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PERFORMANCE OF FREQUENCY-HOPPED SPREAD-SPECTRUM MULTIPLE ACCESS COMMUNICATION SYSTEMS WITH READABLE ERASURES*

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ABSTRACT

As a way to improve user capacity, we introduce and evaluate in this paper the readable-erasure channel for frequency-hopped spread-spectrum multiple access systems using MFSK modulation scheme (FH/MFSK). For an MFSK scheme, in contrast to the erasure-only channel having $M+1$ outputs, the readable-erasure channel has $2M$ outputs, M of which correspond to the outputs when no hits occur and M of which represent the outputs when channel is hit. For comparison purposes, the performances of readable-erasure and erasure-only channels both using Reed-Solomon codes and using BCH codes are evaluated. It is shown that the readable-erasure outputs can help to improve user capacity. By taking the spectral efficiency into consideration, we point out that directly applying RS codes to the FH/MFSK systems is very inefficient in spectrum utilization. Instead, the channel with readable-erasure outputs using BCH codes and 4-ary FSK is suggested for FH/MFSK systems.

1. INTRODUCTION

The frequency-hopped (FH) spread-spectrum multiple access communication systems, one kind of code division multiple access systems, have received intensely attention during the recent two decades. Several different kinds of FH spread-spectrum systems were proposed utilizing M-ary frequency shift keying (MFSK) modulation scheme, collectively named FH/MFSK systems. Among those systems, the one with which we are concerned in this paper was described in [3]-[7]. For this kind of FH/MFSK spread-spectrum communication systems, the entire available bandwidth is divided into q frequency slots, shared by K sender-receiver pairs simultaneously operating in the system. The signal from a given user is hopped from slot to slot by changing the carrier frequency at certain points in time called hop epochs [4]. The sequence of carrier frequencies of a signal is known as its frequency-hopping pattern. Two kinds of frequency-hopping patterns can be used: random patterns or a set of deterministic patterns. One model for a random pattern is a sequence of independent random variables, each of which is uniformly distributed over the set of q frequencies; this kind of pattern is called memoryless pattern in [3]. Several kinds of sets of well-constructed deterministic patterns are known; a detailed description of coded sequences for frequency-hopping patterns can be found in [12]. For performance analysis, the memoryless hopping pattern is usually assumed due to its simplicity [1]-[7]. Whenever two different users hop to a same frequency slot, a hit is said to occur. The probability of the occurrence of hits primarily dominates the performance of the frequency-hopped systems.

For digital communication systems, the error control codes are used to protect the signals against the errors caused by the channels, and among those codes, the Reed-Solomon (RS) code is one of the most preferable codes due to its maximum-distance property. The performance of the FH/MFSK system in conjunction with RS codes was studied in [3]-[6]. In those papers, the RS codes are used as erasure-correcting codes. To do so, the demodulator declares an erasure when a hit occurs. However, under a hit some channel capacity still exists. It is believed that if the decoder can output a decision as well as an erasure, referred to as a readable erasure in [10], then performance improvement is possible. In this paper we refer to the channel with readable erasures as a readable-erasure channel, while the channel with traditional erasures as an erasure-only channel [5]. In addition, for an RS code over Galois field GF(M), the code length n is equal to $M-1$. Hence, to employ an RS code, the channel requires a large value of symbol alphabet M. This in turn results in an inefficient utilization of the channel bandwidth because for

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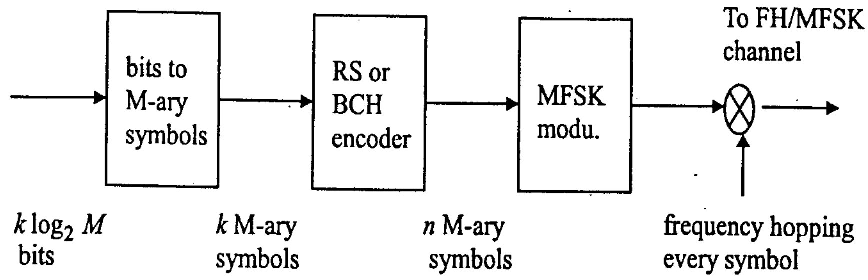

Fig. 1 Model of transmitter with channel code of length n and dimension k.

the MFSK modulation scheme, the bandwidth efficient decreases as M increases. Therefore, in this paper we also consider the BCH codes [11], for which the code length can be arbitrary large while keeping a moderate value of M. Performance evaluation of the readable-erasure channel associated with RS codes and with BCH codes is the primary subject of this paper, and the performance of erasure-only channel is also carried out for comparison purpose.

