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A Generalized Non-redundant 2D String Representation for Symbolic Pictures

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Abstract

In this paper, we generalize the non-redundant (NR) 2D string representation for symbolic pictures to reduce the storage needed and to avoid ambiguity. In image database systems, we can use 2D string representation to efficiently construct iconic indexes for pictures. But, 2D string requires storage for redundant elements. This shortcoming is overcome by the NR2D string representation. However, the ambiguous problem is arised under the NR2D string representation. The NR2D string representation confuses the switch components. Besides, the retrieval efficiency of image queries is degraded. In our generalized NR2D string representation, the different switch components are impossible to be transformed to the same 2D string representation. And thus our method can improve the retrieval efficiency of image queries. So the ambiguous problem can be overcome.

I. Introduction

The image database is widely used in many aspects, such as medicine, geography, and so on. In image database systems, the ways of storage of the images are very important. In 1981, Chang and Kunill offered to use pictorial index to establish databases [Chang and Kunill 1981]. That is, the pictorial indexing technique transforms images into symbolic pictures.

Tanimoto was the first one who introduced an iconic symbolic data structure [Tanimoto 1976]. The concept of the iconic index and the use of pictorial icons as pictorial indices were presented gradually. In 1987, a new pictorial data structure, the two-dimensional strings, was presented [Chang, Shi and Yan 1987]. The abbreviation of the two-dimensional string is 2D string. The idea is based on that an image picture can be transformed into a symbolic picture. That is, an image picture can be represented by some symbols. According to [Chang, Shi and Yan 1987], the 2D string representation of a symbolic picture is derived by orthogonally projecting the symbols in rows and columns [Chang, Shi and Yan 1987]. In this paper, we adopt the 2D string representation to construct iconic indexes for pictures. By using these indices, the retrieval efficiency of image queries can be improved. However, the 2D string representation needs more storage dealing with redundant elements.

To eliminate redundant elements, the NR2D string representation was presented in 1995 [Costaglioa et al 1995]. But, it suffers from the ambiguity of the representation. The ambiguous problem makes a 2D string representation have several corresponding symbolic pictures.

In this paper, we present a variation of the 2D string representation, the generalized NR2D string representation, which improves the traditional 2D string representations. The ambiguous characteristics for symbolic pictures will also be discussed under some 2D string representations. The organization of this paper is as it follows. Section II describes the 2D string representations for the symbolic pictures. Section III defines the generalized NR2D string representation. The reason why symbolic pictures are ambiguous under some 2D string representations is given. Then, we show that our representation has no ambiguity. Section IV shows that the pictorial reconstruction and retrieval by the generalized NR2D strings are as efficient as those by the traditional 2D strings [Chang, Shi and Yan 1987]. Finally, the conclusions are given in Section V.

II. 2D String Representation of Symbolic Pictures

In the section we recall the definitions of symbolic pictures and 2D strings given in [Tanimoto 1976, Chang et al. 1987 and Costagliola et al. 1995].

DEFINITION 1. [Chang, Shi and Yan 1987] Let V be a set of symbols, or the vocabulary. A symbolic picture f over V is a mapping $M * N \rightarrow W$, where $M = \{1, 2, ..., m\}$, $N = \{1, 2, ..., n\}$ and W is the power set of V.

DEFINITION 2. [Costagliola et al. 1995] A 2D string over V, written as (u, v) is defined to be ($x_1\ y_1\ x_2\ y_2 \ldots y_{n-1}\ x_n$, $x_{p(1)}\ z_1\ x_{p(2)}\ z_2 \ldots z_{n-1}\ x_{p(n)}$), where $x_i \in V$ and y_i, z_i are either < or null symbols and $p : \{ 1,2,...,n \} \rightarrow \{ 1,2,...,n \}$ is a permutation function.

Every image picture can be represented by a symbolic picture. In this symbolic picture, every symbol corresponds to a significant element of the image picture. A 2D string can be derived from a symbol picture that projects the symbols of the picture by rows and columns. The symbol "<" is used to separate non-empty columns and rows. And the empty items are ignored.

EXAMPLE 1. Fig.1 shows a symbolic picture. The set of symbols is V = {a, b, c, d}, and m = n = 4. The (reduced) 2D string representation [Chang, Shi and Yan 1987] of the symbolic picture in Fig.1 is (u, v) = (a a < b < c < d, a < c < b < a d), where the permutation function p = 14325, and

x_1 = " a ", x_2 = " a ", x_3 = " b ", x_4 = " c ", x_5 = " d ",
y_1 = " ", y_2 = " < ", y_3 = " < ", y_4 = " < ",
z_1 = " < ", z_2 = " < ", z_3 = " < ", z_4 = " ".

a			d
	b		
		c	
a			

Figure 1 A symbolic picture

There are many variations of the 2D string representation, for example, the non-redundant 2D string representation [Costagliola et al. 1995], the augmented 2D string representation [Chang, Shi and Yan 1987], and so on. And their representations for Fig.1 are (u, v) = (a < b < c < d, a < c < b < a d) and (u, v) = (a a < b < c < d, a < c < b < a d, 14325), respectively. But, some of these 2D string representations may cause ambiguity for some symbolic pictures. In next section, we will discuss the characteristics of ambiguous pictures and propose a new representation, the generalized NR2D string representation, to improve these characteristics.

