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Knowledge Representation Using Extended Fuzzy Petri Nets

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Abstract

In this paper, we present an extended fuzzy Petri net model (EFPN) to model fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules of rule-based systems, where the truth values of the propositions appearing in the rules are represented by vague values in [0, 1]. The vague reasoning process of the rule-based systems also can be modeled by the extended fuzzy Petri nets. The proposed extended fuzzy Petri net model provide a useful way for modeling the fuzzy production rules and the vague reasoning process of rule-based systems.

Keywords: Extended Fuzzy Petri Net, Fuzzy IF-THEN Rule, Fuzzy IF-THEN-ELSE Rule, Rule-Based System, Vague Reasoning.

1. Introduction

It is obvious that knowledge representation is an important research topics of rule-based systems. In [4], we have presented a fuzzy Petri net model (FPN) to represent the fuzzy production rules of a rule-based system and presented an algorithm to perform fuzzy reasoning based on the fuzzy Petri net model, where the truth value of each proposition is represented by a real value between zero and one. However, this single value combines the degree of truth and the degree of false of the proposition, without indicating the degree of truth and the degree of false of the proposition, respectively. Furthermore, the fuzzy production rules used in [4] are restricted to fuzzy IF-THEN rules. If we can allow fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules to be used for knowledge representation and allow the truth values of the propositions appearing in the rules to be represented by vague values [5] in [0, 1] rather than real values between zero and one, then there is room for more flexibility. In [3], we have presented vague reasoning techniques for rule-based systems, where fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules are used for knowledge representation, and the truth values of the propositions appearing in the rules are represented by vague truth values in [0, 1].

In this paper, we extend the works of [3] and [4] to present an extended fuzzy Petri net model (EFPN) to model the fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules of rule-based systems. The vague reasoning process of the rule-based systems also can be modeled by the extended fuzzy Petri nets. The proposed extended fuzzy Petri net model (EFPN) can provide a useful way for modeling the fuzzy production rules and the vague reasoning process of the rule-based systems.

The rest of the paper is organized as follows. In Section 2, we introduce the fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules for knowledge representation. In Section 3, we present the techniques for modeling the vague reasoning process of rule-based systems using extended fuzzy Petri nets. The conclusions are discussed in Section 4.

2. Knowledge Representation

Knowledge representation is one of the important topics of rule-based systems. In order to properly represent the real-world knowledge, fuzzy production rules [2], [4], [9] have been used for knowledge representation. Let R be a set of fuzzy production rules, $R = \{R_1, R_2, \dots, R_n\}$. The general formulation of the the fuzzy production rule R_i, $R_i \in R$, is as follows:

$$R_i\text{: IF } d_j \text{ THEN } d_k \text{ (CF=}\mu_i\text{)}, \quad (1)$$

where

1) d_j and d_k are propositions which may contain some fuzzy terms. The truth value of each proposition is represented by a real value between zero and one.
2) μ_i is the value of the certainty factor (CF), $\mu_i \in [0, 1]$, representing the strength of belief of the rule. The larger the value of μ_i, the more the rule is believed in.

According to [2] and [10, p.28], if the truth value of the proposition d_j of (1) is y_j, $y_j \in [0, 1]$, then the degree of truth of the proposition d_k of (1) is $y_j * \mu_i$.

In [3], we have presented the generalized fuzzy production rules, called fuzzy IF-THEN-ELSE rules, for knowledge representation. Let R_i be a generalized fuzzy production rule shown as follows:

R_i: IF d_j THEN d_k ELSE d_w (CF = μ_i), (2)

where

1) d_j, d_k, and d_w are propositions. The truth value of the propositions d_j, d_k, and d_w are represented by vague values [5] in [0, 1].
2) μ_i is the value of the certainty factor (CF), $\mu_i \in [0, 1]$, representing the strength of belief of the rule. The larger the value of μ_i, the more the rule is believed in.

A vague value x is represented by $[t_x, 1 - f_x]$, where t_x indicates the degree of truth, f_x indicates the degree of false, $1 - t_x - f_x$ indicates the unknown part, $0 \le t_x \le 1 - f_x \le 1$, and $t_x + f_x \le 1$. It is obvious that the vague truth value [0, 0] represents absolutely false and the vague truth value [1, 1] represents absolutely true.

