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Computational Reasoning in Distributed Legal Information Systems

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ABSTRACT
The lattice is a fundamental structure existing at any phenomenological level. Modern electronic networks and links in legal inference are important examples of distributed computational power. A better formal understanding of both legal information systems and legal reasoning is provided by recent advances in theoretical computer science and databases. Computational type theory and intuisionistic logic are by the Curry-Howard isomorphism equivalent. Instances are given in current constructive mathematics using category theory and geometric logic for both intuisionistic legal reasoning and documents in hypermedia. An equivalence for inference is given by the Heyting implication $\mathbf{A} \Rightarrow \mathbf{B}$ and its counterpart the concept of awareness by the universal contravariant functor and natural transformation α^*.

KEYWORDS
Multimedia, Internet, Hypertext Lattice, Intuitionistic Logic

1 Introduction

Digitised legal information in electronic law libraries provide an example of a class of information system where a latent network of meaning and inference lies dormant until it is made patent at run-time by the active context-sensitive informational needs of a particular user. With the advance from bibliographic records, keywords, summaries and abstracts through full-text documents to comprehensive multimedia materials available on-line in distributed libraries and in internet sources the retrieval of information has advanced radically to a form where the semantic links are available in the system and do not have to be supplied by the mind of the user. Law has been somewhat of a pioneer with full text because of the importance of any given word and because the law really has nothing further to support it other than reasoning. Legal sources are therefore particularly well developed body of this kind of reasoning. However other disciplines are rapidly expanding in the same direction, as for instance scientific or medical literature which are a body of distributed knowledge processing and inferencing.

The law operates as a network of relationships at various phenomenological levels. Society is a lattice of interacting relationships between members bound to each other in a network of duties and obligations. Sources of law consist of a network of meaning embedded in a network of interconnected cases, statutes, textbooks, commentaries, etc. Reasoning in law can be shown to be a logic network of legal inferences. It appears that the lattice is a fundamental structure of any system and worthy of close study because of its universal character. Legal reasoning and legal hypertext will be examined here to illustrate this universal equivalence of distributed systems.

Society is governed by the rule of law in the real world. The Law to be consistent with the laws of nature[17] has to be constructive, not axiomatic. Legal reasoning is therefore not axiological but intuitionistic and cannot be adequately formalized in terms of naive set theory. For it has to be remembered that the set-centred theory is based on axioms which do not hold for the logic of legal systems. The main problem is the axiom of choice. Diaconescu[10] has clearly shown that the axiom of choice confines the system of reasoning to Boolean logic where two negatives make a positive:[1]

$$\neg\neg A = A$$

In legal reasoning this $\neg\neg A = A$ is rarely correct. Two wrongs do not make a right, There is a fundamental distinction because legal reasoning is firmly founded in intuitionistic logic which comes from this constructive context of the law, This issue was brought to prominence in the 1920s by the leading Russian mathematician Kolmogorov and the Dutch mathematician who was the founder of intuitionistic mathematics Brouwer. Kolmagorov referred to double negation as a pseudo-truth leading to notions of 'pseudo-existence'[19, 20]. Brouwer[4, 5] thought that $\neg\neg A = A$ amounted to calling 'an absurdity-of-absurdity' true. However, he did recognize that the following holds in intuitionistic logic:

$$\neg\neg\neg A = \neg A$$

or in Brouwer's words[4] (at page 878) an absurdity-of-absurdity-of-absurdity is absurd'.

[1] in symbolic form using the conventional sign $\neg$ for not.

Brouwer's student Heyting was able to show that this informal language represented a much deeper formalism than set theory and he laid the foundations for formal intuitionism. Unfortunately the only formalism available for a long time to deal with intuitionism was an adapted set theory[11]. This has kept back the application of formal intuitionistic theories to topics that needed it like legal theory and legal reasoning until more recently when category theory and geometric logic have emerged as an alternative constructive mathematical workspace.

Constructive mathematics attempts to develop logically what works in practice and can provide the necessary universality on the one hand for the theory of legal reasoning and on the other for understanding the structure of distributed legal information systems now to be found in hypermedia and cyberspace. This turns out to be a very interesting example of the application of the Curry-Howard isomorphism between intuitionistic logic and computational type theory[33]. Geometric logic in category theory is essentially concerned with links between objects and is therefore particularly appropriate for modelling relationships both in legal reasoning[15] and for hypertext[16].

