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An Accurate Method of Measuring Similarities for Chinese Signature Verification System Using Fourier Descriptors

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Abstract

In this study, we present a novel approach to measure the similarities for an on-line Chinese signature verification system. The proposed system is comprised of two stages, namely, the learning stage and the verification stage. The former generates all the reference signatures for the users, and the latter calculates the similarities between the test signature and the corresponding reference signature.

A two-step matching process is proposed including the coarse matching process and the fine matching process. The method developed in [11] is adopted in this study to accomplish the coarse matching process. However, the similarity between the test and reference signatures obtained by the first step is not accurate enough. To achieve a better result, the method developed in [12] is modified in this study to measure the similarity for the two signatures by transforming the corresponding signature strokes into the frequency domain for further minor adjustment. The experimental results reveal the practicability of the proposed methods.

1. Introduction

Personal verification plays an important role in daily life. Examples include automatic banking transactions, building entrance control, e-mail examination, confidential documents access, etc. With the development of computer technology, using computer or electronic tools to assist in developing an automatic system for personal verification becomes more and more compelling. Approaches to verifying an individual person include the following four categories [1]: (1) the verification of personal bodies, e.g., face shapes and fingerprints; (2) the verification of personal possessions, e.g., identification cards, keys, and seal imprints; (3) the verification through specific knowledge forms, e.g., passwords and ciphers; and (4) the verification according to personal behavioral characteristics like signatures. Among the variety of different approaches listed above, the last approach owns the following merits and is considered as one of the best means in verifying a person: (1) signature is not easy to be duplicated; (2) signature cannot get lost, be robbed or stolen; and (3) signature is not necessary to be memorized.

Signing a signature is recognized as a thoughtless movement combining conscious and subconscious writing behaviors [2, 3]. People think what to sign instead how to sign while writing. Signature can be taken as a tool for personal verification because the writing length, velocity, acceleration, and force intensity can vary greatly from person to person. No two signatures done by the same person can be exactly identical, however, signature is considered as a kind of stable behavior [4].

Due to the writing variability exists in signing a signature, even an authentic signature may be rejected by the system, which causes the type-I error. On the other hand, the verification system may accept signatures of the intruders that are almost identical to the true signature. This will allow the intruder to enter the system and cause the type-II error. Hence, the major function of this study is to provide methods that can increase the acceptance rate of the true signatures, and decrease the acceptance rate of the fake signatures (i.e., to decrease both of the type-I and type-II error rates.)

In general, there are two kinds of signature verification system depending on the ways that signatures are entered, namely, the off-line and on-line signature verification system. The former operates after the signature is put on a piece of paper and transmitted into the computer through an optical scanner or video camera for further processing. The latter, on the other hand, receives data from a digital board including the position, velocity, and pressure of writing. Previous studies in related fields are reviewed as follows.