Let x and y be two vague values, where

$x = [t_x, 1 - f_x]$,

$y = [t_y, 1 - f_y]$. (3)

According to [5], the maximum operation between the vague values x and y are defined as follows:

$c = x \otimes y = [t_c, 1 - f_c]$ (4)

where

$t_c = \text{Max}(t_x, t_y)$, (5)

$1 - f_c = \text{Max}(1 - f_x, 1 - f_y)$. (6)

According to [5], the minimum operation between the vague values x and y are defined as follows:

$c = x \oslash y = [t_c, 1 - f_c]$, (7)

where

$t_c = \text{Min}(t_x, t_y)$, (8)

$1 - f_c = \text{Min}(1 - f_x, 1 - f_y)$. (9)

3. Vague Reasoning Using Extended Fuzzy Petri Nets

In [3], we have introduced vague reasoning techniques for rule-based systems. In this section, we present the techniques for modeling the vague reasoning process of rule-based systems using extended fuzzy Petri nets. Let's consider the following fuzzy production rule:

R_i : IF d_j THEN d_k ELSE d_w (CF = μ_i). (10)

Assume that the vague truth value of proposition d_j is $[t_j, 1 - f_j]$, where $0 \le t_j \le 1 - f_j \le 1$ and $t_j + f_j \le 1$, then the vague truth values of propositions d_k and d_w can be evaluated and are equal to $[t_k, 1 - f_k]$ and $[t_w, 1 - f_w]$, respectively, where

$t_k = t_j * \mu_i$, (11)

$1 - f_k = 1 - (f_j * \mu_i)$. (12)

$t_w = f_j * \mu_i$, (13)

$1 - f_w = 1 - (t_j * \mu_i)$. (14)

We can use an extended fuzzy Petri net model (EFPN) for modeling fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules of a rule-based system. The concept of extended fuzzy Petri nets is derived from fuzzy Petri nets [2], [4], [9] and Petri nets [11]. An extended fuzzy Petri net is a bipartite directed graph which contains two types of nodes: places and transitions, where circles represent places, and bars represent transitions. Each place may or may not contain a token associated with a vague truth value in [0, 1]. Each transition is associated with a certainty factor value between zero and one. The relationships from places to transitions and from transitions to places are represented by directed arcs. There are two kinds of directed arcs from transitions to places, i.e., the positive arcs, denoted by "→", and the negative arcs, denoted by "o→". A generalized extended fuzzy Petri net structure can be defined as an 8 -tuple:

EFPN = (P, T, D, I, O, f, δ, β),

where

P = $\{p_1, p_2, \ldots, p_n\}$ is a finite set of places,

T = $\{T_1, T_2, \ldots, T_m\}$ is a finite set of transitions,

D = $\{d_1, d_2, \ldots, d_n\}$ is a finite set of propositions,

$P \cap T \cap D = \varnothing$, $|P| = |D|$,

I: $T \rightarrow P^{\infty}$ is the input function, a mapping from transitions to bags of places,

O: $T \rightarrow P^{\infty}$ is the output function, a mapping from transitions to bags of places,

f: $T \rightarrow [0, 1]$ is an association function, a mapping from transitions to real values between zero and one,

δ: $P \rightarrow [0, 1]$ is an association function, a mapping from places to vague values in [0, 1].

β: $P \rightarrow D$ is an association function, a bijective mapping from places to propositions.

Let A be a set of directed arcs. If $p_j \in I(T_i)$, then there exists a directed arc a_{ji}, $a_{ji} \in A$, from the p_j to the transition T_i. If $p_k \in O(T_i)$, then there exists a directed arc a_{ik}, $a_{ik} \in A$, from the transition T_i to the place p_k. If $f(T_i) = \mu_i$, $\mu_i \in [0, 1]$, then the transition T_i is said to be associated with a real value μ_i. If $\beta(p_i) = d_i$, $d_i \in D$, then the place p_i is said to be associated with the proposition d_i. An extended fuzzy Petri net with some places containing tokens is called a marked extended fuzzy Petri net. In a marked extended fuzzy Petri net, the token in a place p_i is represented by a labeled dot $\overset{\delta(p_i)}{\bullet}$. The token value in a place p_i, $p_i \in P$, is denoted by $\delta(p_i)$, where $\delta(p_i) = [t_i, 1 - f_i]$, $0 \le t_i \le 1 - f_i \le 1$, and $t_i + f_i \le 1$. If $\delta(p_i) = [t_i, 1 - f_i]$ and $\beta(p_i) = d_i$, then it indicates that the degree of truth and the degree of false of proposition d_i are t_i and f_i, respectively.

