動態輸出回授控制器之離散化使用回授神經網路
Discretization of the Dynamic Output Feedback Controllers via Recurrent Neural Networks

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摘要

本論文將提出一個新的方法是使用回授神經網路去離散化動態輸出回授控制器，我們證明所提出的方法在離散化動態輸出回授控制器有非常好的強健穩定性. 此外我們亦證明回授神經網路控制器與無關當滿足取樣定理時. 結果我們所提出的方法不但可離散化動態輸出回授控制器且對取樣時間不確定有非常好的強健性

Abstract

This paper presents a new method which utilizes recurrent neural networks to discretize the dynamic output feedback controller for use in computer-controlled systems. We have shown that the new method does improve the stability robustness in the discretization of dynamic output feedback controllers. Besides. we also have shown that the recurrent neural network controller is independent of the sampling time under the Sample Theorem. Consequently, it can not only discretize the dynamic output feedback controllers, but also tolerate a wider range of sampling time uncertainty.

Keyword : recurrent neural networks, discretization, robustness, dynamic output feedback controller

1. Introduction

When a continuous-time controller is discretized to become a discrete-time controller using the conventional methods, such as design of discrete equivalents by numerical integration, zero-pole mapping equivalents, and hold equivalents, an undesired feature that the discrete-time controller is dependent on the systems sampling time T_s will occur [1]. For linear systems using generalized sampled-data hold function [2], the discrete-time controller also depends on the sampling time T_s. For nonlinear output feedback systems, the discrete-time controller also depends on the sampling time T_s [3]. Therefore, it is difficult to design a digital controller which has robustness against the different sampling time T_s based only on conventional methods in the state space. Recently, Kabamba and Hara [4]. Keller and Anderson [5], applied H_∞ control theory to the worst-case analysis on the sampling time T_s for linear systems; however, their methods also depend on the sampling time. In general, if the sample time changes, the digital controller must be redesign. On the other hand, if we want to obtain a likewise analog controller, one approach to overcome this difficulty is increasing the sample frequency. However. there are loss of reachability and obserability when using the high sample frequency [6].

There are two types of connections in neural networks. Neural networks with only feedforward connections are called feedforward networks, and neural networks with arbitrary connections are recurrent neural networks. The nonlinear dynamical behavior of recurrent networks is suitable for spatio-temporal information processing. Learning algorithms for recurrent neural network which employ the steepest decent method to modify weights have been proposed [7], [8]. Several applications of recurrent neural network have been reported, mainly in dynamic information processing, such as identification of nonlinear and linear dynamic systems [9]. In this work, we propose a method using the recurrent neural networks to overcome the above drawbacks in discrete-time systems. At first, we show that the new method has high stability robustness in the discretization of dynamic output feedback controller. Secondly, when using the recurrent neural networks to learn the desired dynamic output feedback controller, if the error is less than a specified small value, then this new controller satisfies the stability robustness. If the error is nearly equal to zero, then this new controller is an optimal controller in the sense of discretization. We will also show that the new recurrent neural networks controller is independent of the sample time under the Sample Theorem. The recurrent neural networks can use to approximate C^1 functions arbitrary well in this new controller: consequently, it not only discretize the dynamic output feedback controllers, but also tolerate a wider range of sampling time uncertainty. In addition, it is easy to obtain a likewise analog controller. Hence, using recurrent neural networks to the discretization of dynamic output feedback controller is a practical approach.

2. Mathematical preliminaries and definitions

Definition 2.1 (Persistent Excitation) [10]. The set of all functions $g : \Re_+ \to \Re^n$ with g belongs to piecewise-differentiable functions that satisfies Eq. (1)

$$\int_t^{t+T_0} g(\tau) g^T(\tau) d\tau \geq \alpha I \qquad \forall t \geq t_0 \tag{1}$$

where t_0 and α are positive constants over a period T_0, and I is an identify matrix. It is called persistent excitation (P.E.).