The rest of this paper is organized as follows. Section 2 describes the channel models. Section 3 deals with the channel-symbol error probability of the readable-erasure channel. The decoding error probabilities of the codewords for RS codes and for BCH codes are derived in Section 4. Finally, conclusions are drawn in Section 5.

2. DESCRIPTION OF CHANNEL MODELS

The transmitter model of each user with a channel code of length n and dimension k is shown in Fig. 1. Every block of $\log_2 M$ bits constitutes an M-ary symbol, and k symbols are encoded to obtain an n-symbol codeword. Then, every codeword is transmitted over the frequency-hopped channel by using MFSK modulation. At the receiver, the demodulator attempts to determine the transmitted symbols, and then, n received symbols are decoded to obtain $k(\log_2 M)$ bits, which are passed to the data sink.

It is assumed that one symbol is transmitted during one hop interval. By this we mean the system under consideration is a slow frequency-hopped system [3]. Also, we assume a synchronous system; that is, the hop epochs of all the users are the same. Hence, there are no partial hits, and the number of users occupied in any frequency slot is independent from hop to hop.

Under a hit, an erasure is output from the erasure-only channel, while the readable-erasure channel always attempts to determine the transmitted symbol and gives a 'weak' output. If the weak symbol is different from the transmitted one, a channel-symbol error is said to occur. Obviously, the error probability is strongly dependent on the number of users involved in the hit. So, we denote the probability of channel symbols with z users involved in a hit being correctly decoded by $p_{cs|z}$. We assume that the signal strengths from different users are the same. Also, it is assumed that a perfect side information about the total signal strengths of each of the M tones is available for the demodulator [2], so that the demodulator always knows how many signals occupied each of the M tones. By this information the demodulator can choose the tone being occupied by the most number of users as its output. If there are two or more such tones, one of them is randomly chosen. Clearly, this decision rule will result in a $p_{cs|z}$ definitely larger than $1/M$. Notice that under hits, if the demodulator chooses one symbol as its output from among the M symbols at random with no attempts being made to 'pick up' the strongest symbol, then the $p_{cs|z}$ is equal to $1/M$; this is the case of without side information and has been widely studied [3, 5, 6]. Fig. 2 shows the state-transition diagram of the readable-erasure channel, where p_h denotes the probability of a hit and the states with a "-" subscription denote a weak symbol due to a hit. Note that by integrating all the weak states into an erasure state, one can obtain a model of erasure-only channel.

Suppose there are K users operating in the system and there are q frequency slots available. Given a specific user hopping to a specific frequency slot, the probability that

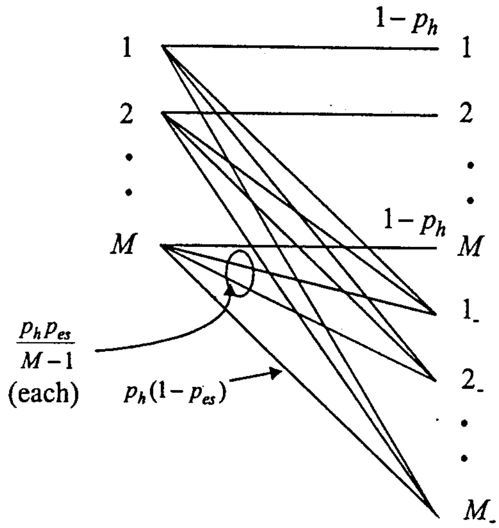

Fig. 2 State-transition diagram of readable-erasure channel.