By using an extended fuzzy Petri net, the fuzzy production rule

R_i: IF d_j THEN d_k ELSE d_w (CF=μ_i)

can be modeled as shown in Fig. 1.

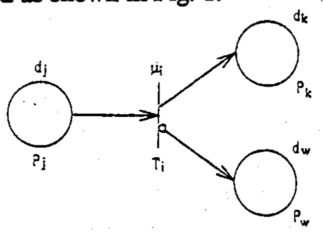

Fig. 1. An extended fuzzy Petri net.

In a marked extended fuzzy Petri net, a transition may be enabled to fire. A transition T_i is enabled if there is a token in each of its input places. A transition T_i fires by removing the tokens from its input places and then depositing one token into each of its output places. Firing fuzzy production rules can be considered as firing transitions. For example, assume that the vague truth value of the proposition d_j of the above fuzzy production rule is $[t_j, 1 - f_j]$, then the vague reasoning process of the above rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 2.

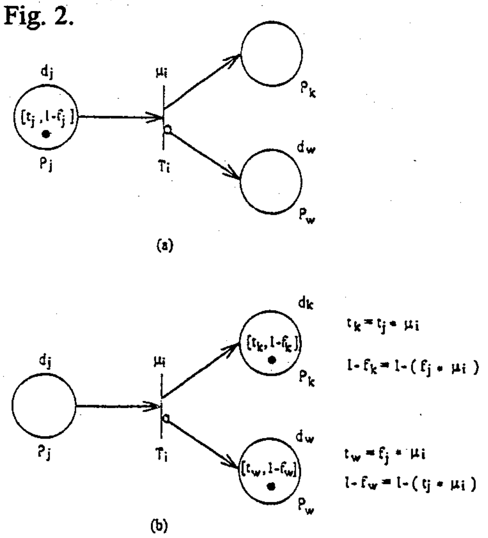

Fig. 2. Firing a marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

For example, consider the following fuzzy production rule:

R_1: IF it is tall THEN it is heavy ELSE it is light (CF = 0.90),

where the vague truth value of the proposition "it is tall" is [0.80, 0.90], then the rule and the fact can be represented by a marked extended fuzzy Petri net as shown in Fig. 3. The vague reasoning process can be modeled by a marked extended fuzzy Petri net as shown in Fig. 4.

EFPN = (P, T, D, I, O , f, δ, β),
P = $\{p_1, p_2, p_3\}$,
T = $\{T_1\}$,
D= {it is tall, it is heavy, it is light},
$I(T_1) = \{p_1\}$, $O(T_1) = \{p_2, p_3\}$, $f(t_1) = 0.90$,
$\delta(p_1) = [0.80, 0.90]$, $\delta(p_2) = [0, 0]$, $\delta(p_3) = [0, 0]$,
$\beta(p_1)$ = it is tall, $\beta(p_2)$ = it is heavy, $\beta(p_3)$ = it is light.

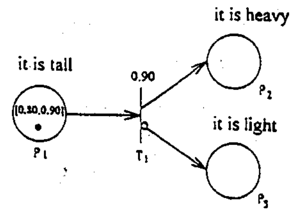

Fig. 3. Knowledge representation with a marked extended fuzzy Petri net.

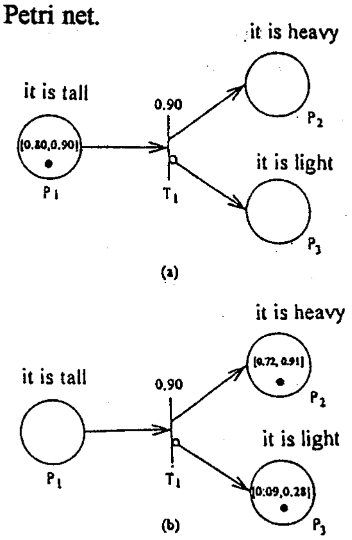

Fig. 4. Firing a marked extended fuzzy Petri net. (a) Before firing transition T_1. (b) After firing transition T_1.