2 The Arrow of Categorial Geometric Logic

The form of constructive mathematics to be found in category theory is based not on the set as a fundamental but on the concept of process. This is generally thought of in terms of the arrow and represented by $\longrightarrow$[26]. The arrow represents any dynamic operation or static condition and can cope therefore with descriptive/ prescriptive equivalent views. For $A \longrightarrow B$ may be a descriptive action or a prescriptive one. That is a norm. Alternatively it may be a probabilistic relationship. There may be any number of different arrows between the same objects. For the arrow may be thought of as a generalization of verbs.

The arrow can never be free–standing: it must have some source and target, often named domain and codomain respectively. A category is a collection of arrows. The concept of a dual category arises from the view of arrows in the reverse direction.

2.1 Adjointness between Text and Image Data

Imaging is rapidly becoming a major industry and the manipulation of image data based on content and meaning is a burning research topic relevant to law. Geometric logic shows well the adjointness between textual and graphical information. Both are mapped into the electronic medium as a bit stream.

Multimedia are logical rather than physical based. They are therefore an abstract category of a document which may be represented as a textual file or as an image file resulting from input by means of a scanner. Clearly the two forms contain equivalent information although they would appear in quite different electronic forms. This is an important example of adjointness as demonstrated in Figure 1. $\mathbf{TXT(X)}, \mathbf{GRF(D)}$ and $\mathbf{E(2)}$ are categories corresponding respectively to text, graphics and electronic form. Each of these categories is a free functor. $\mathbf{TXT(X)}$ is a map from the alphabet X on to finite strings so a character, x, goes to a string,

$x \mapsto <x>$. $\mathbf{E(2)}$ is correspondingly composed of strings of zeros and ones. $\mathbf{GRF(D)}$ is the much more interesting graphical version which contains all the semiotic significance of the text beyond the mere characters (i.e. punctuation, capitalization, italics). There may be a loss of information from the category $\mathbf{GRF(D)}$ to $\mathbf{TXT(X)}$.

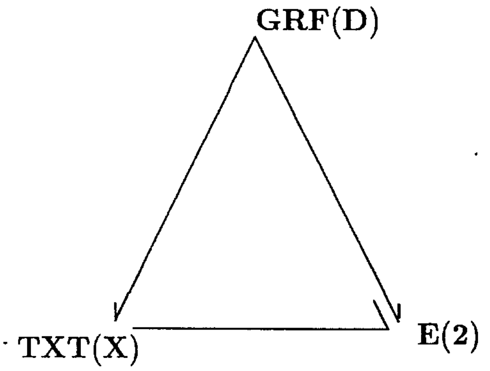

Figure 1: Adjointness of Electronic Forms

2.2 Intension–Extension Mapping

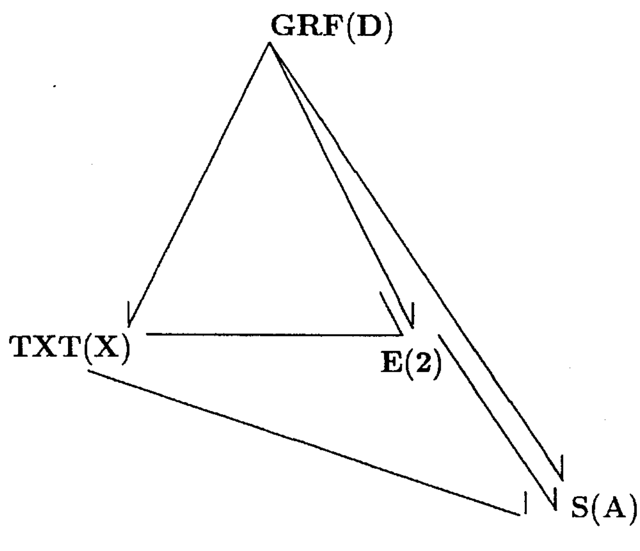

Figure 2: Adjointness in Real–world Semantics

The links in multimedia may be at different levels. The mappings representing the links would therefore need to be typed in geometric logic. There is the simple linking between documents like a citation of a label or name (the intension). A more powerful level of connection is within the semantics (the extension). There is also the intension–extension relationship which has been shown by Lawvere[21] to be composed of adjoint contravariant functors.

The extension level of the abstract document is therefore the same for the three categories of text, graphics and electronic bits. Equality in geometric logic is provided for by composition. The possible relationships between the three categories of documents at the two levels can therefore all be summed up in a simple geometric formal diagram.