Lemma 2.2 [8]. Let σ be a strictly increasing C^1-sigmoid function such that $\sigma(\Re) = (0, 1)$. Let D be an open subset of $(0, 1)^n$, F: $D \to \Re^n$ be a C^1-mapping, and suppose that $\dot{X} = F(X)$ defines a dynamical system on D. Let K be a compact subset of D and we consider trajectories of the system on interval $[0, s_1]$ ($0 < s_1 < \infty$). Then, for an arbitrary $\varepsilon > 0$, there exist an integer h and a recurrent neural network with n output unit and h hidden units such that for any trajectory $\{ X(t) : 0 \leq t \leq s_1 \}$ of the system with initial value $X(0) \in$ K and an appropriate initial state of the network satisfying

$$\max_{t \in s_1} |X(t) - O(t)| < \varepsilon , \tag{2}$$

where $O(t) = (O_1(t), \cdots, O_n(t))^T$ is the output of the recurrent neural networks with the sigmoid output function σ.

Definition 2.3 (Controller is independent of the sampling time). If the sampling time of a system is changed then the controller does not need to redesign again. This is called that the controller is independent of the sampling time.

Definition 2.4 [11]. For each real $p \in [1, \infty)$, the set L_p consists of all measurable function $f(\cdot) : \Re_+ \to \Re$ such that

$$\int_0^\infty |f(t)|^p \, dt < \infty . \tag{3}$$

The set L_p consists of all measurable function $f(\cdot) : \Re_+ \to \Re$ that are essentially bounded on $[0, \infty)$.

Definition 2.5 [11]. For $p \in [1, \infty)$, the function $\| \cdot \|_p : L_p \to \Re_+$ is defined by

$$\|f(\cdot)\|_p = \left[\int_0^\infty |f(t)|^p \, dt \right]^{1/p} . \tag{4}$$

Definition 2.6 [12]. The sub-multiplicative property of the H_∞-norm is defined that satisfies Eq. (5)

$$\|F_1 F_2\|_\infty \leq \|F_1\|_\infty \|F_2\|_\infty , \tag{5}$$

where H_∞ -norm is defined by

$$\|F\|_\infty = \max_{e \in L_2[0,\infty)} \frac{\|Fe\|_2}{\|e\|_2} . \tag{6}$$

Definition 2.7 [11]. The system F is L_p stable without bias (wb) if there exist numbers $\gamma_p(F) \in \Re_+$ such that $\gamma_p(F) \equiv \inf\{\gamma_p : \|Fe\|_p \leq \gamma_p \|e\|_p\}$, where e is the input, $p \in (1, \infty]$, and F is an operator.

Lemma 2.8 (Small Gain Theorem) [11]. Consider the feedback system in Fig. 1, and suppose that $p \in (1, \infty]$ is specified. It is assumed that both F_1 and F_2 are causal and L_p stable wb. Let $\gamma_{1p} = \gamma_p(F_1)$, and $\gamma_{2p} = \gamma_p(F_2)$. Under these conditions, the system in Fig. 1 is L_p stable if $\gamma_{1p} \bullet \gamma_{2p} < 1$.

By a simple transformation of the system in Fig. 1, the range of applicability of Lemma 2.8 can be expanded. The idea is to introduce an additional L_p stable operator C by first subtracting it and then adding it to F_2, as shown in Fig. 2.

Lemma 2.9 (Loop Transformation Theorem) [11]. Consider the system in Fig. 2, and suppose that $p \in (1, \infty]$ is specified. F_1 and F_2 are both causal and L_p stable wb. The system in Fig. 2 is L_p stable wb if there exists a causal operator C which is L_p stable wb, and

(1). $F_1(I + CF_1)^{-1}$ is causal and L_p stable wb,

(2). $\gamma_p(F_2 - C) \bullet \gamma_p\left[F_1(I + CF_1)^{-1}\right] < 1$.