If a proposition d_j is composed of many propositions (i.e., d_{j1}, d_{j2}, ..., and d_{jn}) connected by "and" connectors (i.e., $d_j = d_{j1}$ and d_{j2} and ... and d_{jn}), then d_j is called a composite proposition. If the antecedent portion or the consequence portion of a fuzzy production rule contains "and" or "or" connectors, then it is called a composite fuzzy production rule [9].

In [3], we have presented 15 types of composite fuzzy production rules. In the following, we introduce these types of composite production rules.

Type 1: IF d_{j1} and d_{j2} and ... and d_{jn} THEN d_k (CF=μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 5. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 6.

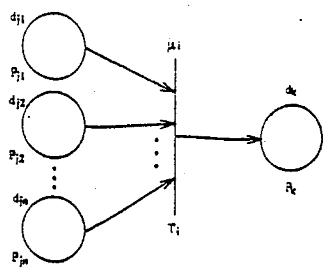

Fig. 5. Extended fuzzy Petri net representation of type 1 rules.

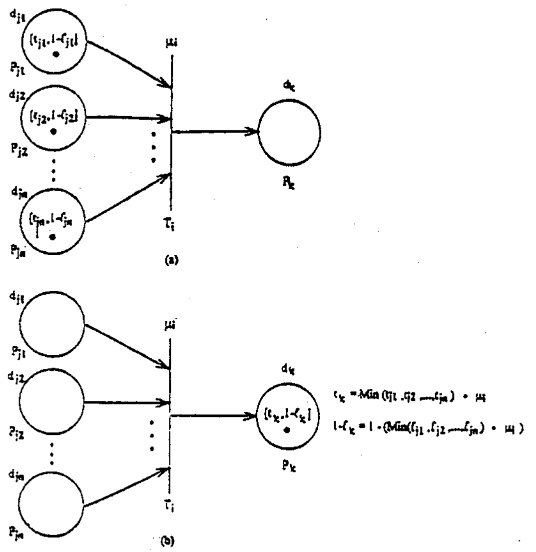

Fig. 6. a type-1 marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

Type 2: IF d_j THEN d_{k1} and d_{k2} and ... and d_{ks} (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 7. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 8.

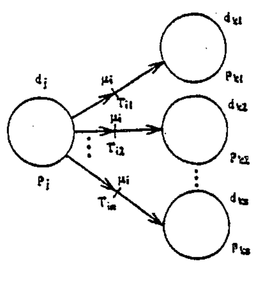

Fig. 7. Extended fuzzy Petri net representation of type 2 rules.

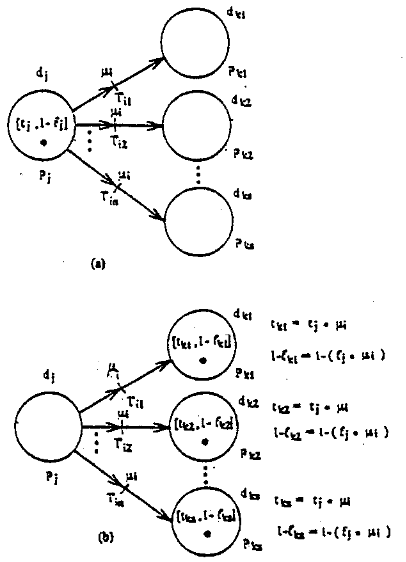

Fig. 8. A type-2 marked extended fuzzy Petri net. (a) Before firing transitions. (b) After firing transitions.

Type 3: IF d_{j1} or d_{j2} or ... or d_{jn} THEN d_k (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 9. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 10.

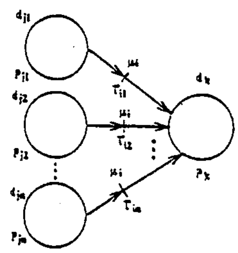

Fig. 9. Extended fuzzy Petri net representation of type 3 rules.

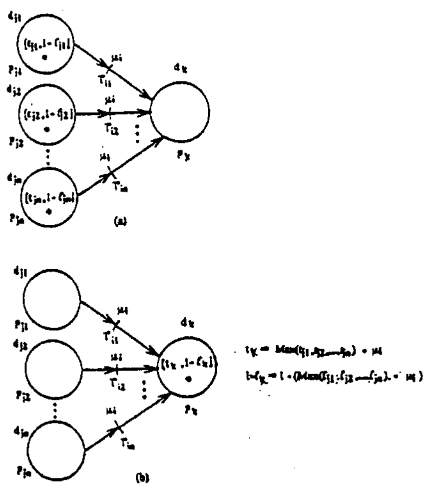

Fig. 10. A type-3 marked extended fuzzy Petri net. (a) Before firing transitions. (b) After firing transitions.

