Fuzzy Control Design for Nonlinear Multiple Time-Delay Systems

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Abstract

To overcome the effect of modeling error between nonlinear multiple time-delay system and Takagi-Sugeno (T-S) fuzzy model with multiple time delays, a robustness design of fuzzy control is proposed in this paper. In terms of Lyapunov's direct method, a stability criterion is hence derived to guarantee the stability of nonlinear multiple time-delay interconnected systems. Based on this criterion and the decentralized control scheme, a set of model-based fuzzy controllers is then synthesized via the technique of parallel distributed compensation (PDC) to stabilize the nonlinear multiple time-delay interconnected systems and the H^{∞} control performance is achieved at the same time. Finally, a numerical example with simulations is given to demonstrate the concepts discussed throughout the paper.

Key words: Interconnected systems, T-S fuzzy models, fuzzy control, Lyapunov theroy.

I. Introduction

During the recent years, a number of research activities have been concerned with the topic of stability analysis and stabilization of interconnected systems, also called large-scale systems or composite systems [1]. In practices, due o the information transmission between subsystems, tine delays naturally exist in interconnected systems. The existence of time delays is frequently a source of instability [2]. Hence, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers wishing to inspect the properties of such systems.

Since the control design of nonlinear systems is a difficult process and the plants are always nonlinear in practical control systems, many nonlinear control methods have been proposed to overcome the difficulty in controller design for real systems. However, the control schemes for

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nonlinear systems are so complicated that they are not suitable for practical application [1]. Therefore, we need to develop a simplified model that can be used to design a controller. In the past few years, fuzzy-rule-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems. In attempt to obtain more flexibility and more effective capability of handling and processing uncertainties in complicated and ill-defined systems, Zadeh [2] proposed a linguistic approach as the model of human thinking, which introduced the fuzziness into systems theory [3]. Unlike conventional modeling, fuzzy rule-based modeling is essentially a multimodel approach in which individual rules (where each rule acts like a "local model") are combined to describe the global behavior of the system [4].

During the last decade, fuzzy control has been successfully applied to the control design of nonlinear systems (see [5-10] and the references therein). In these papers, a so-called Takagi-Sugeno (T-S) fuzzy model was employed to approximate a nonlinear plant, and then a fuzzy controller was designed to stabilize the T-S fuzzy model. All of them, however, neglect the modeling error between nonlinear system and fuzzy model. Existence of the modeling error may be a potential source of instability for control designs that have been based on the assumption that the fuzzy model exactly matches the plant [9]. Over the past two years, Chen et al. [1] and Kiriakidis [9] have proposed novel approaches to overcome the influence of modeling error in the field of model-based fuzzy control for nonlinear systems.

However, a literature search indicates that the effect of modeling error for nonlinear system has not been discussed yet. Therefore, A robustness design of fuzzy control via model-based approach for nonlinear interconnected systems is proposed in this study to overcome the effect of modeling error. Accordingly, the T-S fuzzy model is employed to approximate each nonlinear system. In this type of fuzzy model, each fuzzy implication is expressed by a linear system model, which allows us to use linear feedback control as in the case of feedback stabilization. The control design is carried out based on the fuzzy model via the parallel distributed compensation (PDC) scheme. The idea is that a linear feedback control is designed for each local linear model. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller [5, 8].

This paper is organized as follows. The system description is presented and the T-S fuzzy model is briefly reviewed in section II. In section III, a set of decentralized fuzzy controllers is described. In section IV, a robustness design of fuzzy control and a stability criterion are proposed to overcome the effect of modeling error. The design algorithm is proposed in section V. In section VI, a numerical example with simulations is given to illustrate the results. Finally, the conclusions are drawn in section VII.

II. System Description

Consider a nonlinear multiple time-delay interconnected system ***N*** composed of J subsystems N_j, $j=1,2,\cdots,J$. The *j*th subsystem N_j is described as follows:

$$\dot{x}_j(t) = f_j(x_j(t),u_j(t)) + \sum_{k=1}^{N_j} g_{kj}(x_j(t-\tau_{kj})) + \sum_{\substack{n=1\\ n\neq j}}^{J} C_{nj}x_n(t) + \phi_j(t) \tag{2.1}$$

where f_j f_j and g_{kj} are the nonlinear vector-valued function, $x_j(t)$ is the state vector, τ_{kj} (time delay) $k=1,2,\cdots,N_j$ are positive real numbers, $u_j(t)$ is the input vector, $\phi_j(t)$ denotes the external force and C_{nj} is the interconnection matrix between the *n*th subsystem and *j*th subsystems.

