

## Physical Edge Detection Using the Curvature at the Critical Points

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### Abstract

*This paper introduces a method for three dimensional edge localization using a range sensor based on the triangulation principle.*

*The curvature at points of a digital curve was investigated. A procedure for edges detection that has low sensitivity to noise has been designed.*

*The procedure for edge detection imbeds two steps. The first step is the extraction of the point that has a critical position on the scanned object. The second step is the computation of the curvature at that point and the verification of edge presence.*

*The detection approach presented herein beholds the advantages of being implemented in active exploration of the robotic workcell.*

### 1. Introduction

In a variety of robotics applications, sensing of the exact location and orientation of a workpiece and determination of its detailed surface geometry are needed for robot guidance. Since the robot needs to manipulate and move in the three dimensional workspace, it is essential to derive results of object localization and pose determination in the 3-D coordinate frame. There are several different approaches that can be followed to derive such information. Some of them may acquire 3-D information by using range-sensing approaches such as Laser Range Sensors or Stereo Vision[1].

3-D structure analysis is usually computationally intensive. This computationally expensive processing arise due to the fact that correspondence of points between different views must be established and a complex system of equations must be solved. Therefore, it is of great convenience to explore the possibility of analyzing 3-D structures, that are usually defined

in the 3-D world frame, in a 2-D coordinate system. However, to provide efficient analysis of 3-D objects in the new 2-D frame, the coordinates transformation from the world coordinates system to this new 2-D coordinates system should cause no alteration in such dimensions as tangential deflection (i.e., the angle between two neighboring tangent lines) and curvature at a point.

The informational content of a shape is concentrated along the boundary contours and furthermore, at points on those contours at which the direction changes most rapidly. With additional experiments[2], it was demonstrated that information, sufficient for the recognition of familiar shapes, are contained in the edge-points. A vast amount of literature has been dedicated to the subject of edge detection[3, 4, 5]. Our primary goal in this work is to design and analyze a procedure for physical edge (3-D edge) detection from 3-D information on objects. These 3-D information were acquired by a Laser Displacement Sensor which was fixed to the end-effector of a Spherical SCARA Robot[6, 7]. After fusing the sensor and robot coordinate system together, any point in the sensor coordinate system can be easily defined in the world coordinate system. Moreover, by rotating and translating the sensor around and along a 3-D object, a range map of this object can be obtained. Since the range map is obtained using a predefined scanning mechanism, the neighbors of each point in this map are known. Thus, a comparison between the property of this point and the property of its neighbors can be easily provided. The result of this comparison can be utilized to extract feature-points.

The remainder of this paper is organized as follows:

Section 2 shows how 3-D structures may be analyzed in a 2-D coordinates system. In section 3, the local curvature at points of a digital curve is discussed. Section 4 discusses the smoothed

curvature. Section 5 describes the edge detection procedure. In section 6, experimental results are shown and discussed. Finally, section 7 contains a conclusion.

## 2. 3-D Points data representation

Beginning in this chapter, we shall give a detailed discussion of the scanning mechanism used in this work and conquer the possibility of a reliable analysis of 3-D structures in a 2-D coordinate frame without plaguing notions such as curvature, tangential deflection, distance between two points...etc.

When a shape needs a representation in three dimensions, the three-dimensional information can be either decomposed into three two-dimensional information in the  $XY$ ,  $YZ$  and  $ZX$  planes, or represented by one two-dimensional (eventually, one-dimensional) information associated to a scanning mechanism ( for example, rotation and (or) translation ). When three-dimensional information have to be obtained and analyzed in an active robotic workcell, the method which takes into account the scanning mechanism is more convenient than the one using projection of points on the three perpendicular planes. Hence, we believe that the scanning mechanism utilized should be seriously considered and discussed.

Three-dimensional shape data acquisition by a distance non contact sensor can be possible by:

- moving the scene around the sensor.
- moving the sensor around the scene.
- using multiple sensors in different locations.