3. Problem formulation

In this paper, we consider the problem of discreting a dynamic output feedback controller by recurrent neural networks (*RNN*) controller as shown in Fig. 3.

It is assumed that the dynamic output feedback controller C which has been designed for the control plant is a L_∞ function, and

(1). control input can be persistent excitation, and the sampling time must satisfy the Sample Theory at least, (2). the recurrent neural networks controller can approximate the C^1 function arbitrarily well in the multi-input function approximation problem.

It is desired that the *RNN* controller in Fig. 3 will not alter the conventional computer-controlled implementation structure. It is also required that the proposed *RNN* controller satisfies the stability robustness of discretization of the dynamic output feedback controller and overcomes the difficulty of sampling time uncertainty.

4. Main results

Lemma 4.1 (Stability robustness of discretization of dynamic output feedback controller) [13]. Let *RNN* be a neural networks controller, *C* be a dynamic output feedback controller as the target for neural networks, and *P* be a unstructured nonlinear uncertainty plant. Assume that

(1). *RNN* can approximate *C* arbitrary well,

(2). *C* stabilizes *P*, and

(3). $\left\|(RNN-C)P(I+CP)^{-1}\right\|_{\infty}<1$,

then *RNN* stabilizes the plant *P*.

Definition 4.2 [13]. The *RNN* is optimal in the sense of stability robustness of discretization, if it minimizes

$$\left\|(RNN-C)P(I+CP)^{-1}\right\|_{\infty}. \qquad (8)$$

The Fig. 3 can be represented by Fig. 4 below and can further be expressed by Fig. 5 after loop transformation.

From the Lemma 2.8 and Lemma 2.9, we can derive the following Theorem 4.3.

Theorem 4.3. Let *C* be a continuous, L_∞ and a causal operator which can stabilize *P*. If the following conditions hold

(a). There exists a causal operator *C* which is L_p stable wb,

(b). There exists a causal operator *RNN* which is L_p stable wb,

(c). $P(I+CP)^{-1}$ is causal and L_p stable wb, and

(d). $\gamma_p(RNN-C)\bullet\gamma_p\left[P(I+CP)^{-1}\right]<1$,

then the system in Fig. 5 is L_p stable wb.

Theorem 4.4. Assume the operator *C*, plant *P* and *RNN* controller in Fig. 5 satisfy conditions (a) ~ (d) of Theorem 4.3. In particular, if p = 2, then

$$\gamma_p(RNN-C)\bullet\gamma_p\left[P(I+CP)^{-1}\right]=$$

$$\left\|(RNN-C)\right\|_{\infty}\bullet\left\|P(I+CP)^{-1}\right\|_{\infty}, \qquad (9)$$

and $$\left\|(RNN-C)\right\|_{\infty}\bullet\left\|P(I+CP)^{-1}\right\|_{\infty}<1, \qquad (10)$$

Proof: First, we shall derive $\gamma_p(RNN-C)$. Since

$$\gamma_p(F)\equiv\inf\left\{\gamma_{p1}:\|Fe\|_p\le\gamma_{p1}\|e\|_p\right\}, \qquad (11)$$

and from Fig. 5, we obtain

$$\left\|(RNN-C)\tilde{e}_1\right\|_p\le\gamma_{p1}\left\|\tilde{e}_1\right\|_p,$$

or $$\frac{\left\|(RNN-C)\tilde{e}_1\right\|_p}{\left\|\tilde{e}_1\right\|_p}\le\gamma_{p1}. \qquad (12)$$

From Eq. (11) and Eq. (12), we have

$$\inf\gamma_{p1}=\max_{\tilde{e}_1\in L_p[0,\infty)}\frac{\left\|(RNN-C)\tilde{e}_1\right\|_p}{\left\|\tilde{e}_1\right\|_p}. \qquad (13)$$