A real-world semantics $\mathbf{S(A)}$ can be represented in any of the three forms of graphical, textual and electronic. There will therefore be intension, and extension consisting of contravariant functors between each of the three and $\mathbf{S(A)}$ as in the diagram in Figure 2.

2.3 Geometric Database Models

Hyperspace is equivalent to a multimedia database composed of complex objects in contrast to traditional types of data. Semantic models have developed to meet the need to specify information about the kinds of relationships to capture more meaning through the introduction of rules of relationships and integrity. The most popular of the semantic models is probably the E-R model of Chen but this has been extended in different ways. For instance the type and attribute form had to be added to Chen's original style of representation and Sakai[sak83] added generalization.]. The important deficiency of manipulation and the specification of behavioural characteristics has been satisfied in later models. The complex nature of the objects on the other hand has been satisfied by the development of object-oriented models[2].

These developments show the importance of semantics and some of these models have already been applied to document structures. None of the semantic models even with extensions have all the necessary features. The kinds of relationships needed for databases of self-managed information are extensive:

- abstractions such as aggregation, generalization, specialization, inheritance,classification, definition, designation, associative;
- structural such as models, nets, tables, hierarchies, entity classes, E-relations, P- relations
- statistical such as summation, averages, probabilistic, fuzzy;
- ordering such as sequence, Markov chain, probabilistic, temporal, stochastic;
- reductionist such as projection, parallax, derivation, view;
- behavioural such as dynamic, functional, transaction, operational;
- synthetic such as composition, join, union, cross-product, combined, concatenation, insertion, injection, embedded, tributory;
- analytic such as selection, intersection, adjacency, parametric, attribution;
- parallel such as synchronization, collaborative, collateral, adjacency, adjoin, redundant, orthogonal, anti-parallel, contributory.

Database technology has made progress with some but not all of the above categories. In some relations, the user is not concerned with detailed procedures and these may be the beginnings of automated reasoning. For instance, in aggregation the user is unaware of the way in which subobjects are put together. An example of putting subobjects together is the way that the current version of a section of an English Act of Parliament may be derived from a number of textual amendments in later Acts. Automatic identification of objects and their characteristics is needed to make the selection with the right inter-connections for the aggregation. From a database point of view, the identification is provided by the keys in the system[roh88]. Some universally recognized form is necessary to recognize the keys. This enables documents to be addressed and cross-referenced in a natural manner with a standard identification mechanism. Data typing can be used to characterize components of the keys so that documents can be composed from their underlying subsections (subobjects) in a transparent way. The use of natural keys and relations avoids the unnecessarily reductionist methods of early legal retrieval systems.

[2]such as the Semantic Association Model SAM* for use in statistical databases

Elaborate data management systems are needed to provide the high functionality required for structuring, manipulating and maintaining the data with the necessary integrity to provide professional information systems so that end-users may have access transparently to goal-information in a highly organized state. To do this the management system has therefore to recognize inherent relationships in the data to make the necessary hypertext links.

There are often very many, if not an infinite number of, natural connections that can be made. The author or information provider may predefine certain of these based on some expectation of the user's requirements. Alternatively it may be possible to provide some automatic assistance based on predetermined criteria. It is a simple matter to have a dynamic button to pick up references for a glossary or thesaurus where there is a direct connection usually because the item in the text is itself a simple key to the citation. Where there is a partial or a composite key, the system has to have some awareness functionality of what is needed[rsh90].

But there are limitations. The system needs to be able to follow any potential connection under the control of the user. This requires the system to be conscious of where the user is within the document. A very simple example might be given of anaphora in parliamentary debates like American Congress or the Hansard records of the British Houses of Parliament. When reading from a Hansard CD-ROM, to deal with a sentence beginning with

"As I said in my speech on 28th October to this House ...", the system needs to be aware of the name of the speaker, of whether the current speech is being made in the House of Commons or the House of Lords and of the date of the speech to identify the appropriate year for the 28th October. This necessary awareness required is therefore beyond intelligent hypertext. It also illustrates the practical point that this awareness needs to be a runtime facility. For identifying all possible cross references in advance when only very few of them will ever be required is very inefficient in preprocessing and storage and almost impossible manually.