In this work, objects were scanned by a Laser Displacement Sensor (LDS). The LDS uses the triangulation principle (Fig.1) as optical technique for depth perception. The diffuse backscatter light from the object is received by the Position Sensitive Diode (PSD), the position of the light spot on this receiver causes an output current proportional to the distance to the scanned surface.

This kind of sensors deliver valid range measurement under the condition that the light emitted by the laser diode is reflected only once before being received by the PSD. However, depending on the topography and reflectance of the scanned surface, it can happen that the backscattered light reaches the PSD in a roundbound way, giving wrong data. This problem appears mainly while scanning very smooth surfaces (mirror-like surfaces) from

which the light is specularly reflected. Another bottleneck that threatens the measurement using LDS is the *beam-interrupt*. This is caused by contour discontinuities which stop the backscattered light from reaching the PSD. For a reliable measurement, we have voluntarily restricted ourselves to an application area where the observed objects have relatively simple geometric characteristics which are easily measured.

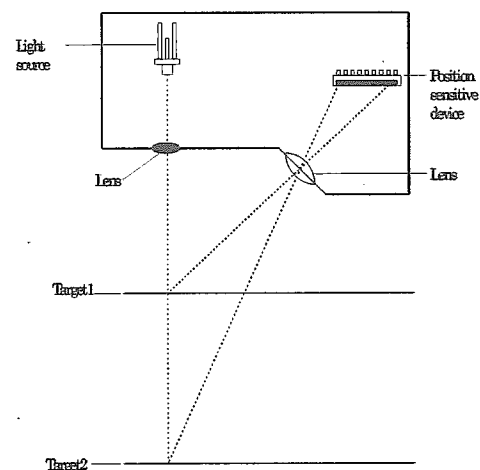


Figure 1: Triangulation principle.

A LDS was fixed to the end-effector of a Spherical SCARA robot and turned around the object of interest using a Spherical Scanning Mode (Fig.2); that is, while acquiring range data, the sensor is rotated around a vertical and a horizontal axis around the object of interest. The simplest version of the algorithm describing this scanning mechanism can be expressed as follows:

*Step 1:* Starting from a certain initial value  $\gamma_1$  of the angle  $\gamma$  (Fig.2).

*Step 2:* rotate the sensor around the scanned object with a certain angular step  $\Delta\gamma$ . If  $\gamma = \gamma_2$  stop.

*Step 3:* Starting from the value of the angle  $\theta = -\pi$ .

*Step 4:* turn the sensor around the object of interest with a certain angular step  $\Delta\theta$ .

*Step 5:* Measure the distance from the sensor to the scanned point.

*Step 6:* Convert the coordinates of the scanned point from the sensor coordinates system to the 3-D world coordinates system.

*Step 7:* Store the 3-D coordinates of the point in a

data-file. If  $\theta \geq \pi$  go to step 2, else go to step 4.  
If  $\gamma = \gamma_2$  stop.

It is indispensable to notice that, as a result of implementing the angular scanning (from step 3 to step 7), the acquired points will define a closed digital curve around the scanned object. The density of the points in this curve depends mainly on the angular step  $\Delta\theta$ . Besides, This curve lies on the plane which normal vector forms an angle  $\gamma$  with the axis  $Z$  of the world frame (Fig.3). Hence, an analysis of the acquired curve in the world 3-D coordinate system is analogous to analyzing this curve in a new 2-D coordinate system ( $X'O'Y'$ ) which belongs to the scanning plane  $P$ . Therefore, 3-D edge-point detection in the world coordinates system can be provided by extracting corner-points of the digital curve in the new 2-D coordinates system ( $X'O'Y'$ ), Then converting these corner-point from this new coordinates system to the world coordinates system.