Now let p = 2 $$\inf\gamma_{p1}=\max_{\tilde{e}_1\in L_2[0,\infty)}\frac{\left\|(RNN-C)\tilde{e}_1\right\|_2}{\left\|\tilde{e}_1\right\|_2}$$

$$=\left\|(RNN-C)\right\|_{\infty}=\|\varepsilon\|_{\infty}=\gamma_p(RNN-C). \qquad (14)$$

Next, we shall derive $\gamma_p\left[P(I+CP)^{-1}\right]$. Since

$$\gamma_p(F)\equiv\inf\left\{\gamma_{p2}:\|Fe\|_p\le\gamma_{p2}\|e\|_p\right\}, \qquad (15)$$

and from Fig. 5, we obtain

$$\left\|P(I+CP)^{-1}\bullet e_2\right\|_p\le\gamma_{p2}\|e_2\|_p,$$

or $$\frac{\left\|P(I+CP)^{-1}\bullet e_2\right\|_p}{\|e_2\|_p}\le\gamma_{p2}. \qquad (16)$$

From Eq. (15) and Eq. (16), we obtain

$$\inf\gamma_{p2}=\max_{e_2\in L_p[0,\infty)}\frac{\left\|P(I+CP)^{-1}\bullet e_2\right\|_p}{\|e_2\|_p}. \qquad (17)$$

Now let p = 2

$$\inf\gamma_{p2}=\max_{e_2\in L_2[0,\infty)}\frac{\left\|P(I+CP)^{-1}\bullet e_2\right\|_2}{\|e_2\|_2}$$

$$=\left\|P(I+CP)^{-1}\right\|_{\infty}. \qquad (18)$$

Therefore,

$$\inf\gamma_{p2}=\left\|P(I+CP)^{-1}\right\|_{\infty}=\gamma_p\left[P(I+CP)^{-1}\right]. \qquad (19)$$

From Eq. (14) and Eq. (19), we can obtain

$$\gamma_p(RNN-C)\circ\gamma_p\left[P(I+CP)^{-1}\right]=$$

$$\left\|(RNN-C)\right\|_\infty\circ\left\|P(I+CP)^{-1}\right\|_\infty. \qquad (20)$$

From Theorem 4.3 and Eq. (20), it follows that

$$\left\|(RNN-C)\right\|_\infty\circ\left\|P(I+CP)^{-1}\right\|_\infty<1. \qquad (21)$$

Q.E.D.

We know that when Eq. (10) holds, it implies that $\left\|(RNN-C)\circ P(I+CP)^{-1}\right\|_\infty<1$ by using the submultiplicative property of the H_∞-norm. The *RNN* controller satisfies stability robustness of discretization of dynamic output feedback controller, and stabilizes the plant *P*. Besides, if we let $\left\|(RNN-C)\right\|_\infty=\varepsilon\approx 0$, then the RNN which minimizes

$$\left\|(RNN-C)P(I+CP)^{-1}\right\|_\infty, \qquad (22)$$

is optimal in the sense of discretization of the dynamic output feedback controller.

When the *RNN* is equivalent to the dynamic output feedback controller *C* in the input-output sense, then $\tilde{e}_2$ in Fig. 5 is equal to zero. From Fig. 5, we have

$$y=P(I+PC)^{-1}(u_2+Cu_1)$$

$$=\frac{P}{I+PC}u_2+\frac{PC}{I+PC}u_1, \qquad (23)$$

which is equivalent to the Fig. 6.

Based on above results, we know that if the error is less than a small constant, and Eq. (10) holds, then this new controller satisfies the stability robustness. If the error is nearly equal to zero and Eq. (10) holds, then this new controller is an optimal controller in the sense of discretization. That is, the Eq. (10) ensure the system's stability robustness in Fig. 3. Besides, for the nonlinear plant the result is the same as Eq. (10), i.e.

$$\left\|(RNN-C)\right\|_\infty\circ\left\|F(C,P)\right\|_\infty<1, \qquad (24)$$

where $F(C,P)$ is the operator from e_2 to y in Fig. 5.