This awareness is now essential in very many areas of business which need continual access to information on changing standards and regulations. This awareness function can be achieved by overlaying another layer of metadata on top of the basic hypertext system. This is a necessary part of intelligence in information retrieval systems[hea85]. We have to provide this additional layer to simulate a human metamemory for any type of document[ros87]. This layer needs to be reliable and comprehensive so that it provides closure to an open system[hea88] in an analagous manner to consciousness

Intuitionistic logic and compuational type theory have the same structure according to the Curry-Howard isomorphism[33]. This means that topics which have been developed in quite different contexts belong to an equivalent class, like, the topos[2], hyperdoctrines[22], Martin-Löf's theory of types[27] and structured information systems in the form of databases. Database operations illustrate the components of intuitionistic geometric logic. The various types of relationships of importance in legal databases may be summed up in Table 1. Fuller details that have been worked through for a product model based on limits are given elsewhere[28, 32].

database operation	categorical construct
abstractions	exactness
structural	adjointness
statistical	subobject classifier
ordering	adjoint functors
reductionist	co–exactness
behavioural	comma category
synthetic	exactness
analytic	co–exactness
parallel	adjointness

Table 1: Database Concepts in Categorical Terms

3 Formal Contextual Sensitivity

3.1 Limits, Colimits and Context

A very fundamental concept that has only been appreciated in the last thirty years is that of *limits* and *colimits*[24]. In arithmetic a limit is constructed by multiplication and colimits by addition, Within set theory, intersection is an example of a limit and disjoint union a colimit. With more general categories, limits and colimits become very powerful. A colimit is a deconstruction and provides no new information other than to make patent the latent components in the limit. The colimit of **A** and **B** is given by the fullest possible combination of taking them together and written **A** + **B**. A partial colimit would be obtained by taking together only certain parts of **A** and **B**. The parts that are significant when taken together may be provided by the context of a different category **C**. The pushout $\mathbf{A} +_{\mathbf{C}} \mathbf{B}$ as shown in Figure 3 then expresses this colimit in context.

This is the geometric logic representation of a hypertext link which brings together documents **A** and **B** through the context **C**. Note that this does not give any new information, but only identifies those parts of **A** and **B** which are relevant together in the context of **C**.

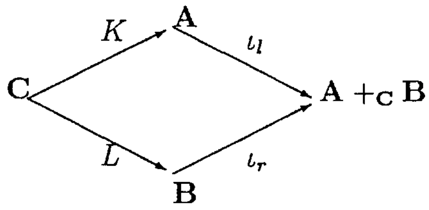

Figure 3: Diagram of Pushout of **A** and **B** over **C**

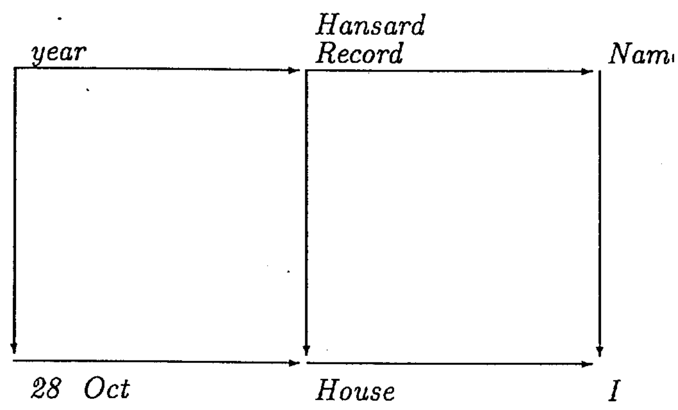

Figure 4: Composition of Pushouts for
"As I said in my speech on 28th October to this House ..."

An example of pushouts can be seen in the geometric logic representation of the remark: "As I said in my speech on 28th October to this House ...". The diagram in Figure 4 shows a pasting together of pushouts in which the result of one pushout *House* (possibly represented by a multimedia icon) is included in turn in another pushout forming *I*.

New information attained by linking **A** and **B** is given by the product limit **A** × **B**. This for a context **C** is the pullback $\mathbf{A} \times_{\mathbf{C}} \mathbf{B}$ shown in Figure 5. In general the difference between a limit and colimit may be summed up in that a limit produces some creative outcome of a link whereas the colimit is a link between standard information.