u of the other $K-1$ users hop to the same slot has the following binomial distribution:

$$P_U(u;K-1)=\binom{K-1}{u}p_q{}^u(1-p_q)^{K-1-u}, \quad (1)$$

where $p_q = 1/q$. It is clear that $u=0$ means that the transmission of the given user is not hit. So, the probability of a hit is

$$p_h = 1-(1-p_q)^{K-1}, \quad (2)$$

and the probability of j of the n symbols of a transmitted codeword being hit is

$$P_H(j)=\binom{n}{j}p_h{}^j(1-p_h)^{n-j}. \quad (3)$$

3. SYMBOL ERROR PROBABILITY FOR READABLE-ERASURE CHANNEL UNDER HITS

In this paper we focus our attention on the multiple-access capacity of the system, so the background noise, the channel fading, and the other factors that might corrupt the transmissions are ignored for simplicity. By this we mean the only factor that affects the transmissions is the interference arising from the other users [1]-[7]. Hence the transmissions are considered to be error free for both readable-erasure and erasure-only channels when no hits. In this section, we derive the $p_{cs|z}$ for readable-erasure channel under hits.

First, we define the two-variable probability mass function (pmf) $P(j,i;z,M)$ as the probability that there are j tones each being occupied by i users' signals and the remaining $M-j$ tones each are occupied by less than j signals, resulting from z users hopping over a same M-ary frequency band. To specify the pmf, we further define variables $t_1,t_2,...,t_M$ as the numbers of users transmitting signal over tones $f_1,f_2,...,f_M$, respectively. Given z users hopping over a same frequency band, we have $t_1+t_2+...+t_M=z$. By the definition of the pmf, j corresponds to $\max(t_1,t_2,..,t_M)$, and i is the number of variables $t_1 \sim t_M$ which have value j. It is clear that, $1 \le i \le M$, and $\lceil z/M \rceil \le j \le z$, where $\lceil x \rceil$ denotes the smallest integer equal to or larger than x. To express $P(j,i;z,M)$ as a closed-form for general values of M and z seems very difficult, but through a series of combinatorial exercises, we can get a recursive form for the expression.

Consider a grid containing M rows and z columns. One O is placed in each column with randomly choosing the row location. This results in M^z distinct patterns of O's. Obviously the number of O's in the xth row means that the total number of users transmitting signal over tone f_x. Here we define $N(j,i;z,M)$ as the number of patterns in the grid with the same definitions for its arguments as those for $P(j,i;z,M)$. Therefore, both functions are related by

$$P(j,i;z,M)=\left(1/M^z\right)N(j,i;z,M). \quad (4)$$

This implies that we can compute $P(j,i;z,M)$ by first determining $N(j,i;z,M)$. For given M and z, j should satisfy that $\lceil z/M \rceil \le j \le z$, where $\lceil x \rceil$ denotes the smallest integer larger than or equal to x. Moreover, the variable i associated with a particular j should satisfy that $z-ij \le (M-i)(j-1)$ and $i \le \lfloor z/j \rfloor$ where $\lfloor x \rfloor$ denotes the integer part of x. These constraint are required because that ij cannot exceed z and, by definition, any one of the remaining $(M-i)$ rows cannot have O's more than or equal to j. For values i and j that cannot satisfy these constraints, $P(j,i;z,M)=0$. To simplify the notation, we denote the set of possible integers for i given j, M, and z by $I\{i \mid j;z,M\}$.

With above ranges for j and i, we have

$$N(j,i;z,M)=\binom{M}{i}\binom{z}{j}\binom{z-j}{j}\cdots\binom{z-(i-1)j}{j}(M-i)^{z-ij},$$
$$i=\left\lfloor \frac{z}{j} \right\rfloor; \quad (5)$$

$$N(j,i;z,M)=\binom{M}{i}\binom{z}{j}\binom{z-j}{j}\cdots\binom{z-(i-1)j}{j}\times$$
$$\left[(M-i)^{z-ij}-\sum_{y=j}^{z-ij}\sum_{x\in I\{x|y;z-ij,M-i\}}N(y,x;M-i,z-ij)\right],$$
$$i<\left\lfloor \frac{z}{j} \right\rfloor. \quad (6)$$

Equation (5) represents that there are $\binom{M}{i}$ ways to choose those i rows, and for the first row there are $\binom{z}{j}$ ways to distribute the i O's, the second row $\binom{z-j}{j}$ ways, and so on. The last term in (6) denotes that there are $(M-i)^{z-ij}$ ways to place the remaining O's (the number of which is definitely less than j) in the rest $(M-i)$ rows, For the case $i<\lfloor z/j \rfloor$ since the number of the remaining O's is larger than j, we must eliminate those patterns appearing in the other $(M-i)$ rows having j or more O's. The recursive term in (6) accounts for this situation.