Definition 2.1 [4]**:** The solution of a dynamic system are said to be uniformly ultimately bounded (UUB) if there exist positive constants β and κ, and for every $\delta \in (0,\kappa)$ there is a positive constant $T = T(\delta)$, such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \leq \beta, \forall t \geq t_0 + T$$

In a little more than a decade ago, a fuzzy dynamical model had been developed primarily from the pioneering work of Takagi and Sugeno [14] to represent local linear input/output relations of nonlinear systems. This dynamical model is described by fuzzy IF-THEN rules and it will be employed here to handle the control design problem of the nonlinear interconnected system ***N***. The *i*th rule of this fuzzy model for the nonlinear interconnected subsystem N_j is proposed as the following form:

Rule *i*: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

$$\text{THEN} \quad \dot{x}_j(t) = A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + B_{ij}u_j(t) + \phi_j(t) \tag{2.2}$$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), \cdots, x_{gj}(t)] \in R^{1\times g}$ denotes the state vector,

$u_j^T(t) = [u_{1j}(t),\ u_{2j}(t), \cdots, u_{mj}(t)] \in R^{1\times m}$ denotes the control input,

$\phi_j^T(t) = [\phi_{1j}(t),\ \phi_{2j}(t), \cdots, \phi_{zj}(t)] \in R^{1\times z}$ denotes the unknown disturbances with a known upper bound $\phi_{upj}(t) \geq \|\phi_j(t)\|$. $i = 1, 2, \cdots, r_j$ and r_j is the number of IF-THEN rules; A_{ij}, A_{ikj}, and B_{ij} are constant matrices with appropriate dimensions; M_{ipj} ($p = 1, 2, \cdots, g$) are the fuzzy sets, and $x_{1j}(t) \sim x_{gj}(t)$ are the premise variables. The final state of this fuzzy dynamic model is inferred as follows:

$$\dot{x}_j(t) = \frac{\sum_{i=1}^{r_j} w_{ij}(t)[A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + B_{ij}u_j(t) + \phi_j(t)]}{\sum_{i=1}^{r_j} w_{ij}(t)}$$

$$= \sum_{i=1}^{r_j} h_{ij}(t)(A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj}) + B_{ij}u_j(t)) + \phi_j(t) \tag{2.3}$$

with

$$w_{ij}(t) \equiv \prod_{p=1}^{g} M_{ipj}(x_{pj}(t)), \quad h_{ij}(t) \equiv \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \tag{2.4}$$

in which $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}. In this paper, it is assumed that $w_{ij}(t) \geq 0, \quad i = 1, 2, \cdots, r_j; \quad j = 1, 2, \cdots, J$ and $\sum_{i=1}^{r_j} w_{ij}(t) > 0$ for all *t*. Therefore, $h_{ij}(t) \geq 0$ and $\sum_{i=1}^{r_j} h_{ij}(t) = 1$ for all *t*.

In the next section, the concept of PDC scheme is utilized to design fuzzy controllers.

III. Parallel Distributed Compensation

According to the decentralized control scheme, a set of model-based fuzzy controllers is synthesized via the technique of parallel distributed compensation (PDC) to stabilize the nonlinear multiple time-delay interconnected system ***N***. The concept of PDC scheme is that each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts [9]. Since each rule of the fuzzy model is described by a linear state equation, a linear control theory can be used to design the consequent parts of a fuzzy controller. The resulting overall fuzzy controller, nonlinear in general, is achieved by fuzzy blending of each individual linear controller.

Hence, the *j*th model-based fuzzy controller can be described as follows:

Rule *i*: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

$$\text{THEN} \quad u_j(t) = -K_{ij} x_j(t), \tag{3.1}$$

where $i = 1, 2, \cdots, r_j$. The final output of this fuzzy controller is

$$u_j(t) = -\frac{\sum_{i=1}^{r_j} w_{ij}(t) K_{ij} x_j(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} = -\sum_{i=1}^{r_j} h_{ij}(t) K_{ij} x_j(t). \tag{3.2}$$

IV. H^∞ Control Design via Fuzzy Control

The purpose of this paper is two-fold: to stabilize the closed-loop nonlinear interconnected systems and to attenuate the influence of the external disturbance $\phi_j(t)$ on the state variable