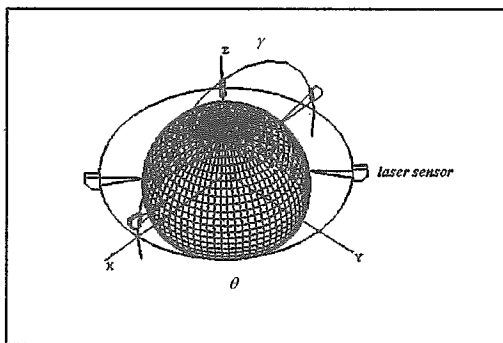


Figure 2: Spherical scanning mechanism.

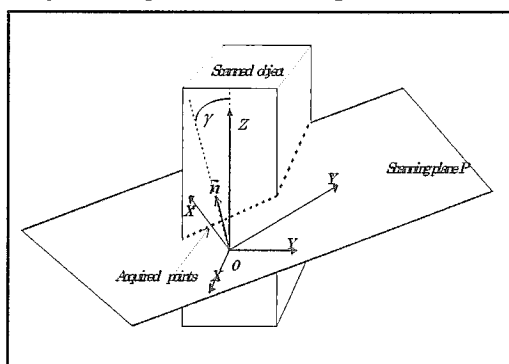


Figure 3: Definition of acquired points in the scanning plane.

### 3. Local curvature of discrete curves

For a piecewise continuous differentiable

boundary curve, the notions of tangent angle or curvature are all unambiguously defined concepts from differential geometry which admit precise analytical formulations and which play an important role in the shape analysis of curves. Consider a continuous planar curve defined parametrically as:

$$r(t) = (x(t), y(t)) \quad a < t < b. \quad (1)$$

For such a curve, the arclength  $s$  satisfies

$$\frac{ds}{dt} = \sqrt{x'^2 + y'^2} \quad (2)$$

Where  $(\cdot)$  denotes differentiation with respect to  $t$ .

The tangent and normal vectors to  $r$  are given by

$$T = \begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} \quad (3)$$

and

$$N = \begin{pmatrix} -y'(t) \\ x'(t) \end{pmatrix} \quad (4)$$

When the parameter  $t$  is the arc-length parameter  $s$  it follows from (2) that these vectors are the unit tangent and normal vectors to  $r$ , and the curvature function  $\kappa(s)$  is thereby defined by the relation

$$\frac{dT(s)}{ds} = \kappa(s)N(s) \quad (5)$$

From this defining relation we have

$$\kappa(s) = \frac{d\theta(s)}{ds} = \lim_{\Delta s \rightarrow 0} \frac{\Delta\theta(s)}{\Delta s} \quad (6)$$

where  $\theta(s)$  is the angle between the tangent line to  $r(s)$  at  $s$  and the positive  $X$ -axis (Fig. 4a). However, if the boundary curve lacks a functional description then the concepts of tangent angle and curvature are no less important to the analysis of shapes. Moreover, these notions no longer enjoy universally accepted definitions, nor do they yield to simple analytical

formulations. This is an unfortunate case, since accurate and efficient calculation of the curvature is essential to a variety of shape analysis.

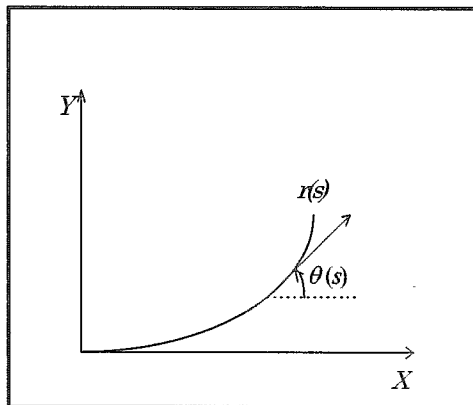
Equation (6) suggests for a discrete curve