Examples of limits abound although they may not be explicitly recognized as such. For instance the subject of information retrieval has relied very heavily on the inner product of document vectors[34]. This is one particular reduced view of the limit $\mathbf{A} \times_{\mathbf{C}} \mathbf{B}$

Patterns of thought as in hypertext consists of a family of trails and it is important to recognize whether two trails are distinct or whether they merge. Thus there may be two par-

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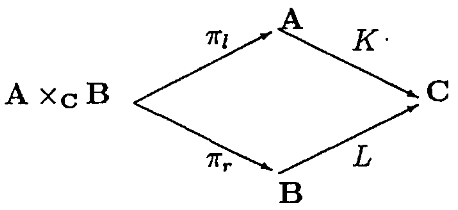

Figure 5: Diagram of Pullback of A and B over C

allel links between the same two documents. The question arises for a hypertext system whether two separate trails arriving at the first document are then merged. For example a legal case may cite a second case more than once during the report but it may be on two quite distinct points of law or even branches of law. Two cases may be connected on a substantive point of law and also quite separately on a point of legal procedure, adjectival law. A document often cites another more than once. Links between two documents in this situation become a limit point in the two trails if they merge there. However, geometric logic shows that there is a duality of limit also in this instance.

A coequalizer is the situation where there are distinct connections between the same two documents so that separate trails can pass across without merger. With the equalizer any separate trails arriving at the first document leave the second document by the same path. An equalizer is a context limit C represented in the diagram of Figure 6. All trails through A and B are merged through context C which will be shared by both A and B.

$$\mathbf{C} \rightarrow \mathbf{A} \rightrightarrows \mathbf{B}$$

Figure 6: The Equalizer C as a context limit on arrows from A to B

The corresponding coequalizer is given by Figure 7. The context of C is null, that is the limits are independent in thought but from the document perspective there is a context of documents where the two trails coexist with local independence. In other words links between documents may be equalizers or coequalizers.

$$\mathbf{A} \rightrightarrows \mathbf{B} \rightarrow \mathbf{C}$$

Figure 7: The Coequalizer C where two distinct trails coexist independently

This same equalizer and coequalizer distinction applies to higher-order links relevant to intellectual property. In organizing methods for payment of access to multimedia objects, it is necessary to have a theory of joint and common ownership. The objects may be quite fragmentary and widely distributed as in the extreme case of digital sampling in the music industry. Until a full theory is available, work on this aspect which is essential to the economic development of digital libraries[9] can only be *ad hoc* and restricted to a literal view of copyright. The payment is an equalizer or a coequalizer depending on whether the objects are subject to intellectual property rights, are in joint or common ownership, or indeed in the public domain. Likewise a systematic approach to attack the problems of policing the Internet to control its use for pornography, organized crime, drug trafficing, etc[23], can only be based on a theoretical structure underpinning the whole.

Two other special limits are the terminal object and (its dual) the initial object. An object in a category C where there is one and only one arrow from every other object to it is known as the final or terminal object of C. This may be denoted by $\top$ which is the last object in the trail. Dually (or oppositely) to the final object there may exist a corresponding initial object where there is an arrow from it to every other object in the category. This is $\perp$, the starting point in the trail and the arrows from it are every potential trail. This has significance for the reasoning and logical content that resides in hypertext links.

In hypertext the initial and terminal objects may have only a local context. There may not be one single starting point, there may be a number of origins for any given trail. Likewise a trail may diverge to more than one finite point. Also natural language is a more general category than that of sets and the trails need not be disjoint. The same words could be used but with two distinct links in thought.

4 Reasoning in Context and the Hypertext Lattice

As pointed out[3] by Ted Nelson in his early idea of hypertext as "non-sequential writing with reader-controlled links", links in hypertext are rarely linear[3] but branch and form a distributive lattice. The internal logic of a lattice is geometric logic which is more general than Boolean logic. The logic of a lattice is well-established. It is equivalent to a Heyting algebra. Any Heyting algebra has a fundamental binary operation of implication $\Rightarrow$: $\mathbf{A} \longrightarrow \mathbf{B}$. This arrow is commonly written in the form $\mathbf{A} \Rightarrow \mathbf{B}$ and this shorthand version will be used here. This implication arrow is defined by the adjunction

$$\frac{(\mathbf{C} \times \mathbf{A}) \leq \mathbf{B}}{\mathbf{C} \leq (\mathbf{A} \Rightarrow \mathbf{B})}$$

$\mathbf{A} \Rightarrow \mathbf{B}$ is the largest category connected with A which is contained in B. In hypertext terms if the current document (A) in its context (C) precedes document B, then B is the next document after A in that context. In terms of concepts rather than documents, the concept may not be represented by a document in existence and from the point of view of a

[3]SGML and derivatives are essentially linear and this can give rise to problems of non-linearity in hypertext for instance when dealing with entities which are shared subobjects.

writer would be the next document to write.