By using (4), (5), and (6), $P(j,i;z,M)$ can be completely determined. Note that for the special case $M=2$, i.e., BFSK scheme, no recursive call is required and the expression can be reduced to

$$P(j,iz,2)=\binom{2}{i}\binom{z}{j}\left(\frac{1}{2}\right)^z, \quad (7)$$

given that $\lceil z/2 \rceil \le j \le z$, and $i=2$ if $j=z/2$,

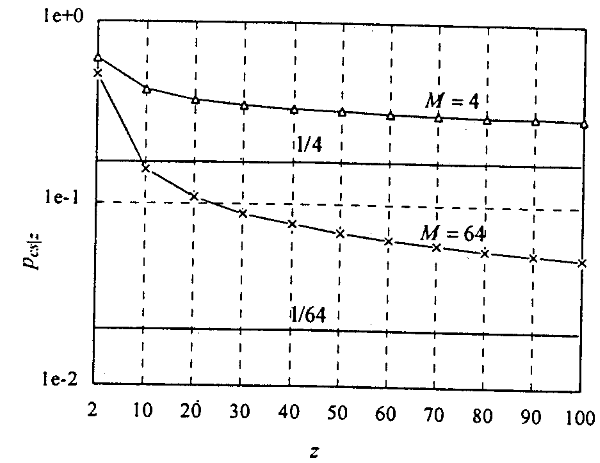

Fig. 3 Probability of channel symbol being correctly decoded under hits.

$i=1$ if $j>z/2$; otherwise, $P(j,i;z,M)=0$.

Having specified the pmf, we now can determine the $p_{cs|z}$ and the channel symbol error probability, denoted by $p_{es|z}$. Since only j out of z signals will be correctly decoded, the $p_{es|z}$ is given by

$$p_{es|z} = 1 - p_{cs|z},$$
$$= 1 - \sum_j \sum_i P(j,i;z,M)\frac{j}{z}. \quad (8)$$

Fig.3 shows the values of $p_{cs|z}$ for $M=4$ and $M=64$ with z ranging from 2 to 100. Two basic lines of the inverse of 4 and 64 are also shown, corresponding to the case without side information. This figure shows that when z is small, the values of $p_{cs|z}$ are quite better than $1/M$, but the gain approaches zero as z becomes large.

By averaging (8) over z, we can evaluate p_{es}; that is,

$$p_{es} = \frac{\sum_{z=1}^{K-1} p_{es|z+1} P_U(z;K-1)}{p_h}. \quad (9)$$

4. DECODING ERROR PROBABILITIES AND CHANNEL THROUGHPUTS

4.1. Decoding error probabilities

For an RS code of length n and dimension k, denoted by (n,k), the minimum distance d^* is equal to $(n-k+1)$, and the code is capable of correcting up to $t=\lfloor (n-k+1)/2 \rfloor$ errors or d^*-1 erasures. The statement is due to the fact that RS codes are maximum distance separable (MDS) codes [11]. Because BCH codes are not MDS codes, the above statement does not hold for BCH codes. For a BCH code to be capable of correcting t errors, k is determined by $(n-s)$, where s is the degree of the generating polynomial of the code. So, for a given code length n, s needs to be determined before k can be determined. The d^* of the BCH code is, therefore, $(2t+1)$. Several possible pairs of k and t for BCH codes of length 63 over GF(4) are given in Table 2.

Depending on the requirements of the application, a received word can be treated using one of the following two algorithms: one is referred to as incomplete (bounded-distance) decoding algorithm and the other is complete decoding algorithm [11]. The incomplete decoding algorithm assigns every received word to a codeword within distance t, if there is one, and otherwise declares the received word to be unrecognizable. The complete decoding algorithm always decodes every received word as a closet codeword. When high reliability performance is required, one may prefer to use incomplete decoder in conjunction with automatic-repeat-request protocol (see [8,9], for example), whereas when retransmissions are not feasible due to delay constraints, such as in most voice-communication systems, a complete decoder is adopted. In the following we derive the decoding error probabilities for readable-erasure and erasure-only channels. For channel coding, we consider both RS codes and BCH codes, and we assume complete decoders. Since the codes to be considered are linear, any codeword is equally likely to be decoded erroneously. With no loss of generality, we can choose the all-zero codeword as the transmitted one and calculate the word error probability P_E caused by all error patterns.