Herbst and Liu [5] took the acceleration curve of signature as a feature in their system. The similarities between signatures can be figured out through a regional correlation algorithm capable of adjusting the signal on the duration axis. Kamins [6] normalized the testing signature, and utilized the technique of interpolation to link the last point and the start point of the signing track together so as to obtain a relevant cyclic expression. The Fourier transform was then applied to transform the cyclic track into the frequency domain, in which the top 15 Fourier descriptors were taken for stepwise discriminant analysis. Parizeau and Plamondon [7] made an evaluation analysis for the following three matching algorithms: the regional correlation algorithm, the dynamic time warping algorithm, and the skeletal tree matching algorithm. The factors considered in the evaluation process include the correctness of verification, the matching time consumed, the number of parameters utilized, and the sensitivity of the parameters. They concluded that each algorithm is with advantages as well as disadvantages, and only the signal of the velocity curve on Y-axis is the most useful tool to verify the truth of signature. Dimauro, Impedovo, and Pirlo [8] verified signatures according to the components embodied. The major components of one's signature are analyzed and saved into a file at the learning stage. A spectrum verification proceeds at the verification stage which takes the references of the available individual signature components and compares them with the target signature parts. Plamondon and Lorette [9] provided an introduction to various kinds of signature verification system, and discussed the problems required to be solved in the system. Various types of pattern matching approaches and signature verification related literature were also examined. According to those matching algorithm, they classified the signature verification systems into two principal groups, namely, the function approach and the parameter approach. The former proceeds with functions as features, e.g., the position function, the velocity function, the acceleration function, and the pressure function. The latter, however, extracts *m* parameters as features from the measured signals, e.g., the geometric and timing parameters. Some mathematical transformations have also been applied in this group to signals to derive the coefficients as features, e.g., the Walsh transform, the Haar transform, and the Fourier transform. In general, the parameter approach is comparatively faster and easier than the function approach but the stability is inferior. The function approach takes all signal data contained in the signature into account and achieves a better stability as well as verification result; however, it is quite time-consuming. Leclerc and Plamondon [10] provided a clear look of signature verification related approaches and results appeared from 1989 till 1993.

Previous studies regarding Chinese signature verification systems are reviewed as follows. Lin and Chen [11] proposed a Chinese signature verification system. They took the key points, e.g., the pen-down and pen-up points, and the break points, to characterize a Chinese signature, and utilized the relaxation technique and the A* searching method to find out the best matches between the key points of the test signature and the reference signature. The best matches were then taken to figure out the affine transformation matrix which is responsible for adjusting the test signature and solving the problem resulted from different signature sizes and orientations. However, the transformation process is not accurate enough, since only few key points are taken to characterize the entire test signature.

Based on the weaknesses exist in [11] mentioned above, this paper presents a novel method to obtain a more accurate result in calculating the similarity between the test signature and the reference signature. The best feature matches of the two signatures determined by [11] are first transformed into the correspondences of signature strokes. By transforming the corresponding strokes into the frequency domain, a more elaborate transformation matrix can be obtained by utilizing the algorithm of LMSE (Least Mean Square Error) approach. Finally, the matrix is further applied to the test signature for more accurate adjustment, and a better result is obtained accordingly.

The rest of the paper is organized as follows. Section 2 presents a detailed look of the concept of curve matching using the Fourier descriptors. Section 3 presents the proposed signature verification approach and the method for finding the similarities of static writing tracks and dynamic velocity curves. Section 4 contains the discussion of experimental result. Finally, Section 5 provides a conclusion of this study, and prospects further studies and future development in related fields.

2. Curve Matching by Fourier Descriptors

To measure the similarity between two signatures, the method proposed by [12] is modified in this study, in which the Fourier descriptors are utilized to match two 2-D curves. The detail is presented as follows.

A closed 2-D curve can be characterized by two 1-D periodic functions $x(t)$ and $y(t)$, which can be expressed by the Fourier descriptors as

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{i=1}^{\infty} \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix} \cdot \begin{bmatrix} \cos it \\ \sin it \end{bmatrix}, \qquad (1)$$

where

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} x(t)dt,$$

$$c_0 = \frac{1}{2\pi} \int_0^{2\pi} y(t)dt,$$

$$a_i = \frac{1}{\pi} \int_0^{2\pi} x(t)\cos it\,dt,$$

$$b_i = \frac{1}{\pi} \int_0^{2\pi} x(t)\sin it\,dt,$$

$$c_i = \frac{1}{\pi} \int_0^{2\pi} y(t)\cos it\,dt, \text{ and}$$

$$d_i = \frac{1}{\pi} \int_0^{2\pi} y(t)\sin it\,dt.$$

$t = 2\pi l / L$, L is the perimeter of the closed curve, and l is the arc length from the start point s to an arbitrary point p as shown in Figure 1.