$$C = \{X_i = (x_i, y_i) / i = 0, 1, 2, \dots, n\}$$

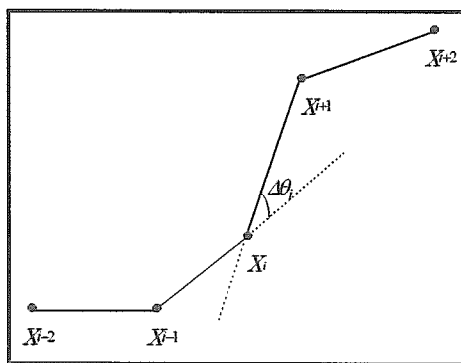
that we define the local curvature at a point

$$X = X_i \text{ as: } \kappa(X) = \frac{\Delta\theta_i}{\Delta s} \quad (7)$$

where  $\Delta\theta_i$  is the angle between the segments  $(X_i, X_{i-1})$  and  $(X_i, X_{i+1})$  (Fig. 4(b)).  $\Delta s$  is the sum of the distance  $d_{i-1}$  between  $X_i$  and  $X_{i-1}$  and the distance  $d_{i+1}$  between  $X_i$  and  $X_{i+1}$ .



(a)



(b)

Figure 4: (a) Continuous curvature.  
(b) Discrete curvature.

It is important to notice that the local curvature

defined above in eq.(7) for discrete points is very vulnerable to noise and measurement errors (Fig. 5). The source of noise in the data obtained by a LDS can originate, as already mentioned, from factors such as quantization due to the robot and sensor limited accuracy, instability of the power supply, induction, electromagnetic field, intensive light (for example, light from an electric welding machine). Hence when the data come from physical measurements then it is necessary to examine more closely the wisdom of trying to define a new curvature which tends to be more robust in order to reduce the influence of noise and measurement inaccuracy.

#### 4. The q-smoothed curvature

Let  $A$  and  $B$  be two data sequences as shown in Fig. 6. The points shown do not lie exactly on the indicated boundary, reflecting some possible errors in measurement. Let  $\Delta\theta_q(A, B)$  denote the angle between the lines of best fit (in the sense that the sum of the squared orthogonal distances from the data points to their respective lines is minimum) to the data sets  $A$  and  $B$ . Let  $l_{i+q}$  and  $l_{i-q}$  be the lines parallel respectively to the lines of best fit of the sequences  $A$  and  $B$  and passing through the point  $X_i$ . Let  $p_{i+q}$  and  $p_{i-q}$  be respectively the orthogonal projection of the points  $X_{i+q}$  and  $X_{i-q}$  on the lines  $l_{i+q}$  and  $l_{i-q}$ .

*Definition 1:* The  $q$ -smoothed curvature of the discrete curve  $C$  at a point  $X = X_i$  is :

$$\kappa_q(X) = \frac{\Delta\theta_q(A, B)}{\Delta_q(s)} \quad (8)$$

where  $\Delta_q(s) = d_{ql}(s) + d_{qr}(s)$

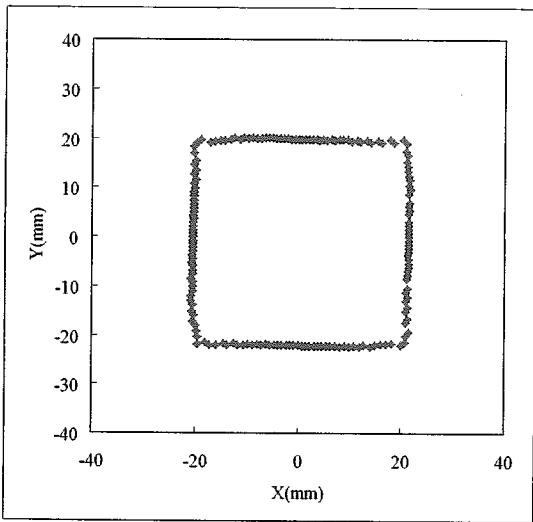
$d_{ql}$  is the Euclidian distance between the points  $P_{i-1}$  and  $X_i$ ,  $d_{qr}$  is the Euclidian distance between the points  $p_{i+1}$  and  $X_i$ .