III. Generalized Non-redundant 2D String Representation

Many symbolic pictures are ambiguous under the reduced 2D string representation, the normal 2D string representation [S.K. Chang, Q.Y. Shi and C.W. Yan 1987] and the non-redundant 2D string representation. Chang, Shi and Yan [1987] and Costaglioa et al. [1995] clearly have described the characteristics of ambiguous pictures under these representations.

DEFINITION 3. Let f and g be two different symbolic pictures. Let (u_f, v_f) and (u_g, v_g) be 2D string representations of f and that of g respectively. We call that f and g are ambiguous under 2D string representation if (u_f, v_f) and (u_g, v_g) are the same.

DEFINITION 4. Let f be a symbolic picture over V. If there exist indices i_1, i_2, j_1, j_2 with $1 \le i_1 < i_2 \le m$, $1 \le j_1 < j_2 \le n$ such that one of the following conditions holds:

(1) $a \in f(i_1, j_1)$, $a \in f(i_2, j_2)$, $a \notin f(i_1, j_2)$ and $a \notin f(i_2, j_1)$;

(2) $a \notin f(i_1, j_1)$, $a \notin f(i_2, j_2)$, $a \in f(i_1, j_2)$ and $a \in f(i_2, j_1)$,

we call that the symbolic picture f has a switching component on a symbol 'a' of V.

EXAMPLE 2. Figure 2 shows that the symbolic pictures, f and g, are different. After carefully examining their two 2D string representation, we find that (u_f, v_f) = (a < a, a < a) = (u_g, v_g) = (a < a, a < a). So we call that f and g are ambiguous under 2D string representation.

f:

	a
a	

g:

a	
	a

Figure 2 Ambiguous symbolic pictures

Now, we will introduce a variation of the 2D string representation for symbolic pictures, the generalized NR2D string. In this representation, it contains two strings, x-coordinate string and y-coordinate string. The first string is the same as the first string of other 2D string representations, but the second string is different from others. The permutation indices are used under this representation. By the definition of the generalized NR2D string representation, we may find it is more compact. But it does not make symbolic pictures ambiguous. In Theorem 2, we will prove that.

DEFINITION 5. Local Substring [Costaglioa et al. 1995]

Let f be a symbolic picture, and its 2D string representation be (u, v). Each substring between consecutive "<", or before the first or after the last "<" of u (or v) is called local substring of u (or v), e.g., in Example 1, "aa", "b", "c", and "d" are called local substrings of u.

DEFINITION 6. Generalized Non-redundant 2D String

Let f be a symbolic picture, and let its reduced string representation (u_0, v_0) be given. The generalized NR2D string representation of f is (u, p), where u is obtained from u_0 by replacing multiple occurrences of a same symbol in each local substring of u by exactly one occurrence of the symbol. The second string p is the permutation index string. We use the permutation indices of u, the symbols in v_0 are replaced by the permutation indices.

EXAMPLE 3. Fig. 3 shows a picture, and its reduced 2D string representation is (u_0, v_0) = (a a b < a c < b d < d d, a a d < a < b b d < c d). We have its generalized NR2D string representation is (u, p), where u = a b < a c < b d < d (permutation indices : 1 2 < 3 4< 5 6 < 7) and p = 1 3 7 < 1 < 2 5 7 < 4 6. Then (u, p) = (a b < a c < b d < d, 1 3 7 < 1 < 2 5 7 < 4 6).

	c	d	
b		b	d
a			
a	a		d

Figure 3 A symbolic picture

THEOREM 1. [Costaglioa et al. 1995] A symbolic picture f over V is ambiguous under the non-redundant 2D string representation if and only if its non-redundant 2D string contains (a < a, a < a) as a subsequence, where 'a' is some symbol in V.

Now, we will use the above theorem to prove that the generalized NR2D string representations are unambiguous for symbolic pictures.

LEMMA 1. Every unambiguous symbolic picture under the NR2D string representation over V is unambiguous under the generalized NR2D string representation.

Proof :

Let the symbolic picture f over V be unambiguous under the NR2D string representation. Its NR2D string representation and the generalized NR2D string representation are (u_n, v_n) and (u, p), respectively. By the definitions of the two representations, we obtain u = u_n .

We easily find a one-to-one mapping F such that F(u, p) = (u_n, v_n). Since the symbolic picture f over V is unambiguous under the NR2D string representation. We use the permutation indices of u, the permutation indices in p are replaced by the symbols. That is, there exists a function F_1 such that F_1(p) = v_0, and (u_0, v_0) is the reduced 2D string representation of the symbolic picture f. And the symbolic picture f is unambiguous under the NR2D string representation or the reduced 2D string representation. Thus, there also exists a one-to-one mapping F_2 such that F_2(u_0, v_0) = (u_n, v_n). Then the mapping F is composed by functions F_1 and F_2. We define that F(x, y) = F_2(x, F_1(y)), where x and y are strings, and find that F(u, p) = F_2(u, F_1(p)) = F_2(u, v_0) = (u_n, v_n).