An open 2-D curve can be traced from the start point to the end point, and then traced back to form a closed curve. The Fourier descriptors of an open curve can be expressed as

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{i=1}^{\infty} \begin{bmatrix} a_i \cos it \\ c_i \cos it \end{bmatrix}, \qquad (2)$$

where

$$a_0 = \frac{1}{\pi} \int_0^{\pi} x(t)dt,$$
$$c_0 = \frac{1}{\pi} \int_0^{\pi} y(t)dt,$$
$$a_i = \frac{2}{\pi} \int_0^{\pi} x(t)\cos itdt, \text{ and}$$
$$c_i = \frac{2}{\pi} \int_0^{\pi} y(t)\cos itdt.$$

The mean square difference (*MSD*) defining the shape difference of two curves can be characterized in terms of the above Fourier descriptors as:

$$MSD = \frac{1}{2} \sum_{i=1}^{k} \left(V_{a_i}^2 + V_{b_i}^2 + V_{c_i}^2 + V_{d_i}^2 \right) \qquad (3)$$

for a closed curve, and

$$MSD = \frac{1}{2} \sum_{i=1}^{k} \left(V_{a_i}^2 + V_{c_i}^2 \right) \qquad (4)$$

for an open curve, where k is the maximum harmonic used, and V_{a_i}, V_{b_i}, V_{c_i}, and V_{d_i} are the residual differences of the Fourier descriptors. The detail discussion about how to derive Equations through (1) to (4) can be referred to [12].

By treating the entire signature as an open curve and employing the above method, the problem of measuring similarity between the test and reference signatures can be solved accordingly. However, the result is poor due to the following reasons. As mentioned previously, signing a signature is considered as a thoughtless writing behavior. There are many factors that may cause the variation of writing tracks, e.g., writing on different pieces of paper, with different pens, or in different moods. Due to the above variations, two Chinese signatures done by a single person may have different number of "signature strokes", i.e., some strokes existing in one signature may disappear in the other signature, and vise versa. Taking entirely the two signature curves into account for measuring their similarity may cause incorrect and unpredictable results, since the missing strokes will bias greatly the measuring result. The problem is well solved in this study by presenting a two-step matching process that will be discussed in the next section.

3. Proposed Methods

The proposed method for signature verification is comprised of the following two stages, namely, the learning stage and the verification stage, as shown in Figure 2. In the former stage, the user is required to write down his signatures several times so as to study his (or her) signature characteristics, and one of the signatures is selected as a reference and saved into a disk file. In the latter stage, the test signature entered by the same user will be examined and matched with the reference created in the former stage, and the similarities between the two signatures are calculated by the proposed method for verifying the truth of the test signature. The method of selecting the reference signatures is discussed in Section IV, and the method of matching is presented in the following.

The matching process proposed in this paper is a two-step matching process, in which the first step is a coarse matching process, and the second step is a fine matching process as depicted in Figure 2. The method developed in [11] is adopted in this study to find the transformation matrix used for coarse matching. However, the similarity between the test and reference signatures obtained from the first step is not accurate enough. To achieve a better result, we thus propose a novel approach that takes into account the stroke correspondences between the two signatures found by the first process, and transforms those corresponding strokes into the frequency domain for further minor adjustment of the signatures. The detail of the proposed method is elaborated in the following subsequent sections.

3.1Signature verification via coarse matching

The method presented in [11] is employed in this paper to accomplish the process of coarse matching. Signatures are written on a tablet using a wireless pen, and entered into a personal computer via an RS-232 communication line. Each signature is digitized by the tablet and represented as a series of point data. Some important points including the pen-down points (i.e., the points obtained in the instant that the pen is handed down to touch the tablet,) the pen-up points (i.e., the points obtained in the instant that the pen is handed up to leave the tablet,) and the critical points (i.e., the points having locally the greatest curvature) are detected for each signature signed in the learning stage as well as the verification stage. An example is depicted in Figure 3. The signature is treated as a point pattern consisting of those points, and the problem of matching two signatures is thus transformed into the problem of matching two point patterns. The problem was well tackled in [11] by utilizing the relaxation approach and the A^* searching algorithm. The similarities between the test and reference signatures were obtained by finding the transformation matrix through the available matching correspondences. Finally, by applying the matrix to the test signature, the size and rotation problems resulted from the natural writing deviation were solved accordingly. An illustrative example is given in Figure 4.