$x_j(t)$ [24, 25]. The influence of $\phi_j(t)$ will worsen the performance of fuzzy control system. So, to guarantee the control performance by eliminating the influence of $\phi_j(t)$ is a significant problem in the control system. In this work, not only the stability of fuzzy control system is advised but also the H^∞ control performance is satisfied as follows:

$$\sum_{j=1}^{J} \int_0^{t_f} x_j^T(t) Q_j x_j(t)\, dt \le \eta_j^2 \sum_{j=1}^{J} \int_0^{t_f} \phi_j^T(t)\phi_j(t)\, dt \tag{4.1}$$

where t_f denotes the terminal time of the control, η_j is a prescribed value which denotes the effect of $\phi_j(t)$ on $x_j(t)$, and Q_j is a positive definite weighting matrix. The physical meaning of (4.1) is that the effect of $\phi_j(t)$ on $x_j(t)$ must be attenuated below a desired level η_j from the viewpoint of energy.

If the initial condition is also considered, the inequality (4.1) can be modified as

$$\sum_{j=1}^{J} \int_0^{t_f} x_j^T(t) Q_j x_j(t)\, dt \le \sum_{j=1}^{J} \mathrm{x}_j^T(0) P_j \mathrm{x}_j(0) + \eta_j^2 \sum_{j=1}^{J} \int_0^{t_f} \phi_j^T(t)\phi_j(t)\, dt \tag{4.2}$$

where P_j are some positive definite matrices.

IV. Robustness Design of Fuzzy Control

In this section, the stability of the nonlinear multiple time-delay interconnected system $\boldsymbol{N}$ is examined under the influence of modeling error. In subsection 4.1, the issue of modeling error is addressed and the guarantee of stability of $\boldsymbol{N}$ is given in subsection 4.2.

4.1 Modeling Error

Substituting Eq. (3.2) into Eq. (2.1) yields the *j*th ($j = 1, 2, \cdots, J$) closed-loop nonlinear subsystem $\overline{N}_j$ as follows:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j} \sum_{l=1}^{r_j} h_{ij}(t) h_{lj}(t) [(A_{ij} - B_{ij} K_{lj}) x_j(t) + \sum_{k=1}^{N_j} A_{ikj} x_j(t - \tau_{kj})]$$

$$+\phi_j(t)+\overline{f_j}(x_j(t))+\sum_{k=1}^{N_j} g_{kj}(x_j(t-\tau_{kj}))+\sum_{\substack{n=1\\ n\neq j}}^{J} C_{nj}x_n(t)$$

$$-\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)[(A_{ij}-B_{ij}K_{lj})x_j(t)+\sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})]$$

$$=\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)[(A_{ij}-B_{ij}K_{lj})x_j(t)+\sum_{k=1}^{N_j} A_{ikj}x_j(t-\tau_{kj})]$$

$$+\phi_j(t)+e_j(t)+\sum_{k=1}^{N_j}\bar{e}_j(t-\tau_{kj})+\sum_{\substack{n=1\\ n\neq j}}^{J} C_{nj}x_n(t) \tag{5.1}$$

where $\overline{f_j}(x_j(t))\equiv f_j(x_j(t),u_j(t))$ with $u_j(t)=-\sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t)$

$$e_j(t)\equiv[\overline{f_j}(x_j(t))-\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)(A_{ij}-B_{ij}K_{lj})x_j(t)], \tag{5.2}$$

$$\bar{e}_j(t-\tau_{kj})\equiv g_{kj}(x_j(t-\tau_{kj}))-\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)A_{ikj}x_j(t-\tau_{kj})$$

$$=g_{kj}(x_j(t-\tau_{kj}))-\sum_{i=1}^{r_j} h_{ij}(t)A_{ikj}x_j(t-\tau_{kj}) \tag{5.3}$$

and $\Delta\Phi_j(t)\equiv e_j(t)+\sum_{k=1}^{N_j}\bar{e}_j(t-\tau_{kj})$ denotes the modeling error between the jth closed-loop nonlinear subsystem (4.1) and the close-loop fuzzy model ((2.3)+(3.2)).