Case 1: Reed-Solomon codes

- P_E for erasure-only channel

Suppose that a word with a specific pattern containing v erasures is received. Then the codeword which differs in the least components from the unerased portion of the received word is decoded as the transmitted word. If there are several codewords which differ in the same components from the unerased portion of the received word, then the decoder chooses as it output from among those codewords at random. It is clear that for $v<d^*$, since the all-zero codeword is always chosen, $P_E=0$. If $v \ge d^*$, a decoding error might occur.

Usually P_E of an erasure-only channel is approximately estimated as the probability that the number of erased symbols of a received word exceeds or equals d^* [3]-[6]. However, the MDS property of RS codes can help us calculate P_E accurately. To determine P_E, we need the following corollary:

Corollary: Consider an (n,k) MDS code over GF(q). The number of codewords which contain a specific set of v erasures, where $d_{\min} \le v \le n$, and the rest $n-v$ components are all zeros is equal to $q^{k-(n-v)}$.

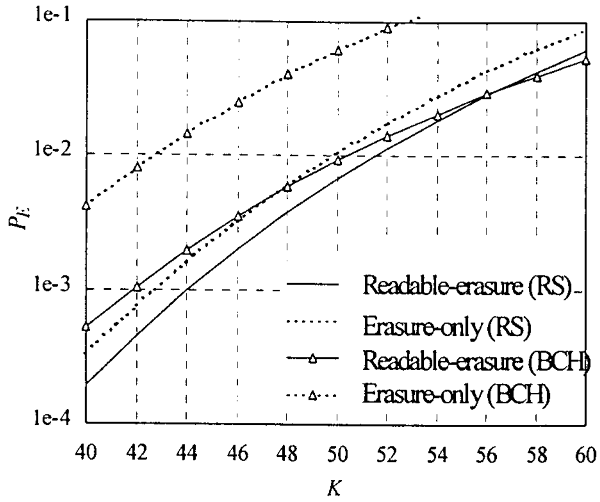

Fig. 4 Decoding error probabilities for (63 ,30) RS code and (63,11) BCH code with readable-erasure and erasure-only channels.

Proof: For an (n,k) MDS code, any set of k components uniquely specifies all the q^k codewords [11]. Since there are v erasures, the unerased components are $n-v$. Choose these $n-v$ components plus additional $k-(n-v)$ components as information symbols. If all the $n-v$ components are set to zero, then we can specify exactly $q^{k-(n-v)}$ codewords. This completes the proof.

By this Corollary, it is known that there are $q^{k-(n-v)}$ codewords of which the unerased components are all zeros. Since the decoder is equally likely to decode the received word as any of these codewords, P_E, as a function of v, is given by

$$P_E(v) = 1 - \frac{1}{q^{k-(n-v)}}. \tag{10}$$

By averaging over v, we can obtain P_E, given that

$$P_E = \sum_{v=d^*}^{n} P_E(v) P_H(v), \tag{11}$$

where $P_H(v)$ is given in (3).

• P_E for readable-erasure channel

With a specific erasure pattern, the possible error symbols of a received word are constrained in those erased words, and the unerased components are known to be error free. Since a code-symbol error can occur only when there are at least d^* erased symbols and at least $t+1$ of them are in error. To exactly compute P_E using a complete decoder seems somewhat complicated. Instead, we upper bound P_E, as a function of v, by

$$P_E(v) \le \sum_{i=t+1}^{v} \binom{v}{i} (p_{es})^i (1-p_{es})^{v-i}, \quad v \ge d^*, \tag{12}$$

where p_{es} is given in (9). Similar to (11), we have

$$P_E \le \sum_{v=d^*}^{n} P_E(v) P_H(v). \tag{13}$$

Fig. 4 shows the P_E s as a function of K for (63,30) RS code with both channels considered. The curves for (63,11) BCH code shown in this figure will be discussed later. The number of frequency slots q is assumed to be 100 throughout this paper. We observe from this figure that for given a constraint that $P_E \le 10^{-2}$, the maximum values for readable-erasure and erasure-only channels are 51 and 49, respectively. At this point, it is realized that the readable-erasure channel indeed has larger user capability than its erasure-only counterpart. This argument still holds for other values of k of interest, as will be shown below. Notice that the P_E for readable-erasure channel is an upper bound, whereas that for erasure-only channel is an exact value.