By the application of this implication we can obtain the more generalized type of negation found in natural systems. Indeed in natural language it is often possible to represent negative concepts in a positive way. This is also true in hypertext where falsity and truth are not simple atomic entities. These are geometric concepts. Truth is given by $\mathbf{B} \Rightarrow \top$ and falsity by $\mathbf{A} \Rightarrow \bot$, sometimes written $\neg\mathbf{A}$.

Truth and falsity are relative to context. In hypertext, $\mathbf{A} \Rightarrow \bot$ is (usually back) in the direction of the initial document, a state of ignorance, whereas $\mathbf{B} \Rightarrow \top$ is forward in the direction towards the last document to be viewed in the lattice, the state of enlightenment. Knowledge and ignorance in hypertext are the counterparts of true and false.

The nature of the pseudocomplement then $\mathbf{A} \Rightarrow \bot$, that is not $\mathbf{A}$, may be further understood by substituting the special instance $\bot$ for $\mathbf{B}$ in the definition of the adjunction above. We then get

$$\frac{(\mathbf{C} \times \mathbf{A}) \leq \bot}{\mathbf{C} \leq (\mathbf{A} \Rightarrow \bot)}$$

In the real world two negatives do not always make a positive. This is familiar in natural language which opposes the principle of *tertium non datur.* The pseudocomplement is so important that natural languages often make it a separate word. For example the concept *relevant* has the pseudocomplement *irrelevant* which results in the further concept of *not irrelevant.* So *not irrelevant* is not equivalent to *relevant.* In fact there is a Heyting ordering:

	$\mathbf{B} \Rightarrow \bot$	$\leq (\mathbf{B} \Rightarrow \bot) \Rightarrow \bot$	$\leq \mathbf{B} \Rightarrow \top$
i.e.	irrelevant	$\leq$ not irrelevant	$\leq$ relevant

In hypertext terms, this gives a ranking of the relevancy of the documents in general terms for **B** the next possible document. It is an irrelevant document, if it is in the direction of the first document. It is the required next relevant document, if it is in the direction of the final document in the trail. Note when it is not irrelevant. That is, if it is not in the direction of the first document, whether or not it is in the direction of the final document. It is this three-level ordering which is the basis of much fuzzy thought and a generalization of fuzzy sets.

In terms of the Heyting algebra, $\mathbf{C} \Rightarrow \bot$ is another special case of $\mathbf{A} \Rightarrow \mathbf{B}$. As noted above $\mathbf{A} \Rightarrow \mathbf{B}$ is itself a concept/document and $\mathbf{C} \Rightarrow \bot$ is an *irrelevant* context concept/document. A fundamental feature is that the pseudocomplement $\mathbf{A} \Rightarrow \bot$ is the largest category disjoint from **A**.

5 Consciousness and Legal Cognition

The human brain is able to handle well the integration of multimedia stimuli and hypermedia navigation is comparable to mental processes. Consciousness is increasingly being recognized as an inherent feature of any cognitive process[35] and needs to figure in any computational model involving human-computer interaction at the level of the mind[14, 30] to counteract critics of machine understanding.

From a taboo subject in orthodox scientific circles, consciousness is fast becoming an essential ingredient to be considered in any research involving human cognition[6, 12].

5.1 Contextual Awareness in Hypertext

The earlier discussion on context with pullbacks and pushouts deals with the simpler straight-forward type of static and objective contextuality but it is perhaps worth looking at the example previously raised:

"As I said in my speech on 28th October to this House ..."

A simple form of contextual awareness can be attained in this example by state of the art database techniques using fields, relations or keys. Thus the information identifying *I*, *House*, and *year* can be anaphorically resolved by reference to meta-records in the database system. Fuller details on how this works using keys in a Hansard database are given elsewhere[18, 31] where partial or composite keys are examples of colimits.