3.2 Signature verification via fine matching

The process of coarse matching mentioned in the last section can be used to solve the problems of size and orientation deviation while checking whether a test signature is authentic or not. However, the result may not be accurate enough because the matching process is implemented based on few points of the entire signature. By finding the correspondences of signature stroke that is defined later between the test signature and the reference signature, and then transforming the corresponding strokes into the frequency domain, a more accurate transformation matrix can be obtained through the LMSE approach. To achieve a better verification result, the matrix is further applied to the test signature for more accurate adjustment. The detail is elaborated as follows.

Let the point sets detected in the last section of the test signature and the reference signature be denoted by $U=\{u_1,u_2,\cdots,u_m\}$ and $V=\{v_1,v_2,\cdots,v_n\}$, respectively. Also let $S=\{s_1,s_2,\cdots,s_k\}$ be the set of match pairs between U and V obtained after the process of coarse matching. Each element s in S is denoted by a (u_i,v_j) pair, where $u_i \in U$ and $v_j \in V$. Element s is called a pen-down pair and a pen-up pair if both u_i and v_j are pen-down points and pen-up points, respectively. The matching strokes of the two signatures are defined in this study to be the strokes described by a sequence of consecutive elements in S that starts with a pen-down pair and ends with a pen-up pair. More specifically, assume that $s_i=(u_j,v_k),\cdots,s_x=(u_y,v_z)$ is a sequence of elements in S, where s_i is a pen-down pair and s_x is the first occurrence of pen-up pair in the sequence. Then the stroke described by the points through u_j to u_y in U and the stroke described by the points through v_k to v_z in V are two match strokes. In the following subsequent paragraphs, only a single pair of match strokes, say, P and Q of the test and reference signatures, respectively, is discussed. The method we proposed can be easily extended to the process of the entire signature.

Due to strokes of the test signature may vary greatly in size, length, location, and orientation, it is not easy to measure accurately the similarity between P and Q in the spatial domain. Thus transforming this problem into the frequency domain for further processing is suggested in this study. Since a signature stroke belongs to a kind of open curve, it must be converted to a closed curve using the method mentioned in Section 2, and is characterized by the Fourier descriptors in the frequency domain as

$$P=\begin{bmatrix} p_{x(t)} \\ p_{y(t)} \end{bmatrix}=\begin{bmatrix} a_0 \\ c_0 \end{bmatrix}+\sum_{i=1}^{k}\begin{bmatrix} a_i\cos it \\ c_i\cos it \end{bmatrix}, \tag{5}$$

$$Q=\begin{bmatrix} q_{x(t)} \\ q_{y(t)} \end{bmatrix}=\begin{bmatrix} a'_0 \\ c'_0 \end{bmatrix}+\sum_{i=1}^{k}\begin{bmatrix} a'_i\cos it \\ c'_i\cos it \end{bmatrix}, \tag{6}$$

where k is the maximum harmonic used. The mean square distance is defined by

$$MSD(P,Q)=\frac{1}{2}\sum_{i=1}^{k}\left((a_i-a'_i)^2+(c_i-c'_i)^2\right) \tag{7}$$

To calculate the MSD value defined above, instead of adopting the Newton-Raphson method proposed in [12], we present the LMSE method because the former is more time-consuming and requires a good initial value. If a good initial value is not available, it will cause a wrong result or divergence. The method is stated as follows.