Indeed, one would notice that eq.(8) will be equivalent to eq.(7) for  $q = 1$ .  $q$  is called the *smoothing factor* for the curvature.

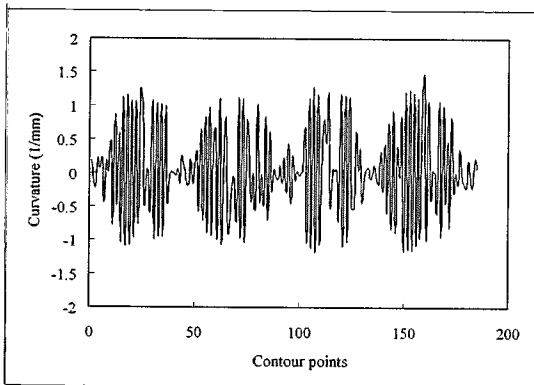
The choice of  $q$  is dictated mainly by the degree of contamination of the data represented in the discrete curve. Furthermore, the data contamination depends (as stated above) on the experimental environment. The  $q$ -smoothed curvature of the discrete curve shown in Fig. 5(a) was computed for  $q = 6$ . The smoothing effect of

$q$  is shown in Fig. 7.

In practice, we choose  $q$  as follows. A cylinder is scanned in the cross-section so that the obtained closed digital curve defines a circle. The curvature at each point of this curve is computed for different values of  $q$  ( $q = 1, 2, 3, \dots$ ). The chosen value of  $q$  corresponds to the smallest value of  $q$  which shows more constancy of the curvatures at the points of the digital circle. To illustrate the choice of  $q$ , we adopt the example shown in Fig. 8.



(a)



(b)

Figure 5:(a) Acquired contour points.  
 (b) Local curvature at the points.

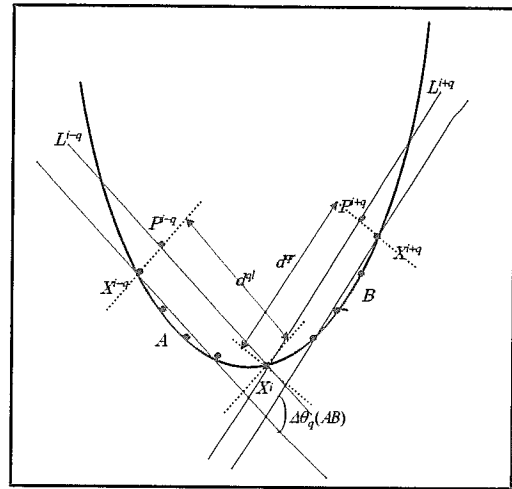


Figure 6:  $q$ -smoothed curvature definition.

