it is a more common situation that multiple sessions are in progress at the same time. That is, a lot of sources want to communicate with their individual destinations simultaneously. Thus, in this paper, we will consider the problem that given multiple pairs of sources and destinations in a MANET, find a route between each pair of source and destination satisfying the power requirements, namely, guaranteeing that the transmission of data packets during a session can be supported by the residual power capacity of each node along the route. We will call the problem *the power-aware multiple routing (PAMR) problem*. In this paper we will prove that the PAMR problem is an NP-complete problem.

The rest of the paper is organized as follows. In Section 2, the formal definition of the PAMR problem is given. In Section 3, the PAMR problem is shown to be an NP-complete problem. Section 4 concludes the whole research.

2. The Definition of the PAMR Problem

2.1 Assumptions

Power Model

In the following, we assume that the MANET's topology would not change, i.e. no node gets move. The power required by each mobile host can be classified into two categories: communication-related power and non-communication-related power [7]. The former can be further divided into two parts, namely: processing power and transceiver power. Each mobile host spends some processing power to execute network algorithms and run applications. Transceiver power refers to the power used by the radio transceiver to communicate with the other mobile hosts (namely, to transmit packets to and receive packets from others). In the following, for simplicity, we only consider transceiver power and assume that each node is able to estimate its current residual power capacity.

To provide sufficient power for the transmission of packets along a path from a source to a destination such that each node in the path does not run out of its residual power while the data transmission takes place, the required amount of power needs to be reserved along the path. Before the power can be reserved, the transmission power consumption of each node along the path has to be calculated in order to determine whether the current residual power is enough or not. Namely, the transmission power consumption of each node must be no more than its current residual power capacity

As described in Section 1, a method has been proposed in [8] to estimate the transmission power that each node along path will be consumed during the transmission of packets. According to the path-loss model [8] [9], the transmission power consumption for node v_i can be expressed as a function of the distance between the endpoints. To be more specific, the power required to transmit a packet along link (v_i, v_j) is approximately equal to $l(v_i, v_j)^4 \pi$, where $l(v_i, v_j)$ denotes the distance between node v_i and node v_j, π is a constant in *mW*. The path-loss model also indicates that the power required to receive a packet by node v_i does not depend on the length of the link, and can be modeled by a constant ρ in *mW*. Thus, for a given node v_i, to ensure the transmission of σ packets to node v_j, it must have $l(v_i, v_j)^4 \pi \times \sigma$ units of power (the power is multiplied by the number of packets) for transmission and $\rho \times \sigma$ units of power for reception.

Traffic Model

In the paper, we are concerned with the connection-oriented traffic. To be more specific, given a pair of source and destination nodes, a fixed path connecting the two nodes should be established to transmit all the packets originated from the source to the destination. We will define the time period of the whole transmission as *a session*. Therefore, in our PAMR problem, it is required to guarantee that the transmission of packets along a single path can be supported by the residual power capacity of each node along the path until the completion of the session. For simplicity, in the following, it is assumed that the number of packets transmitted during any session should be a constant σ and only the transmitting power be considered (i.e., the reception power is ignored). As a result, the power required to transmit packets along link (v_i, v_j) during a session is approximately equal to $l(v_i, v_j)^4 \pi \sigma$, where $l(v_i, v_j)$ denotes the Euclidean distance between node v_i and node v_j, π is a constant in *mW*. The value $l(v_i, v_j)^4 \pi \sigma$ will be denoted as $\beta(i, j)$ in the following discussions.

2.2 Problem Formulation

We represent a MANET by a weighted graph $G = (V, E)$, where V denotes the set of mobile nodes and E denotes the set of communication links connecting the nodes. For V, we define a *residual power capacity function* $\alpha : V \rightarrow R^+$ that assigns a nonnegative weight to each node in the MANET. The value $\alpha(v_i)$ associated with node $v_i \in V$ represents the current residual power capacity of node v_i. For E, we define a *transmission power consumption function* $\beta : E \rightarrow R^+$ that assigns a nonnegative weight to each link in the network. The value $\beta(i, j)$ associated with link $(v_i, v_j) \in E$ represents the transmission power that one session will consume on that link.