The hypertext system of awareness is one that identifies for the user the next document to see. This is available from the implication $\mathbf{A} \Rightarrow \mathbf{B}$. Thus awareness is the contravariant natural transformation $\phi : \mathbf{B} \longrightarrow \mathbf{A}$. Awareness in hypertext is therefore the self identification of the document **B** in $\mathbf{A} \Rightarrow \mathbf{B}$. In Figure 4, it is the document records. The awareness to identify *I* is the *Hansard Record*, the *House* and the *Year* as given in Figure 8. This figure shows how the awareness works. The identity of the speaker *I* is given by ϕ_3, the identification of which house (Lords or Commons) by ϕ_2 and the awareness of the date of the speech from ϕ_1. These can be obtained algebraically. For example $f \circ \phi_2 = \phi_1$. f is the meta-record giving the house where the speech is given. In a database implementation f consists of those parts of the composite key which uniquely identify the *House.*

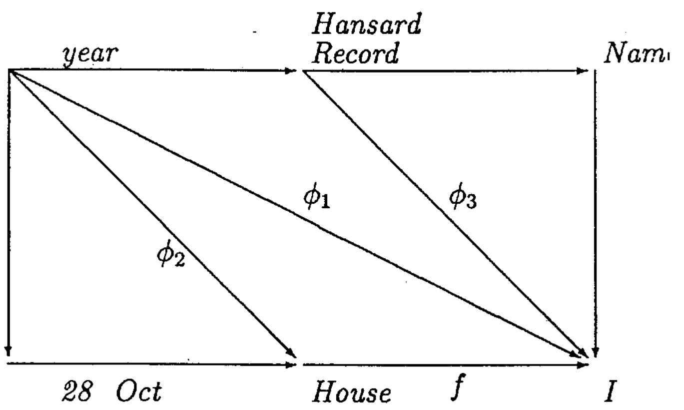

Figure 8: Awareness to identify *I* and *House* in "As I said in my speech on 28th October to this House ..."

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5.2 Computational Model of Conscious Legal Reasoning

While the basic purpose of hypertext awareness is for that next document **B** in **A** $\Rightarrow$ **B** to identify itself to the user, the position is complicated by the fact that the user is operating at two levels: the intension (represented by the document) and the extension (represented by the meaning). This identification is a precompositional contravariant arrow $\alpha^* : \mathbf{B} \longrightarrow \mathbf{A}$ which is a backward selection from the relevant documents of the document to which they are related.

Hypertext links are really connections between the semantic objects in the current document with related semantic objects in the documents to be retrieved. The connection is between objects $A_1, A_2, \ldots$ in the category $\mathbf{S(A)}$ (that is the meaning of the contents of the document under examination which is **A**) with objects $B_1, B_2, \ldots$ in the category $\mathbf{S(B)}$ which are the meanings in the documents to be retrieved **B**. The identification of the relevant documents depends upon the purpose and intentions of the user. This is a natural transformation $\eta_A : A \longrightarrow \mathbf{S(B)}$ as shown in Figure 9. The awareness is given by the inverse natural transformation α^*, preserving limits, colimits and implications.

$\alpha^*(\top) \cong \top$,
$\alpha^*(B \times B') \cong \alpha^*(B) \times \alpha^*(B')$,
$\alpha^*(\bot) \cong \bot$,
$\alpha^*(B + B') \cong \alpha^*(B) + \alpha^*(B')$,
$\alpha^*(B \Rightarrow B') \cong \alpha^*(B) \Rightarrow \alpha^*(B')$

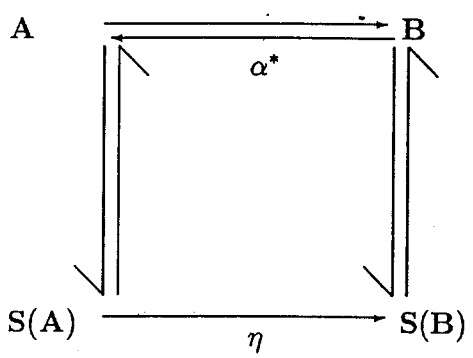

Figure 9: Commuting Target Square for Awareness as a Natural Transformation

The signs for products and sums are again used to represent generally limits and colimits respectively. Table 1 shows that none of the database relationships and structures in section 5 require any operations beyond these. Therefore $\alpha^*(\mathbf{B})$ can claim to be a general awareness relationship.

This relationship is valid for reasoning by analogy only when implications are preserved. The test for the preservation of implications is well–established and known as the Frobenius identity[25]:

$$\frac{\alpha^*(B \times \alpha A)}{\alpha^* B \times A}$$

This is equivalent to analagous reasoning because the inverse natural transformation α^* preserves limits and colimits as well as implications.