Type 4: IF d_j THEN d_{k1} or d_{k2} or ... or d_{ks} (CF = μ_i). Because rules of this type do not make specific implications, they are unsuitable for deducing control. Thus, we do not allow this type of rule to appear in the knowledge base.

Type 5: IF d_{j1} or d_{j2} or ... or d_{jn} THEN d_k ELSE d_w (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 11. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 12.

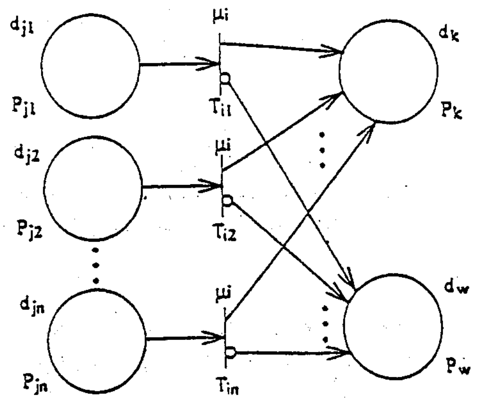

Fig. 11. Extended fuzzy Petri net representation of type 5 rules.

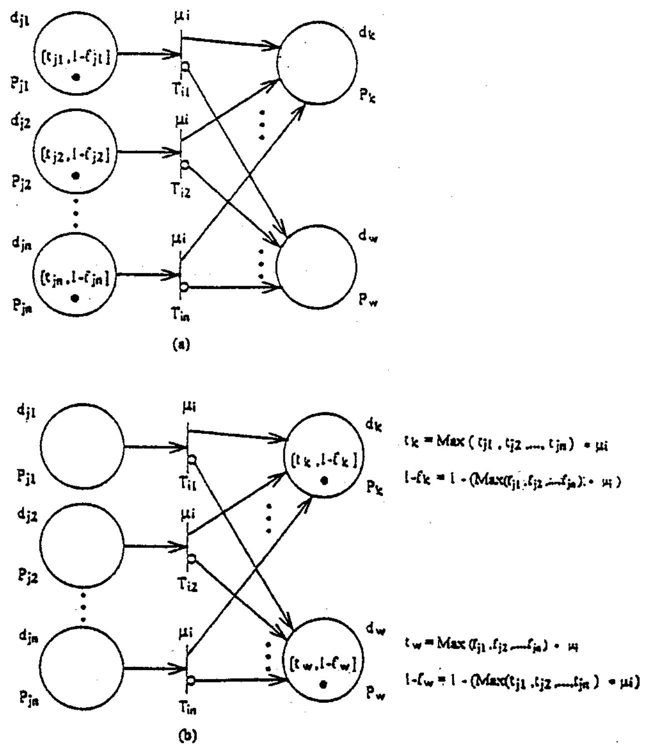

Fig. 12. A type-5 marked extended fuzzy Petri net. (a) Before firing transitions. (b) After firing transitions.

Type 6: IF d_j THEN d_{k1} and d_{k2} and ... and d_{ks} ELSE d_w (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 13. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 14.

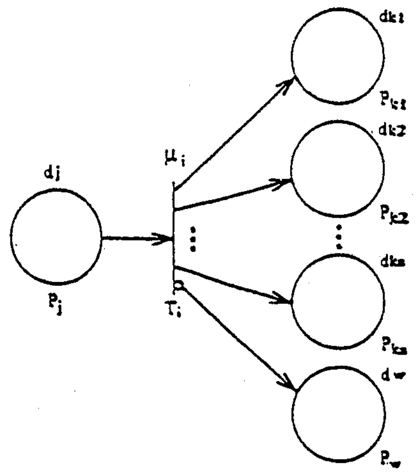

Fig. 13. Extended fuzzy Petri net representation of type 6 rules.