Under the PARM problem we are considering, k sessions originating from k *source* nodes $v_{s_i} \in V$ in the MANET have to be connected to their individual destination nodes $v_{d_i} \in V$. A set R of k routes r_i for the k sessions is *feasible* if and only if each route r_i in R is a simple path connecting source v_{s_i} to destination v_{d_i}, and each node $v_i \in V$ satisfies the constraint that $\alpha(v_i) \geq \sum_{(i,j)\in E} \beta(i, j) \times \mu(i, j)$, where $\mu(i, j) =$ the number of the sessions passing link (i, j).

Based on these notations and definitions, we can now formally describe the PAMR problem in our paper: given a weighted graph $G=(V,E)$, k pairs of *source* and destination nodes (v_{s_i}, v_{d_i}), $v_{s_i}, v_{d_i} \in V$, *a residual power capacity function* $\alpha : V \rightarrow R^+$ is defined in V, *a transmission power consumption function* $\beta : \ell \rightarrow R^+$ is defined in E, find a set R of k feasible routes r_i that sets up a session between each source v_{s_i} to its corresponding destination v_{d_i}.

In other words, a set of feasible routes must guarantee that the transmission of packets along each route r_i from its source v_{s_i} to its destination v_{d_i} do not run out of power of any node in r_i until the completion of the session.

The General PAMR Problem and the Geometric PAMR Problem

Given a PAMR problem, when its *transmission power consumption function* β is an arbitrary function, i.e., the weight assigned to a link is independent of the length of link, we say the considered PAMR problem to be *a general PAMR problem*. On the other hand, if the weight assigned to a link is related to the Euclidean distance of link, the associated PAMR problem is defined to be *a geometric PAMR problem*.

Example

As an illustration of the above definitions and notations, let us consider the example shown in Figure 1. In Figure 1, let nodes v_{s_1}, v_{s_2}, and v_{s_3} be the source nodes and nodes v_{d_1}, v_{d_2}, and v_{d_3} be their corresponding destination nodes, respectively. The number within a node represents the current residual power capacity of the node while the number next to a link represents the transmission power consumption of the link. It is not difficult to observe that there is no feasible solution in Figure 1 for the 3 sessions to be established. In fact, at most two feasible paths can be discovered (as shown by the two bold-faced lines which connect v_{s_1} to v_{d_1} and connect v_{s_2} to v_{d_2}). On the other hand, if the current residual power capacity of node v_1 is increased from 10 to 15, then there exist a set of 3 feasible paths for the current PARM problem (the new path connecting v_{s_3} to v_{d_3} is shown by the dotted line). Note that $\mu(1,3) = 2$.

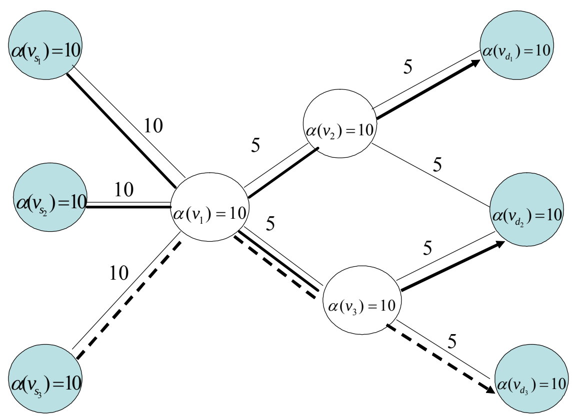


Figure 1. An example to illustrate the PAMR problem

3. The Complexity of the General PAMR Problem

In this section, we will show that the general PAMR problem is NP-complete.

First let introduce the *k disjoint paths* (*k*DP) problem

Instance: A graph $G = (V, E)$ and k distinct pairs of vertices $(v_{s_1}, v_{d_2}), (v_{s_2}, v_{d_2}) \ldots (v_{s_k}, v_{d_k})$ of G.