It is well known that the translation and rotation transformations of a point in coordinate (x',y') to a new coordinate (x'',y'') can be expressed as

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix}=\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}+\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\cdot\begin{bmatrix} x' \\ y' \end{bmatrix}, \tag{8}$$

where $(\Delta x, \Delta y)$ represents the translation vector, and θ is the rotation angle. If a curve is transformed into the frequency domain and is characterized by the Fourier descriptors, then Equation (8) can be expressed as

$$\begin{bmatrix} a''_i \\ c''_i \end{bmatrix}=\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}+\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\cdot\begin{bmatrix} a'_i & b'_i \\ c'_i & d'_i \end{bmatrix} \tag{9}$$

for a closed curve, and

$$\begin{bmatrix} a''_i \\ c''_i \end{bmatrix}=\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}+\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\cdot\begin{bmatrix} a'_i \\ c'_i \end{bmatrix} \tag{10}$$

for an open curve. Equation (10) can be rewritten as

$$\begin{bmatrix} a''_i \\ c''_i \end{bmatrix}=\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}+\begin{bmatrix} r_3 & -r_4 \\ r_4 & r_3 \end{bmatrix}\cdot\begin{bmatrix} a'_i \\ c'_i \end{bmatrix}, \tag{11}$$

and then homogenized by

$$\begin{bmatrix} a''_i \\ b''_i \\ 1 \end{bmatrix}=\begin{bmatrix} r_3 & -r_4 & r_1 \\ r_4 & r_3 & r_2 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} a'_i \\ b'_i \\ 1 \end{bmatrix}, \tag{12}$$

or expressed in a more simple form:

$$\begin{bmatrix} a''_i \\ b''_i \\ 1 \end{bmatrix}=M\cdot\begin{bmatrix} a'_i \\ b'_i \\ 1 \end{bmatrix}. \tag{13}$$

Given two correspondent signature strokes P and Q, in which the Fourier descriptors are defined in Eqs. (5) and (6). The best transformation matrix M is expected to be found such that the MSD is minimized, i.e., to minimize

$$E = MSD = \frac{1}{2}\sum_{i=1}^{k}\left[(a_i - a_i'')^2 + (c_i - c_i'')^2\right], \tag{14}$$

where

$$\begin{bmatrix} a_i'' \\ b_i'' \\ 1 \end{bmatrix} = M \cdot \begin{bmatrix} a_i' \\ b_i' \\ 1 \end{bmatrix} = \begin{bmatrix} r_3 & -r_4 & r_1 \\ r_4 & r_3 & r_2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_i' \\ b_i' \\ 1 \end{bmatrix}. \tag{15}$$

M can be found by setting the partial derivatives of E with respective to r_1, r_2, r_3, and r_4, respectively, to zero, i.e.,

$$\frac{\partial E}{\partial r_1} = \frac{\partial E}{\partial r_2} = \frac{\partial E}{\partial r_3} = \frac{\partial E}{\partial r_4} = 0, \tag{16}$$

and solving the four simultaneous equations. The similarity measure (SM) between the test and reference signatures is thus obtained by the following equation:

$$SM = \sum_{i=1}^{h} E_i, \tag{17}$$

where h represents the number of matching strokes, and E_i is defined in Eq. (14).

4. Experimental Result

The experimental result of the proposed approach will be elaborated in this section. The user of the established system will be asked to write down five times of his (or her) name at the learning stage, and these input signatures will be compared with each other using the method presented in the last section (so there will be in total 20 comparisons and hence 20 SM computations required for each user.) Sum up those SMs for each signature as compared with the other signatures, and select one of them with the least sum of SM as the reference for the user. The reference accompanied with an identification (ID) number given by the user is then saved into a disk file for later use. In the verification stage, the user will be asked to input an ID number and sign his name as a test signature. The ID number is used to extract the reference signature of the user, and a comparison between the two signatures is made.