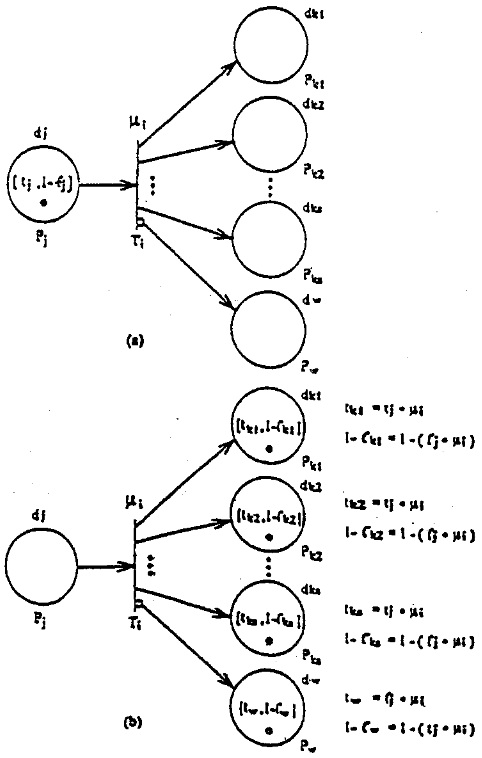

Fig. 14. A type-6 marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

Type 7: IF d_j THEN d_k ELSE d_{w1} and d_{w2} and ... and d_{wy} (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 15. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 16.

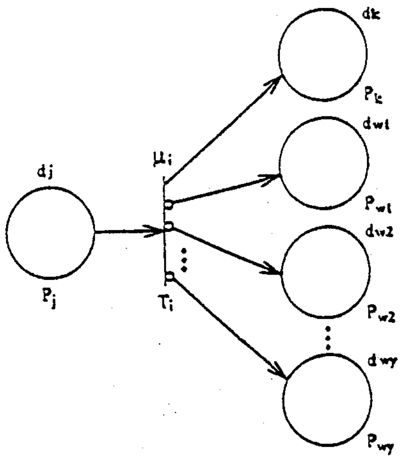

Fig. 15. Extended fuzzy Petri net representation of type 7 rules.

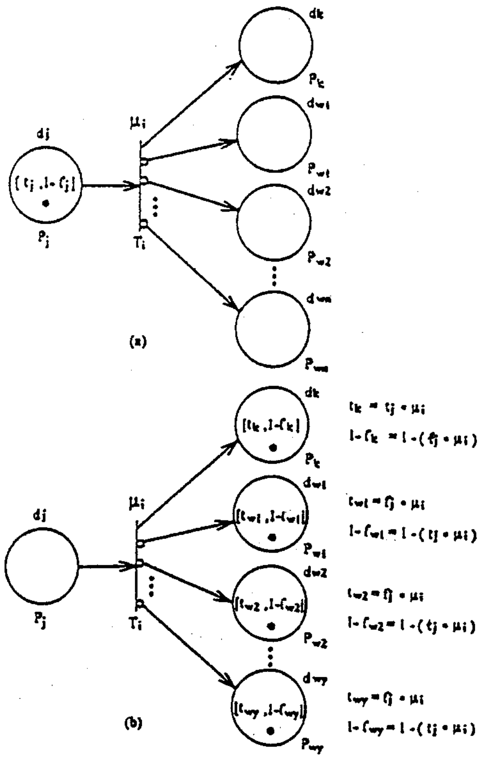

Fig. 16. A type-7 marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

Type 8: IF d_{j1} and d_{j2} and ... and d_{jn} THEN d_k ELSE d_w (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 17. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 18.

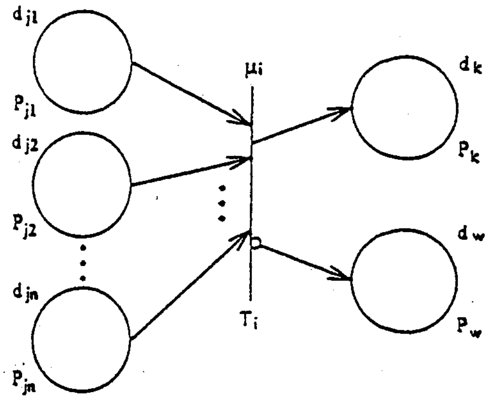

Fig. 17. Extended fuzzy Petri net representation of type 8 rules.