Now, we shall show that the *RNN* controller in the Fig. 5 is independent of the sampling time T_s. Fig. 5 can be simplified as Fig. 7 below, where $\tilde{e}_1$ is controller input, x is controller output, $\hat{x}$ is the output of recurrent neural networks controller, and $\tilde{e}_2$ is the error. The sample time T_s must satisfy the Sample Theorem. We can construct $f(\cdot)$ by $\hat{f}(\cdot)$ based on a *RNN*.

From Fig. 7, we have

$$\tilde{e}_2(n+1)=x(n+1)-\hat{x}(n+1)$$

$$=f(\tilde{e}_1(T_{i+1}),x(T_{i+1}))-\hat{f}(\tilde{e}_1(T_{i+1}),x(T_{i+1})). \quad (25)$$

According to Lemma 2.2 the *RNN* can approximate C^1 functions arbitrarily well. When we use any one of the *RNN* to learn $f(\cdot)$ by $\hat{f}(\cdot)$, and the sample time T_s satisfies the Sample Theorem. Then the *RNN* can approximate C^1 functions $f(\cdot)$ arbitrarily well. Therefore, we can obtain the following result from the Lemma 2.2

$$\left|f(\tilde{e}_1,x)-\hat{f}(\tilde{e}_1,x)\right|<\sum_{n=1}^{N}\tilde{e}_2^2(n)=\varepsilon, \qquad (26)$$

and

$$f(\tilde{e}_1(T_i),x(T_i))=\hat{f}(\tilde{e}_1(T_i),x(T_i))+\varepsilon/N, \qquad (27)$$

where T_i is a constant in the interval $[0\ s]$ ($0<s<\infty$), N is the number of sample points, and ε is a arbitrary small constant. Therefore, we conclude

$$f(\tilde{e}_1(s),x(s))\approx\hat{f}(\tilde{e}_1(s),x(s)). \text{ for all input}$$

or $$\left\|\hat{f}-f\right\|_\infty<\varepsilon \text{ for all input} \qquad (28)$$

That is, $\hat{f}$ converges to f uniformly on $(\tilde{e}_1(s),x(s))$. Since there is no restriction on the step size of the *RNN* controller output, we can conclude that the recurrent neural network controller $\hat{f}(\cdot)$ is independent of the sampling time T_s.

5. Computer simulations

The propose method of using the *RNN* to discretize the dynamic output feedback controller is simulated by following example. The simulation programs are running under the Matlab.

Consider the third-order plant [14].

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0&1\\1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}z+\begin{bmatrix}0\\1\end{bmatrix}u$$

$$\varepsilon\dot{z}=x_1-4z+u$$

$$y=x_1+z. \qquad (29)$$

The corresponding reduce order plant is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0&1\\1.25&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\1.25\end{bmatrix}u$$

$$y=1.25x_1+0.25u. \qquad (30)$$

The dynamic output feedback controller is

$$\begin{aligned} \dot{v} &= -1.5v - 2(y - 0.25u) + 1.25u \\ \hat{x}_1 &= 0.8(y - 0.25u) \\ \hat{x}_2 &= v + 2(y - 0.25u) \\ u &= -2\hat{x}_1 - 2\hat{x}_2 \end{aligned} \qquad (31)$$

Eq. (31) can be simplified as Eq. (32) when use $y = 1.25x_1 + 0.25u$.

$$\begin{aligned} \dot{v} &= 7.25v + 22.5y \\ u &= 5v + 14y \end{aligned} \qquad (32)$$

Fig. 8 shows that the *RNN* controller is independent of sampling time. From the results we can know that the new method has very good robustness for the different sampling times.