Two different kinds of experiments were done for demonstrating the efficacy of the proposed method. The first kind applies the coarse matching process to the test signature before measuring their similarities, while the second kind applies both of the coarse matching and the fine matching processes to the test signature. Both of the two kinds of experiments were performed based on a database containing 2025 signatures of 45 signers. Each of the signers was asked to sign his (or her) signatures 25 times, of which 5 times were used for training, and the remaining 20 times were used for measuring the type-I error rate. For each signer, 20 imitated signatures made by deliberate forgers were also collected to evaluate the performance of the type-II error rate. The acceptance threshold λ_i is defined as follows.

Let μ_i and σ_i denote the mean and standard deviation of the SMs calculated for the ith signer in the learning stage. The acceptance threshold λ_i is defined in this study by

$$\lambda_i = \mu_i + k\sigma_i, \tag{18}$$

where k is a positive constant. If the SM between the reference and the test signatures of the ith signer is less than or equal to threshold λ_i, the latter is accepted as a genuine signature; otherwise, it is rejected. The type-I and type-II error rates for the two kinds of experiments with different k values ranging through 0 to 5 at intervals of 0.5 are calculated and summarized in Table 1. It is noted from this figure that the type-II error rate can be reduced if the test signatures are transformed utilizing our method before measuring their similarities.

5. Conclusion

As elaborated in Brault and Plamondon [13], there are two problems in establishing a signature verification system, namely the unstable personal signatures, which cause difficulties in deducting the characteristics of personal signatures or reject the real signatures, and the intelligent intruders, who may imitate perfect signatures like the real ones in static writing tracks, dynamic velocity, and pressure. The proposed approach which undergoes a global signature calibration as well as a minor refinement of individual signature line has been proved to be accurate and stable in solving the problem of natural divergence resulted from the size and angle changes while inputting Chinese signatures. The approaches applied in this study, such as transforming the curve lines into the frequency domain, characterizing the curves through the Fourier descriptors, avoiding the problems of searching for correspondent points while verifying signatures, and figuring out the similarities between the reference signature and the testing signature through the Fourier descriptors, have all significantly enhance the accuracy of signature verification.

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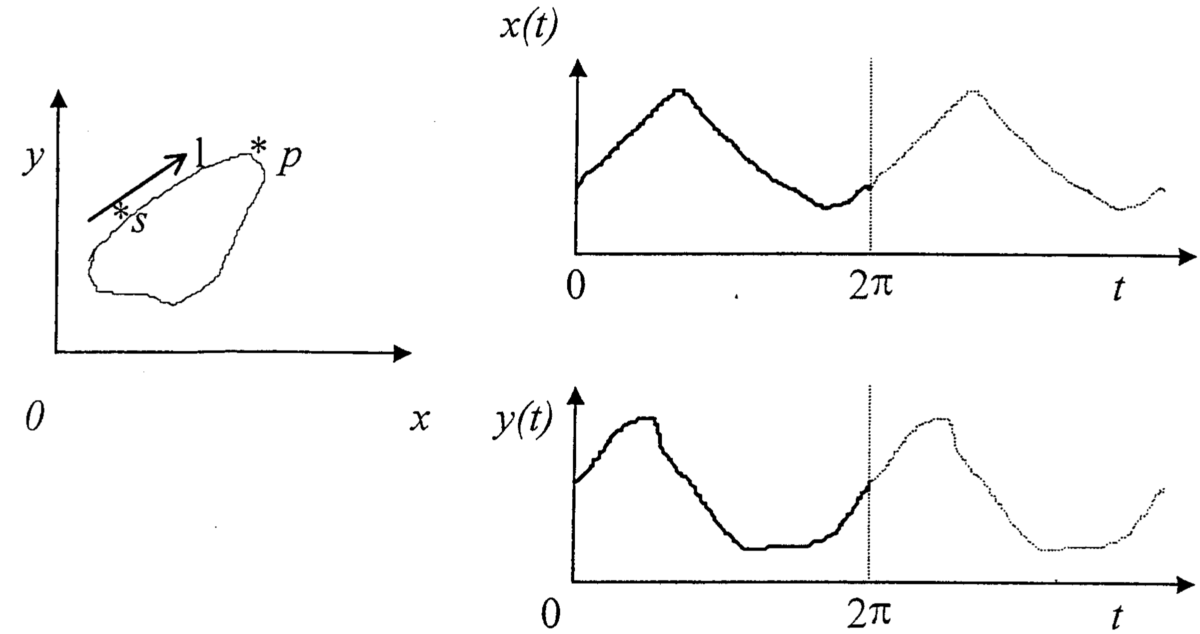