5.3 Relative and Dynamic Contexts

Simple categories may be built–up to represent the greater complexity found in hypertext systems. For instance a concept, that emerges from a structure of related documents, itself is a diagram as previously indicated and may be used to replace a single object A. These can be employed to give hypertext the facility to deal with dynamic, subjective content. In geometric logic this amounts to manipulating more sophisticated structures for diagrams are a more general form of objects and simple categories.

For example the comma category has attracted considerable attention in computing science[1] and can provide general contextuality. The comma category can add structure to an ordinary category by considering the arrows from the point of view of a particular object. Given a category **A** with a variable object A which may be represented by A' (when we want to distinguish different instances), the arrows $f : A \longrightarrow A'$ relative to C are objects in the comma category $\mathbf{A/C}$ (sometimes written $\mathbf{A} \downarrow \mathbf{C}$) as shown in Figure 10. It should be emphasised that the objects in the comma category are arrows; the comma category arrows are triangles. For a map of the domain A and codomain A' together onto C specifies $f : A \longrightarrow A'$.

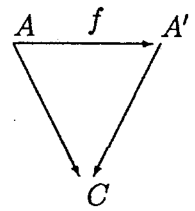

Figure 10: Diagram of Comma Category

In practice hypertext does not just relate two documents but two documents in their respective categories. Therefore the hypertext link between document **A** (the one being viewed) and the next document **B** is given in Figure 11. The functor K is the hypertext link between the categories where the objects in each category are triangles composed of lower–level arrows. This shows up the dynamical aspects of context. Figure 12 shows the corresponding contravariant functor α^* between comma categories. Consciousness, with relative and dynamic context, is obtained by generalizing from the following relationships shown in Figures 11, 12:

$$K : \mathbf{A/C} \longrightarrow \mathbf{B/C'}$$

$$\alpha : f \longrightarrow g$$

$$\alpha^* : \mathbf{B/C'} \longrightarrow \mathbf{A/C}$$

The whole collection can be viewed as analagous reasoning thus confirming the equivalence of legal reasoning and hypertext.

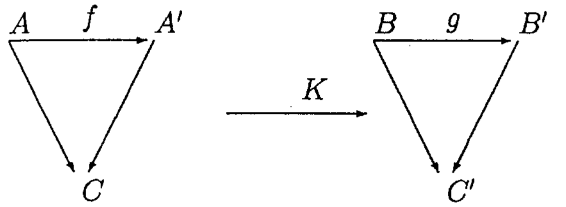

Figure 11: Covariant Functor K between Comma Categories

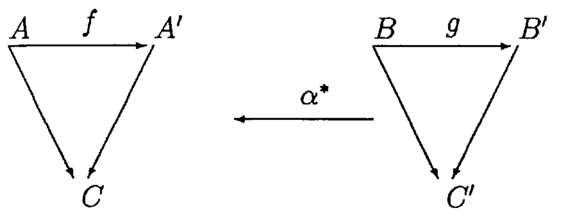

Figure 12: Contravariant Functor α^* between Comma Categories

6 Conclusions

The Law has developed into a very sophisticated information system, one that is populated by a variety of distributed continuously-changing heterogeneous materials scattered across documents and unwritten in the thoughts and minds of participants in the legal process like lawyers, legislators and litigants. These dynamic physical and non-physical objects of different forms and formats are pulled together at any given time to create a contemporaneous information subsystem to be navigated by established principles of legal reasoning like as it were a virtual hypertext system.

The view of the instant hypertext document in cyberspace is shaped by the real–world perceptions of the user interacting with the various documents encountered on the way. With the power to backtrack and its inherent branches, the perception of the document is more than just shaped by a linear sequence of links. It is a Heyting lattice. Likewise legal reasoning consists of a Heyting algebra of open concepts where the structure is shaped by the client's viewpoint and ultimate objectives. Inferences of legal reasoning are comparable to the links and connections for document navigation in a legal hypermedia information system where one document follows on from the next. This is a prime example of the Curry-Howard isomporphism between intuisionistic logic and computational type theory. The inference required with law and for the legal document sought is both given by the same Heyting implication $\mathbf{A} \Rightarrow \mathbf{B}$ while awareness in both is representable in geometric logic by the contravariant pull-back functor α^*.

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