Case 2: BCH codes

For a noncoherent MFSK modulation communication system and a given source data rate R_b, the channel symbol rate is $R_s = R_b/\log_2 M$. In order to guarantee the outputs of these M tones are orthogonal when noncoherent demodulation applies, the bandwidth required for each M-ary frequency slot is at least [1]

$$W = M \frac{R_b}{\log_2 M}. \tag{14}$$

For a given R_b, $M=2$ and $M=4$ require the same amount of bandwidths, and the cases $M>4$, W is a monotonically increasing function of the channel alphabet M. The length n of an (n,k) RS code over GF(M) is equal to $M-1$. Thus, when n requires large so does M. This, in turn, leads to inefficiency in spectrum utilization. By using (14) and assuming $Q = M^\gamma$, the ratio of bandwidth required for a Q-ary system to an M-ary system is determined by

$$\mu = \frac{1}{\gamma} M^{\gamma-1}. \tag{15}$$

For example, the bandwidth required for a $Q=64$ system is 16/3 times as large as that required for an $M=4$ system. So, using a modulation scheme of large value of M to accommodate the symbol size required for RS codes seems very inefficient in spectrum utilization. Although, as shown in [8]-[9], we can use consecutive γ M-ary symbols to obtain a code symbol alphabet $Q = M^\gamma$, this approach leads to a poor channel-symbol error probability because a correct symbol is possible only when all the γ M-ary symbols are correctly decoded. Another approach is to use a reasonable value of M associated with BCH codes over GF(M). Since BCH codes are no longer maximum distance codes, the weight distributions are not known for most codes, and the exact analysis of decoding error probability for erasure-only channel seems very troublesome. So, it is better to resort to an upper bound; that is,

$$P_E \leq \sum_{v=d^*}^{n} P_H(v). \tag{16}$$

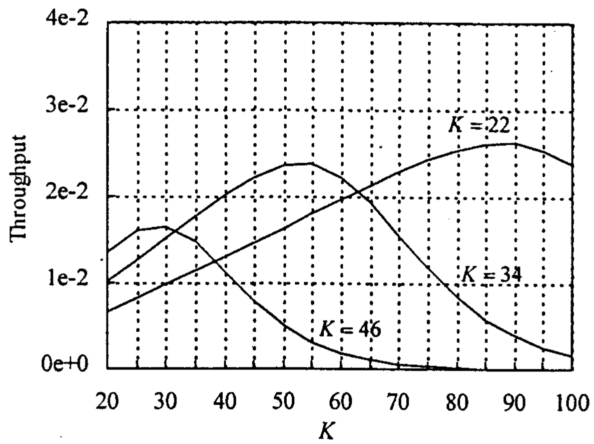

Fig 5 Maximum throughputs obtained directly using (17) for several $(63,k)$ RS codes with readable-erasure channel.

By this expression we mean that as long as d^* or more code symbols are erased, a decoding error occurs. As for RS codes, the P_E for readable-erasure channel can be evaluated by using (13). The P_E's for (63,11) BCH code are shown previously in Fig. 4. These curves shows that the maximum numbers of users for readable-erasure and erasure-only channels are 50 and 42, respectively. Clearly, there is a significant improvement in user capacity by using readable-erasure channel.