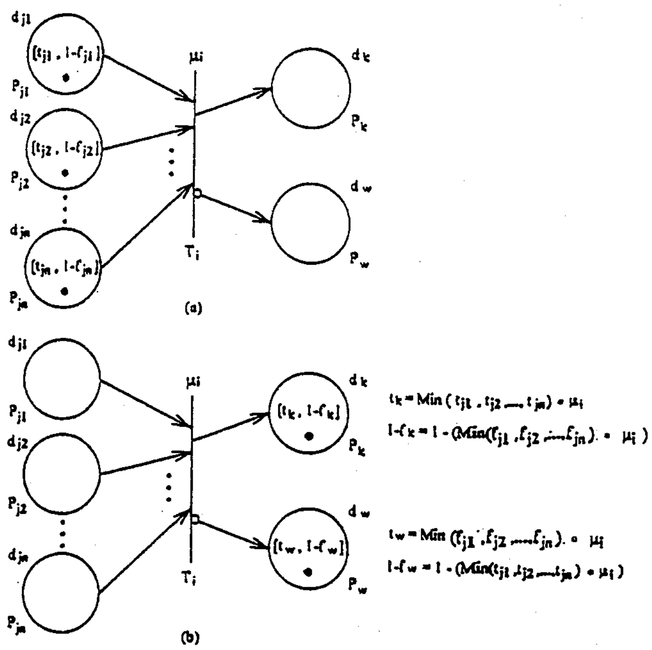

Fig. 18. A type-8 marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

Type 9: IF d_{j1} and d_{j2} and ... and d_{jn} THEN d_{k1} and d_{k2} and ... and d_{ks} ELSE d_w (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 19. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 20.

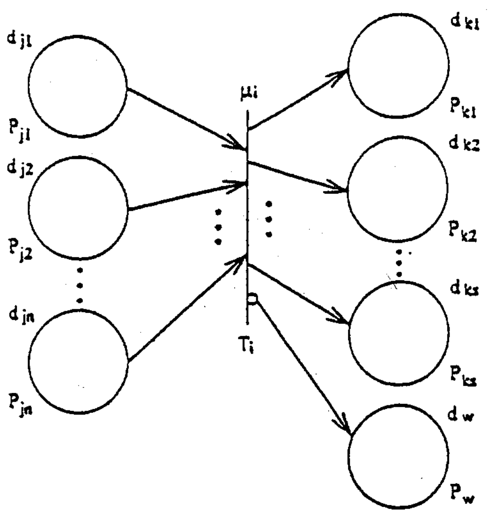

Fig. 19. Extended fuzzy Petri net representation of type 9 rules.

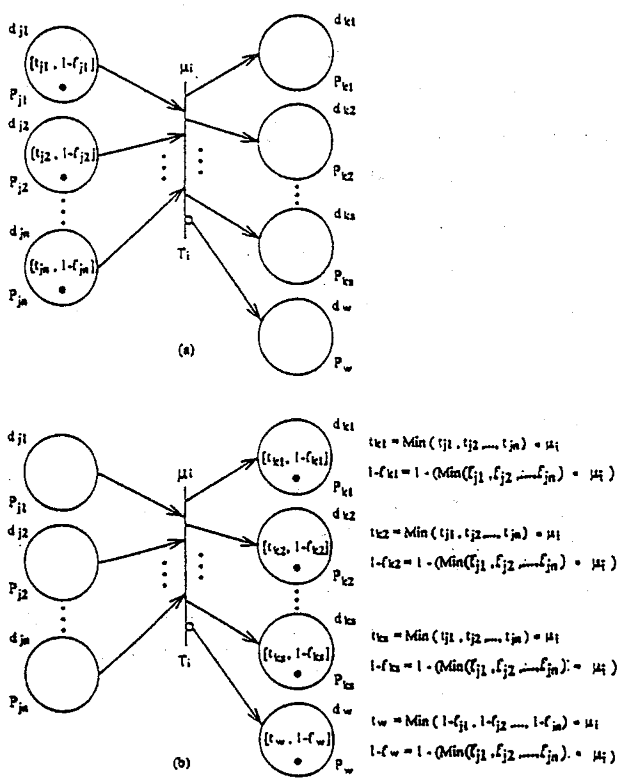

Fig. 20. A type-7 marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

Type 10: IF d_{j1} and d_{j2} and ... and d_{jn} THEN d_{k1} and d_{k2} and ... and d_{ks} ELSE d_{w1} and d_{w2} and ... and d_{wy} (CF = μ_i). This rule type can be modeled by an extended fuzzy Petri net as shown in Fig. 21. The vague reasoning process of this type of rule can be modeled by a marked extended fuzzy Petri net as shown in Fig. 22.