Question: Do there exist k pairwise vertex-disjoint paths $P_1, P_2, \ldots, P_k$ of G such that P_i is a path connecting v_{s_i} and v_{d_i}, for each $1 \le i \le k$?

This problem was shown to be NP-complete by Karp [10], if k is a variable part of the input.

Theorem 1. The general PAMR problem is NP-complete.

Proof. At the first, the general PAMR problem can be easily seen to be in the class NP. We next transform the *k*DP problem [10] to the general PAMR problem in polynomial time. Let graph $G = (V, E)$ and its k distinct pairs of vertices $(v_{s_1}, v_{d_2}), (v_{s_2}, v_{d_2}) \ldots (v_{s_k}, v_{d_k})$ be an instance of the *k*DP problem. We will construct an instance of the general PAMR problem as follows: The weighted graph $G' = (V', E')$ has a node v'_t $(1 \le t \le |V|)$ for each corresponding v_t in G and has an edge (v'_i, v'_j) for each corresponding edge (v_i, v_j) in G. All the nodes are assumed to have a residual power capacity of 1. All the edges are assumed to have a transmission power consumption of 1. It is easy to see that this transformation can be finished in polynomial time. An illustration of G' constructed from G is shown in Figure 2.

We next show that the *k*DP problem has a solution if and only if a feasible solution for the general PAMR problem exists.

First, suppose we have a solution to the *k*DP problem. Let $(P_1, P_2, \cdots, P_k)$ be one of the possible solutions in G. Let $(P_1', P_2', \cdots, P_k')$ be the corresponding paths in G'. Because these k paths: $(P_1, P_2, \cdots, P_k)$, are pairwise vertex-disjoint, any link *(i, j)* belongs to at most one of these k paths. Therefore, for each link $(i', j') \in E'$, $\mu(i', j') \le 1$. Similarly, any node v_i belongs to at most one of these k paths. So, for any $v'_i \in V'$, $\sum_{(i',j') \in E'} \mu(i', j') \le 1$. As a result, for any $v'_i \in V'$, we have $\sum_{(i',j') \in E'} \beta(i', j') \times \mu(i', j') \le \sum_{(i',j') \in E} 1 \times \mu(i', j') \le 1 = \alpha(v'_i)$

This implies that each node $v'_i \in V'$ has enough power until the completion of the session. As a result, $(P_1', P_2', \cdots, P_k')$ is a feasible solution for the corresponding general PAMR problem in G'.

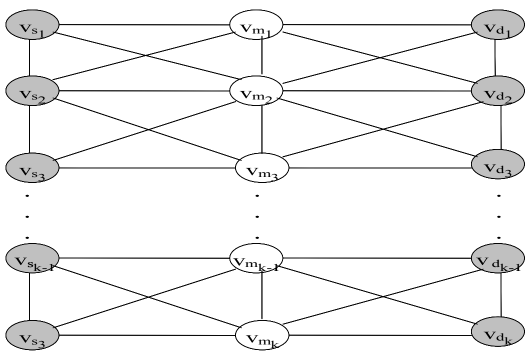
Next, suppose we have a feasible solution for the general PAMR problem in the weighted graph G'. Let $(P_1', P_2', \cdots, P_k')$ be one of the possible solutions in G'. Because each node has a residual power capacity of 1 and each link has a transmission power consumption of 1, each node v'_i belongs to at most one path P'_l. Thus $(P_1', P_2', \cdots, P_k')$ are pairwise node-disjoint. Let $(P_1, P_2, \cdots, P_k)$ be the corresponding paths in G. Clearly these paths: $(P_1, P_2, \cdots, P_k)$, are also pairwise node-disjoint and thus are one solution of the *k*DP problem associated with G.