Suppose that there exists a bounding matrix ΔH_{ilj} such that

$$\|\Delta\Phi_j(t)\|\le\left\|\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t)\right\| \tag{5.4}$$

for the trajectory $x_j(t)$, and the bounding matrix ΔH_{ilj} can be described as follows [15, 16]:

$$\Delta H_{ilj}=\delta_{ilj}\overline{H}_j \tag{5.5}$$

where $\|\delta_{ilj}\|\le 1$, for $i,\ l=1,2,\cdots,r_j$, $j=1,2,\cdots,J$. From Eqs. (4.3–4.4), we have

$$\Delta\Phi_j^T(t)\,\Delta\Phi_j(t)\le\{\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t)\}^T\{\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\Delta H_{ilj}x_j(t)\}$$

$$= \{\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\delta_{ilj}\overline{H}_j x_j(t)\}^T \{\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\delta_{ilj}\overline{H}_j x_j(t)\}$$

$$\leq \left\|\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\delta_{ilj}^T\right\| \left\|\sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t)h_{lj}(t)\delta_{ilj}\right\| [\overline{H}_j x_j(t)]^T[\overline{H}_j x_j(t)]$$

$$\leq [\overline{H}_j x_j(t)]^T[\overline{H}_j x_j(t)]. \tag{5.6}$$

That is to say the modeling error $\Delta\Phi_j(t)$ is bounded by the specified structured bounding matrix $\overline{H}_j$.

Remark 4.1: The procedures for determining δ_{ilj} and $\overline{H}_j$ are described by the following simple example. Assuming that the possible bounds for all elements in ΔH_{ilj} are

$$\Delta H_{ilj} = \begin{bmatrix} \Delta h_{ilj}^{11} & \Delta h_{ilj}^{12} \\ \Delta h_{ilj}^{21} & \Delta h_{ilj}^{22} \end{bmatrix} \tag{5.7}$$

where $-\varepsilon_j^{rs} \leq \Delta h_{ilj}^{rs} \leq \varepsilon_j^{rs}$ for some ε_j^{rs}, $r, s = 1, 2$ and $i,\ l = 1, 2, \cdots, r_j$; $j = 1, 2, \cdots, J$.

One possible description for the bounding matrix ΔH_{ilj} is

$$\Delta H_{ilj} = \begin{bmatrix} \delta_{ilj}^{11} & 0 \\ 0 & \delta_{ilj}^{22} \end{bmatrix} \begin{bmatrix} \varepsilon_j^{11} & \varepsilon_j^{12} \\ \varepsilon_j^{21} & \varepsilon_j^{22} \end{bmatrix} = \delta_{ilj}\overline{H}_j \tag{5.8}$$

where $-1 \leq \delta_{ilj}^{rr} \leq 1$ for $r = 1, 2$. It is noticed that δ_{ilj} can be chosen by other forms as long as $\|\delta_{ilj}\| \leq 1$. Then, we check the validity of Eq. (4.3) in the simulation. If it is not satisfied, we can expand the bounds for all elements in ΔH_{ilj} and repeat the design procedures until Eq. (4.3) holds.

4.2 Stability in the Presence of Modeling Error

In the following, a stability criterion is proposed to guarantee the stability of the closed-loop nonlinear interconnected system $\overline{\boldsymbol{N}}$ which consists of *J* closed-loop subsystems described in Eq. (4.1). Prior to examination of stability of $\overline{\boldsymbol{N}}$, a useful concept is given below.

Lemma 5.1 [27, 28]**:** For any *A*, *B*$\in \boldsymbol{R}^n$ and for any symmetric positive definite matrix G$\in \boldsymbol{R}^{n\times n}$

or $\boldsymbol{R}$, we have

$$-2A^T B \le A^T GA + B^T G^{-1} B .$$

Theorem 4.1: The closed-loop nonlinear interconnected system $\overline{\boldsymbol{N}}$ is stable, if there exist symmetric positive definite matrices P_j and positive constants β_j, λ, γ and the feedback gains K_{ij}'s shown in Eq. (3.2) are chosen to satisfy

(I) $\hat{\lambda}_{inj} = \lambda_M (Q_{inj}) < 0$ for $i = 1, 2, \cdots, r_j$; $n, j = 1, 2, \cdots, J$ (4.10a)