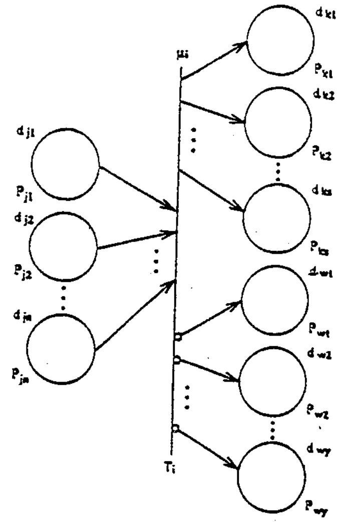

Fig. 21. Extended fuzzy Petri net representation of type 10 rules.

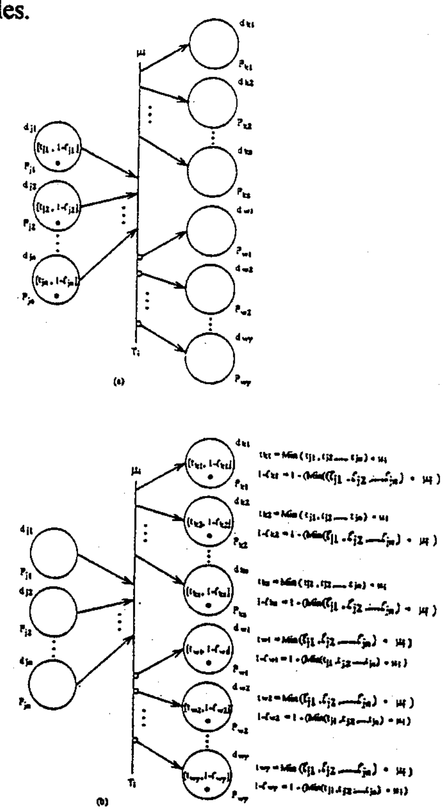

Fig. 22. A type-7 marked extended fuzzy Petri net. (a) Before firing transition T_i. (b) After firing transition T_i.

Type 11: IF d_j THEN d_{k1} or d_{k2} or ... or d_{ks} ELSE d_w (CF = μ_i), where d_j and d_w are either a simple proposition or a composite proposition. Rules of this type are unsuitable for deducing control due to the fact that they do not make specific implications. Thus, we do not allow this type of rules to appear in the knowledge base.

Type 12: IF d_j THEN d_k ELSE d_{w1} or d_{w2} or ... or d_{wy} (CF = μ_i), where d_j and d_k are either a simple proposition or a composite proposition. Rules of this type are unsuitable for deducing control due to the fact that they do not make specific implications. Thus, we do not allow this type of rules to appear in the knowledge base.

Type 13: IF d_j THEN d_{k1} or d_{k2} or ... or d_{ks} ELSE d_{w1} or d_{w2} or ... or d_{wy} (CF = μ_i), where d_j is either a simple proposition or a composite proposition. Rules of this type are unsuitable for deducing control due to the fact that they do not make specific implications. Thus, we do not allow this type of rules to appear in the knowledge base.

Type 14: IF d_{j1} or d_{j2} or ... or d_{jn} THEN d_{k1} or d_{k2} or ... or d_{ks} ELSE d_{w1} or d_{w2} or ... or d_{wy} (CF = μ_i). Rules of this type are unsuitable for deducing control due to the fact that they do not make specific implications. Thus, we do not allow this type of rules to appear in the knowledge base.

Type 15: IF d_{j1} or d_{j2} or ... or d_{jn} THEN d_k ELSE d_{w1} or d_{w2} or ... or d_{wy} (CF = μ_i), where d_k is either a simple proposition or a composite proposition. Rules of this type are unsuitable for deducing control due to the fact that they do not make specific implications. Thus, we do not allow this type of rules to appear in the knowledge base.

It should be noted that the number of tokens in a place is always restricted to one as shown in Figs. 5-22.

4. Conclusions

In this paper, we have extended the works of [3] and [4] to present an extended fuzzy Petri net model (EFPN) to model fuzzy IF-THEN rules and fuzzy IF-THEN-ELSE rules of rule-based systems. The vague reasoning process of the rule-based systems also can be modeled by the extended fuzzy Petri nets. The proposed extended fuzzy Petri net model provide a useful way for modeling the fuzzy production rules and the vague reasoning process of the rule-based systems.

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