4.2 Channel throughputs

To account for the effect of the code rate in performance and the bandwidth required for different modulation schemes, we define the normalized channel throughput as

$$\eta = K\left[1 - P_E(K,n,k,q)\right]\frac{r}{wq}, \tag{17}$$

where $r = k/n$ is the code rate, and w is obtained by normalizing (14) with respect to R_b, that is $w = M/(\log_2 M)$, meaning that the bandwidth require for transmitting one bit per unit time. For a given q, η is a function of K and r. So, for a given code rate, it is believed that there exists an optimum value of K which maximizes η. Fig. 5 shows the normalized channel throughputs as a function of K for $(63,k)$ RS codes with readable-erasure channel obtained using (17). As shown, the optimum values of K for $k = 46$, 34, 22 are 29, 53 and 88, respectively. The values of P_E for $k = 46$, 34, 22 at the optimum throughputs are 1.6e-1, 1.08e-1, and 8.74e-2, respectively. Obviously, these values are too large relative to a suitable value, say 10^{-2}, for an applicable system. So, the system cannot operate at such conditions. For this reason, we impose an extra constraint of $P_E \leq 10^{-2}$ on (17). The optimum values of K and the corresponding throughputs obtained thereby are denoted by K^* and η^*, respectively. Table 1 depicts these values for several $(63,k)$ RS codes. We observe from this table that for all values of k, the values of K^* for readable-erasure channel is at least greater than that for erasure-only channel by one. It can also be seen that the optimum throughputs for readable-erasure is 2.4e-2 achieved when $k = 22$.

Similar to Table 1, the values of K^* and η^* for several $(63,k)$ BCH codes over GF(4) are depicted in Table 2. By carefully examining these results, we can see

Table 1 Maximum numbers of users and the corresponding throughputs for several $(63,k)$ RS codes over GF(64).

k	Readable-erasure		Erasure-only	
	K^*	η^*	K^*	η^*
18	88	2.33e-2	87	2.3e-2
22	74	2.4e-2	72	2.33e-2
26	61	2.34e-2	60	2.3e-2
30	51	2.25e-2	49	2.17e-2
34	42	2.1e-2	40	2e-2
38	33	1.85e-2	32	1.79e-2
42	26	1.61e-2	25	1.54e-2
46	20	1.35e-2	18	1.22e-2

Table. 2 Maximum numbers of users and the corresponding throughputs for several $(63,k)$ BCH codes over GF(4).

$k(t)$	Readable-erasure		Erasure-only	
	K^*	η^*	K^*	η^*
8(20)	81	5.09e-2	69	4.34e-2
11(15)	50	4.32e-2	42	3.63e-2
20(12)	36	5.38e-2	30	4.72e-2
27(10)	27	5.41e-2	23	4.88e-2
30(7)	17	4.01e-2	14	3.3e-2
36(6)	14	3.96e-2	11	3.12e-2
39(5)	11	3.37e-2	9	2.76e-2
45(4)	8	2.83e-2	6	2.13e-2

that the improvement of readable-erasure channel over erasure-only channel is quite prominent. On the other hand, the maximum throughput for readable-erasure channel is 5.41e-2 achieved when $k=27$. This value is 2.14 times that of RS coding case. This finding shows that, normalized to a unit of bandwidth, the amount of information the channel can accommodate by using BCH coding is at least 2.14 times that the channel can accommodate by using RS coding. As shown above, the ratio of w for BCH coding case to that for RS coding case is $3/16$. The reduction of the ratio from $16/3$ to 2.14 shows that the correcting capability of RS codes themselves is essentially better than that of BCH codes. Since the value $M=2$ is as spectrally efficient as $M=4$, we also evaluate the performances for several values of $(63,k)$ BCH codes over GF(2). The results show that the maximum value of η^* is slightly smaller than $M=4$ case, so those results are not shown. Consequently, we recommend readable-erasure channel utilizing BCH over GF(4) for FH/MFSK systems.

5. CONCLUSIONS

We have introduced and evaluated the user capacity of readable-erasure and erasure-only channels for FH/MFSK multiple access systems both using RS codes and using BCH codes. It has been shown that the readable-erasure channel can improve user capacity over erasure-only channel, especially for BCH coding case. In terms of maximum throughput, we have shown that the readable-erasure channel utilizing BCH over GF(4) is 2.14 times that of the same channel utilizing RS code over GF(64).

Although only synchronous systems were considered in this paper, it is believed that the trend of improvement of readable-erasure channel over erasure-only channel can hold for systems in which the hop epochs are not perfectly synchronous. We have not yet considered the asynchronous case because the exact analysis for channel-symbol error probability under hits seems very troublesome.

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