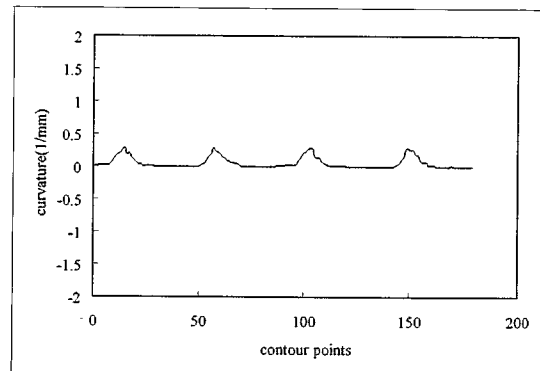
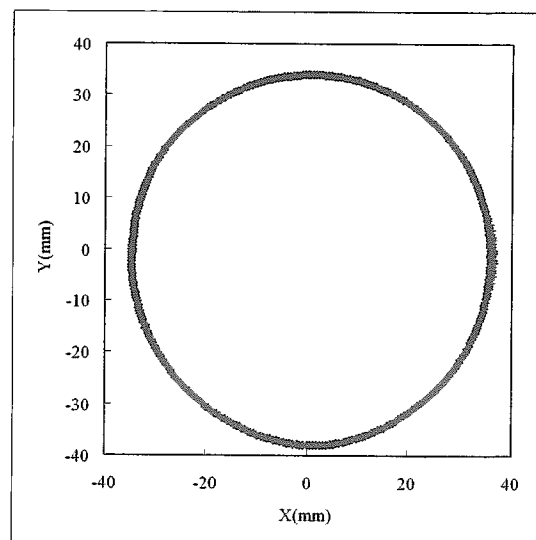
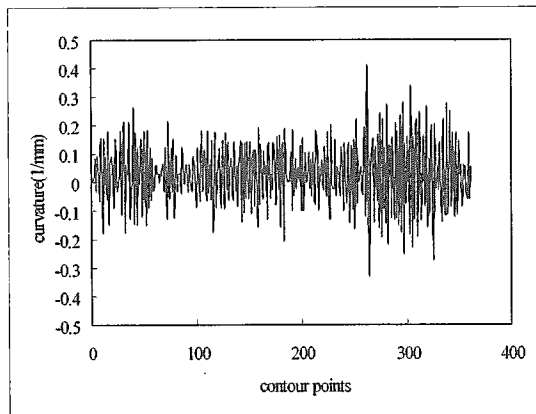


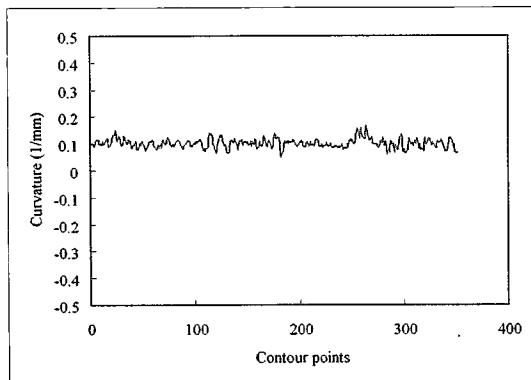
Figure 7:  $q$ -smoothed curvature at the points of Fig.5(a) for  $q=6$ .



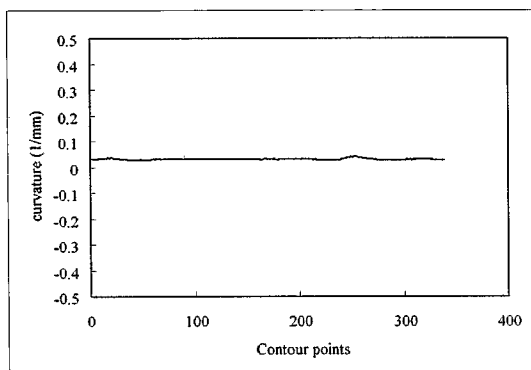
(a)



(b)



(c)



(d)

Figure 8: (a) Digital circle. (b) q-smoothed curvature for  $q=1$ . (c) q-smoothed curvature for  $q=3$ . (d) q-smoothed curvature for  $q=6$ .

## 5. Steps for edge detection

The procedure for edge detection contains basically 2 steps: extraction of critical points and corner-point determination. Those steps are discussed below in more detail.

### 5.1 Localization of critical points

Critical points are local extrema of the gradient depth in the gradient direction. These boundaries can be detected by comparing range values at neighboring points (Fig.9 and Fig.10). Since the scanning mechanism uses an angular scanning around the object of interest, for every value of  $\theta$  (see Fig.2) will correspond a set of scanned points  $p_i$  ( $0 \leq i \leq \frac{\theta}{\Delta\theta}$ ), where  $\Delta\theta$  is the angular

step of the sensor around the object of interest. Note that points  $p_i$  define a contour curve (see Fig.3). Since every point of this curve has two possible neighbors, we can compare the depth of this point with the depth of its two neighbors.