Figure 1. A closed 2-D curve and its corresponding two 1-D periodic functions.

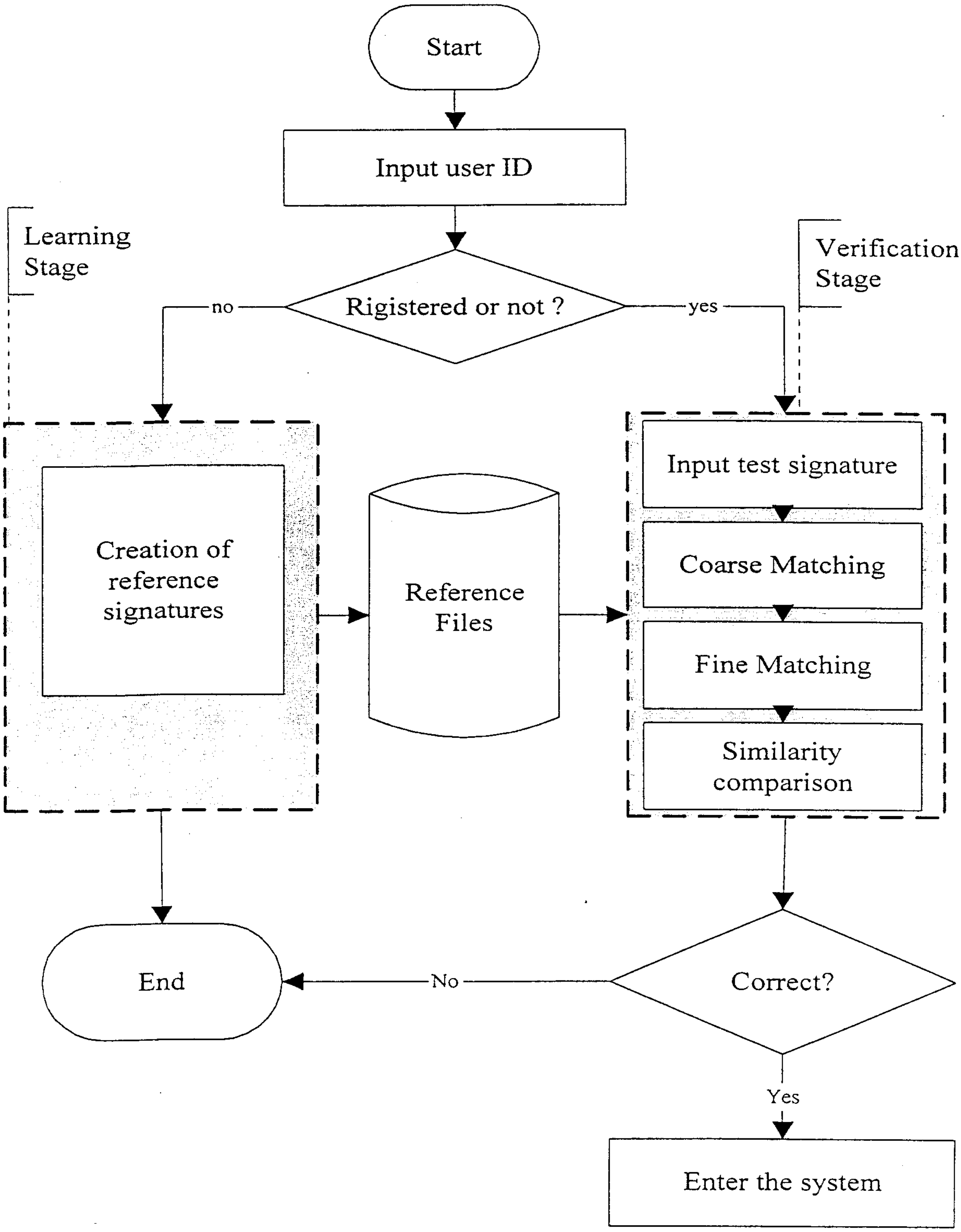


Figure 2. The flowchart of the proposed system.

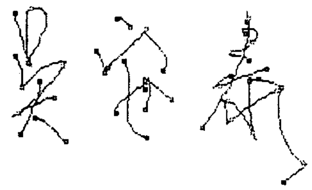
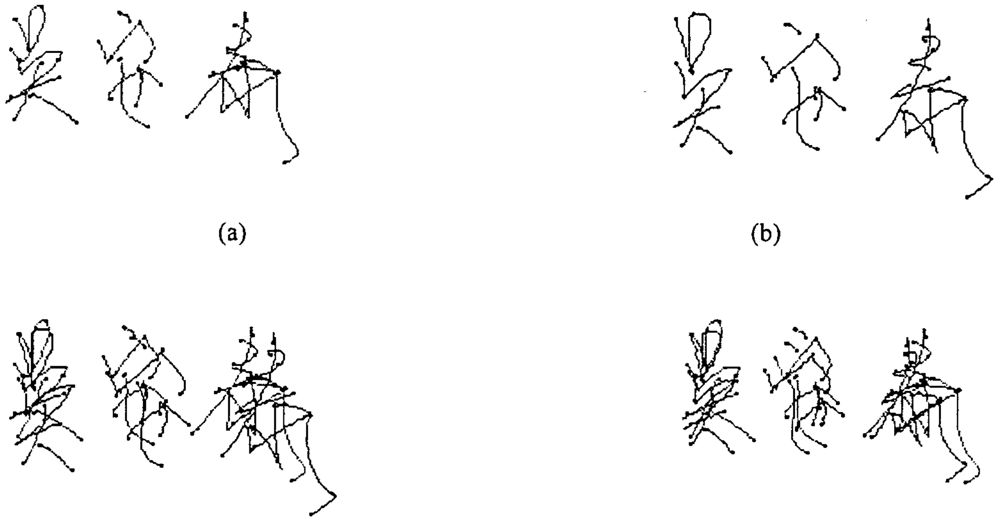


Figure 3. The important points of a signature.



(a) (b) (c) (d)

Figure 4. The result of coarse matching of the reference signature and the test signature. (a) The reference signature. (b) The test signature. (c) The superimpose of the two signatures before the process of coarse matching. (d) The superimpose of the two signatures after the process of coarse matching.

Table 1. A summarized table of the type-I and type-II error rates for the two kinds of experiments.

k	Experiment of kind 1		Experiment of kind 2	
	type-I error rate (%)	type-II error rate (%)	type-I error rate (%)	type-II error rate (%)
0	58.9	0.8	57.6	0.2
0.5	44.6	3.2	45.9	1.3
1.0	32.6	5.9	37.1	2.2
1.5	23.7	7.7	29.9	3.8
2.0	17.2	8.9	24.2	4.6
2.5	12.2	11.7	18.3	6.1
3.0	9.3	14.4	14.7	7.9
3.5	6.4	16.0	10.7	8.9
4.0	5.4	18.4	7.9	9.6
4.5	4.3	21.1	5.6	10.3
5.0	3.7	22.3	3.6	11.8