Thus, we have completed the proof. ■

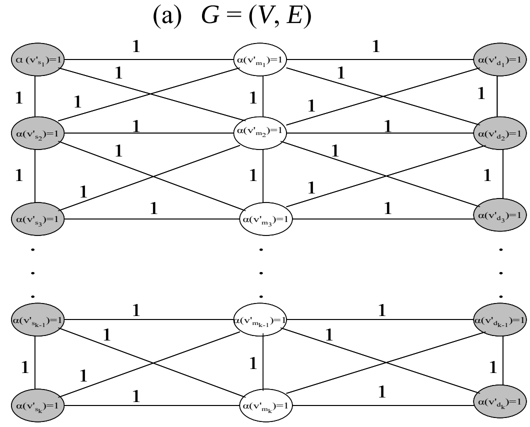
Corollary 1. The general PAMR problem is strongly NP-complete. That is, the problem remains NP-complete even if the power of node and the transmission power of line are constrained to be below a given constant.

Corollary 2. The general PAMR problem is still NP-complete even when the power of node and the transmission power of line are the same.

Note that when $k = 1$, the geometric PAMR problem is reduced to the power-aware unicast routing problem considered in [8] and can be solved in polynomial time. When each node has very large power, such as the case in the wired network, the general and geometric PARM problems can be easily solved by Dijkstra's algorithm. Finally, let us notice that although the general PAMR problem has been proved to be an NP-complete problem, the complexity of the geometric PAMR problem is unknown.



(a) $G = (V, E)$



(b) $G' = (V', E')$

Figure 2. An illustration of Theorem 1

4. Conclusions

In this paper, we have considered the problem of searching multiple routes satisfying the power requirement in a mobile ad hoc network. We have shown that the PAMR problem is NP-complete. When a problem is proved to be NP-complete, the follow-up quest will be to search for various heuristic algorithms and evaluate them by computer simulations. In the future, we will design efficient heuristic algorithms for the PAMR problem.

References

[1] C. E. Perkins and E. M. Royer, “Ad-hoc on demand distance vector routing,” *Proc. of IEEE WMCSA’99*, pp. 90-100, Feb. 1999.

[2] J. Broch, D. B. Johnson, and D. A. Maltz, “The dynamic source routing protocol for mobile ad hoc networks,” *Internet-Draft*, draft-ietf-manet-dsr-01.txt, Dec. 1998.

[3] V. D. Parh and M. S. Corson, “Temporally-Ordered Routing Algorithm (TORA) version 1 functional specification,” *Internet-Draft*, raft-ietf-manet-tora-spec - 03.txt, Nov. 2000.

[4] A. Michail and A. Ephremides, “Energy efficient routing for connection-oriented traffic in ad-hoc wireless networks,” *Personal, Indoor and Mobile Radio Communications, 2000. PIMRC 2000. The 11th IEEE International Symposium on*, vol. 2, pp.762-766, Sep. 2000.

[5] A. Misra and S. Banerjee, “MRPC: maximizing network lifetime for reliable routing in wireless environments,” *Wireless Communications and Networking Conference, WCNC2002. 2002 IEEE*, vol. 2, pp.800-806, Mar. 2002.

[6] S. Singh, M. Woo, and C. S. Raghavendra. “Power-aware routing in mobile ad hoc networks,” *The Fourth Annual ACM/IEEE International Conference on Mobile Computing and Networking*, pp. 181-190, 1998.

[7] C. K. Toh, ”Maximum battery life routing to support ubiquitous mobile computing in wireless ad hoc networks,” *Communications Magazine, IEEE*, vol. 39, pp.138-147, Jun. 2001.

[8] S. Tragoudas and S. Dimitrova, “Routing with energy considerations in mobile ad-hoc networks,” *Wireless Communications and Networking Conference, 2000. WCNC. 2000 IEEE*, vol. 3, pp.1258-126, Sep. 2000.

[9] R. Sivakumar, P. Sinha, and V. bhagravan, “CEDAR: A core extraction distributed ad hoc routing algorithm,” *IEEE JSAC*, vol. 17, no. 8, pp.1454-1465, Aug. 1999.

[10] M. R. Garey and D. S. Johnson, *Computers and intractability: a guide to the theory of NP-completeness*, San Francisco: W. H. Freeman & Company, Publishers, 1979.