Laser sensors are prone to intervention from many internal and external sources of noise; such as induction, instability of the power supply, electromagnetic field, intensive light (for example, light from an electric welding machine) etc. It is of great importance to notice that any possible occurrence of noise will cause a spurious critical point. We are faced then with the problem of distinguishing between critical points which arise due to the presence of a physical edge and critical points which arise due to the presence of noise. To eliminate those false extrema, a local computation of critical points using large neighborhoods should be used, instead of point-wise computation.

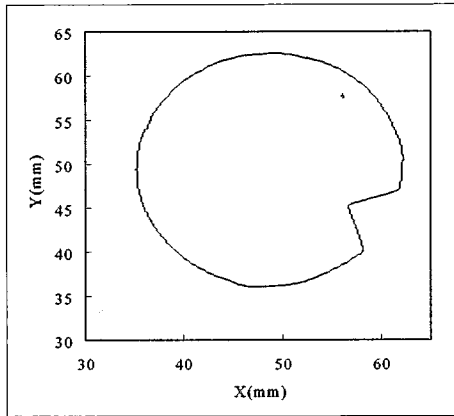
### 5.2 Corner-point determination

The q-smoothed curvature at each maximal point is computed using equation (8). A corner-point is said to be present at a maximal point in a sequence if :

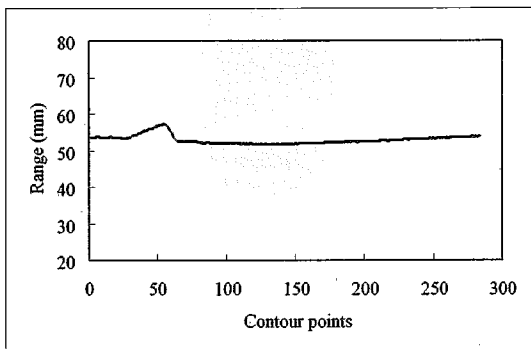
- the curvature at the maximal point exceeds a preset threshold value.
- the curvature at that point is larger than the curvature at its neighbors.

It is important to draw attention to the fact that the application of this step (step for corner point determination) is by itself sufficient for the edge detection. However, the price paid using this

single step is an increasing computation time. Hence, in order to speed up this process, an extra step (step for critical points localization) is added.

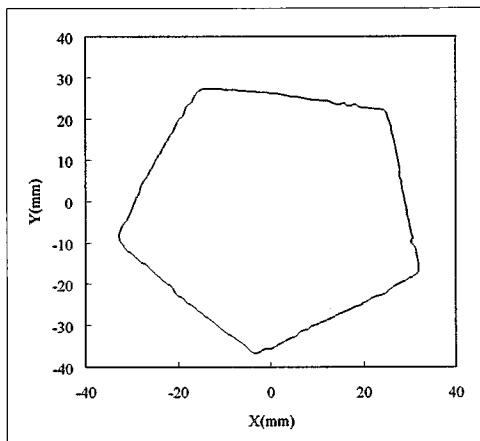


(a)

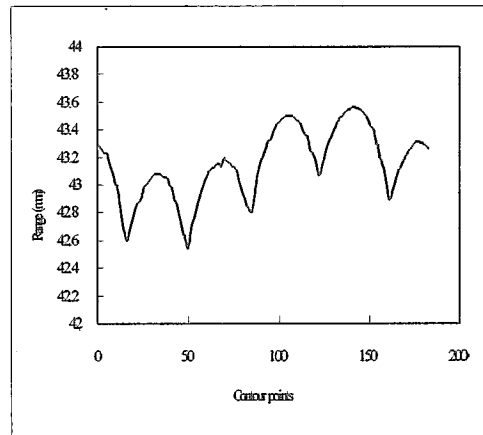


(b)

Figure 9: (a) Contour points. (b) Range at the contour points.