$\tilde{\lambda}_{ilnj} = \lambda_M (Q_{ilnj}) < 0$ for $i < l \le r_j$; $n, j = 1, 2, \cdots, J$ (4.10b)

or

(II)
$$\Lambda_j = \sum_{n=1}^{J} \begin{bmatrix} \hat{\lambda}_{1nj} & \tilde{\lambda}_{12nj} & \cdots & \tilde{\lambda}_{1r_jnj} \\ \tilde{\lambda}_{12nj} & \hat{\lambda}_{2nj} & \cdots & \tilde{\lambda}_{2r_jnj} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\lambda}_{1r_jnj} & \tilde{\lambda}_{2r_jnj} & \cdots & \hat{\lambda}_{r_jnj} \end{bmatrix} < 0 \quad \text{for} \quad j = 1, 2, \cdots, J \tag{4.11}$$

or

(III)
$$\sum_{n=1}^{J} Q_{inj} + Q_j < 0 \tag{4.11}$$

where

$$Q_{inj} = \{\frac{1}{J}[(A_{ij} - B_{ij}K_{ij})^T P_j + P_j (A_{ij} - B_{ij}K_{ij}) + \sum_{k=1}^{N_j} R_{kj} + \sum_{k=1}^{N_j} P_j A_{ikj} R_{kj}^{-1} A_{ikj}^T P_j]$$
$$+ \frac{1}{J}[\beta_j \overline{H}_j^T \overline{H}_j + \beta_j^{-1} P_j^2 + \gamma_j^{-1} P_j^2] + \lambda (\frac{J-1}{J}) I + \lambda^{-1} P_j C_{nj} C_{nj}^T P_j \} \tag{4.12}$$

$$Q_{ilnj} = \{\frac{1}{J}[(H_{ilj}^T P_j + P_j H_{ilj}) + \sum_{k=1}^{N_j} R_{kj} + \sum_{k=1}^{N_j} P_j A_{ikj} R_{kj}^{-1} A_{ikj}^T P_j]$$
$$+ \frac{1}{J}[\beta_j \overline{H}_j^T \overline{H}_j + \beta_j^{-1} P_j^2 + \gamma_j^{-1} P_j^2] + \lambda (\frac{J-1}{J}) I + \lambda^{-1} P_j C_{nj} C_{nj}^T P_j \} \tag{4.13}$$

with $$H_{ilj} = \frac{(A_{ij} - B_{ij}K_{lj}) + (A_{lj} - B_{lj}K_{ij})}{2} . \tag{4.14}$$

Moreover, $\lambda_M (Q_{inj})$ and $\lambda_M (Q_{ilnj})$ denote the maximum eigenvalues of Q_{inj} and Q_{ilnj}, respectively.

V. Algorithm

Based on the above analysis, the complete design procedure can be summarized in the following algorithm.

Problem: For a given nonlinear interconnected system $\boldsymbol{N}$, how do we synthesize a set of decentralized fuzzy controllers to stabilize $\boldsymbol{N}$?

The problem described above can be solved in the following steps.

Step 1: Select the fuzzy plant rules and membership functions for each nonlinear subsystem N_j to establish its fuzzy model.

Step 2: Synthesize a set of decentralized model-based fuzzy controllers via the concept of PDC scheme.

Step 3: Based on Remark 4.1, the bounding matrix $\Delta H_{ilj}\ (= \delta_{ilj}\overline{H}_j)$, for $i, l = 1, 2, \cdots, r_j$, $j = 1, 2, \cdots, J$, are chosen to satisfy Eq. (4.3).

Step 4: If there exist some positive definite matrices P_j and the feedback gains K_{ij}'s to satisfy the stability conditions of Theorem 4.1 via LMI (linear matrix inequality) optimization algorithms, the nonlinear interconnected system $\boldsymbol{N}$ can be stabilized by the synthesized fuzzy controllers in Step 2. Otherwise, repeat Steps 2-3 to find appropriate fuzzy controllers and the bounding matrix $\Delta H_{ilj}\ (= \delta_{ilj}\overline{H}_j)$ such that the stability criterion is satisfied.

VI. Conclusions

In order to ensure the stability of nonlinear interconnected TMD systems, a stability criterion is derived from Lyapunov's direct method. According to this criterion and the decentralized control scheme, a set of model-based fuzzy controllers is synthesized to stabilize the nonlinear interconnected TMD system and overcome the influence of modeling error. Similarly, the common P matrix of the criterion is obtained by using linear matrix inequality (LMI) optimization algorithms to solve the robust fuzzy control problem. So, the proposed fuzzy control can be applied to any robust control design of nonlinear interconnected systems. Finally, a numerical example with simulations is provided to demonstrate the results.

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