Therefore, the symbolic picture f over V is unambiguous under the generalized NR2D string representation. The proof is concluded.

LEMMA 2. Every ambiguous symbolic picture under the non-redundant 2D string representation over V is unambiguous under the generalized non-redundant 2D string representation.

Proof :

Let the symbolic picture g over V be ambiguous under the NR2D string representation. In Theorem 1, the NR2D string representation (u_n, v_n) must contain (a < a, a < a) as a subsequence, where 'a' is some symbol in V. Under the generalized NR2D string representation, the representation of the subsequence becomes (a < a, p_1< p_2), where p_1 and p_2 are the permutation indices of the symbol 'a's in the x-coordinate string and p_1 is different from p_2. That is, the switching component is clearly distinguished under the generalized NR2D string representation.

Thus, the symbolic picture g over V is unambiguous under the generalized NR2D string representation. We conclude the proof.

THEOREM 2. Every symbolic picture over V is unambiguous under the generalized NR2D string representation.

Proof:

By **LEMMA 1** and **LEMMA 2**, we find that every symbolic picture over V is unambiguous under the generalized NR2D string representation. Therefore, we have the proof.

Table 1 makes a summary of various 2D string representations and a comparison of their results. We can find that the ambiguous problem is solved under the generalized NR2D string representation, which is more precise and compact than the other representations including the reduced 2D string representation, the augmented 2D string representation and the absolute 2D string representation. Since the generalized NR2D string representation uses more elements to record the symbolic positions precisely, the the number of stored elements in the generalized NR2D strings may be greater than that of the NR2D strings for a symbolic picture. Thus, it is unambiguous that every symbolic picture is represented by the generalized NR2D string representation.

IV. Pictures Reconstruction Using Generalized Non-Redundant 2D Strings

In the image database system, at first, a picture or a graph is translated into a symbolic picture, and then translated in an iconic index representation (e.g., a 2D string). The results are used by image queries. It is a very important issue in the image database systems that pictures are reconstructed and retrieved under an iconic index representation after our matching pictures. In this section, we show how a symbolic picture can be reconstructed from its generalized NR2D string. A sparse matrix representation of a symbolic picture from its generalized NR2D strings is produced in the following algorithm. And we use a table to record the data that contains the number of columns at the first string in the generalized NR2D strings.

Algorithm A :
Pictures Reconstruction using generalized NR2D strings

Input : A generalized NR2D string (u, p)
Output : R, the set of triples (a symbol, x-coordinate, y-coordinate)'s

Step 0
An empty set R = ∅
A generalized NR2D string (u, p)
A table is used for storing the data containing (a symbol, column-number, row-number).

Step 1
Input the string u and store it in the table.

Step 2
By the permutation-index string p, search the table and record the row-number of each item.

Step 3
Get the symbol, the column-number and row-number of the table.

Step 4
Add {symbol}*{column-number}*{row-number} to R.

EXAMPLE 4. Let us consider the generalized NR2D string (u, p) = (a b < a c < b d < d, 137 < 1 < 257 < 46) of Fig.4. From Algorithm A, firstly, we input the string u and construct the table.

	c^4	d^6	
b^2		b^5	d^7
a^1			
a^1	a^3		d^7

Figure 4 A symbolic picture

Table 2 The storage of the data

Index	Symbol	Column-number	Row-number
1	a	1	1,2
2	b	1	3
3	a	2	1
4	c	2	4
5	b	3	3
6	d	3	4
7	d	4	1,3

Table 1 Summary of 2D string representations for an n*n symbolic picture

Variations / Factors	Absolute 2D string	Reduced 2Dstring	Non-redundant 2D string	Augmented 2D string	Generalized NR2D string
Number of storage elements	$2*n^2$	$2*n^2$	$n+n^2$~$2*n^2$	$2*n^2$	$n+n^2$~$2*n^2$
Ambiguous condition	unambiguous	ambiguous	ambiguous	unambiguous	unambiguous

Secondly, we search the table by the permutation-index string p and record the row-number of each item.

Finally, we add {symbol}*{column-number}*{row-number} to R and obtain the result R= { (a, 1, 1), (a, 2, 1), (d, 4, 1), (a, 1, 2), (b, 1 ,3), (b, 3, 3), (d, 4, 3), (c, 2, 4), (d, 3, 4) }.

V. Conclusions

In this paper, we show the characteristics of ambiguous representations for some pictures. It may make mistakes in symbolic picture retrieval and pattern matching. We present a variation of the 2D string representation, the generalized NR2D string representation, which can solve the ambiguous problem. The generalized NR2D string representation is more precise and compact than the other representations, the number of stored elements in the generalized NR2D strings is greater than that of the NR 2D strings for symbolic pictures. Because the generalized NR2D strings can precisely record the symbolic positions, it is unambiguous in its pictorial representation.

Another advantage of the generalized NR2D string representation is that its picture retrieval algorithm is run in linear time, thus we can easily retrieve the pictures.

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