(a)



(b)

Figure 10: (a) Contour points. (b) Range at the contour points.

## 6. Experimental results and discussion

The procedure introduced above was implemented for a large set of 3-D objects. Some results are shown in Fig.11. Results presented here were obtained entirely autonomously by the procedure; there was no human intervention whatsoever.

Fig.11.(a) shows an object made from plastic. This object was scanned using a spherical scanning mode with angular steps  $\Delta\theta = \Delta\gamma = 1^\circ$ . The smoothing factor  $q$  was set to 6. The time required to extract the edge-points shown in Fig.11.(b) is about three hours. It is of great significance to mention that, depending on the angular step, it can happen that edges are omitted by the sensor. Hence, angular steps should be chosen taking into account the shape of the scanned object.

Range image acquisition speed is a critical issue. Since photons are quantized, the speed of data acquisition is limited by the number of photons that the PSD can gather during the dwell time and by the sensor's averaging rate. Another issue in the speed of data acquisition is the scanning mechanism; most time is spent to move from point A to point B. All issues fused together and added to the computational time issue (computational time required for localizing critical points and curvature computation), cause that the method for edge detection described above is time consuming. The detection time can be reduced by reducing the sensor

averaging rate and by reducing the noise in the experimental environment so that smaller values of the smoothing factor  $q$  can be used to compute the curvature. This time also can be reduced by increasing the speed of the robotic manipulator. However, there is a physical limit to the fastest possible speed for scanning at which vibration of the robot manipulator becomes prominent.

## 7. Conclusion

This paper has described a method for detecting corners and edges in noisy scenes using information from a laser displacement sensor attached to the end-effector of a Spherical SCARA Robot. It was shown that it is possible to design a procedure that incorporates all the available knowledge about the scanning mechanism and the type of sensor utilized, resulting in an effective edge detection performance.

The edge-point detection procedure is accomplished in two stages. In the first stage, a critical point is localized. In the second stage, the corner-point is determined by computing the  $q$ -smoothed curvature at the critical point.

All the necessary computer and robot controller interfaces, calibration, data acquisition and processing algorithms have been developed and extensively tested. The spatial output resolution (i.e. density of acquired points) can be easily controlled. For higher spatial resolution (i.e. more points for the same scanned distance) the robotic manipulator is made to move with a very small step angle. However, because of the robot repeatability constraint, there exists a limit to the smallest possible step angle. Faster edge detection can be reached by reducing the sensor averaging rate, reducing the noise in the experimental environment and increasing the speed of the robot manipulator.

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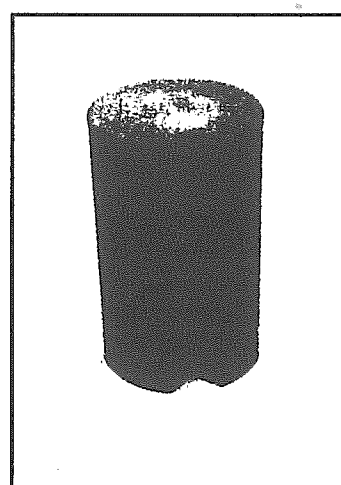
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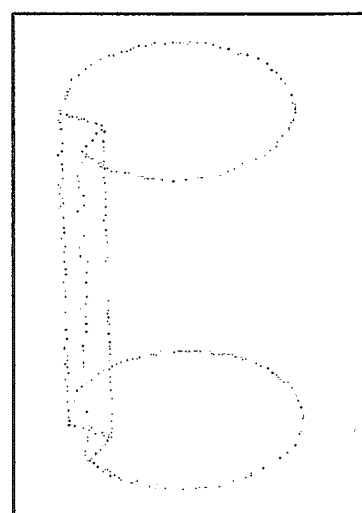
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(a)



(b)

Figure 11: (a) Scanned-object.  
(b) Extracted edge.