6. Conclusions

This paper presents a new method to discretize the dynamic output feedback controller by the recurrent neural networks which has very good robustness for different sampling times. All dynamic output feedback controllers can be implemented by the proposed method without changing the computer-controlled structure under the Sample Theorem. We show that the recurrent neural networks controller is independent of the sampling time under the Sample Theorem. That is, the recurrent neural networks are used to approximate C^1 functions. Consequently, it is not only discretize the dynamic output feedback controllers, but also tolerate a wider range of sampling time uncertainty. Hence, using the recurrent neural network to the discretization of dynamic output feedback controller is more practical in some applications. Simulation results show that the new method has very good performance in the discretization of dynamic output feedback controller.

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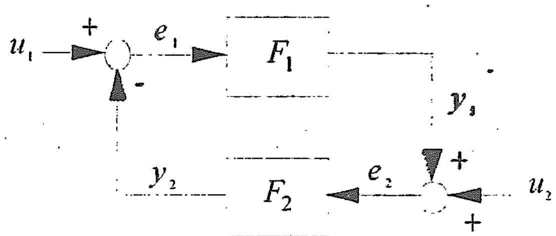

Fig. 1. The general form of a feedback system.

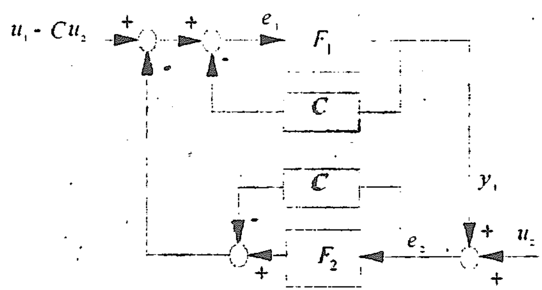

Fig. 2. The loop transformation of Fig. 1.

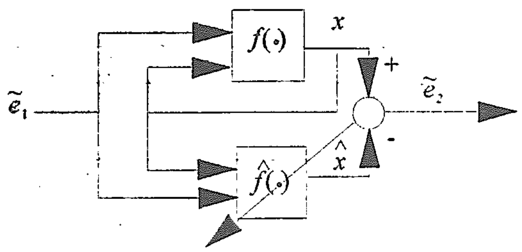

Fig. 7. Simplification of the Fig. 5.

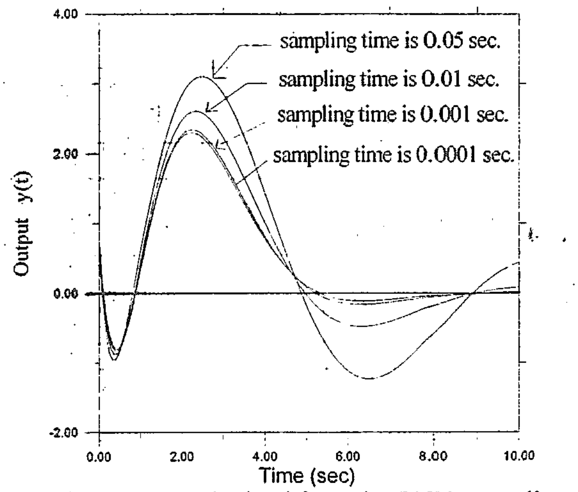

Fig. 8 The response obtained from the *RNN* controller with different sampling time.

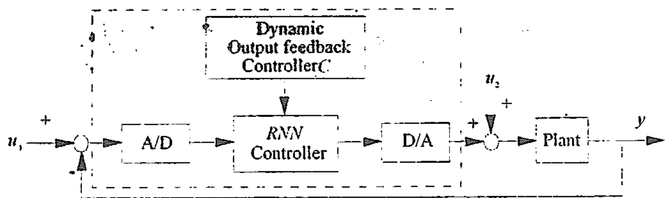

Fig. 3. Discretization of the dynamic output feedback controller by *RNN* controller.

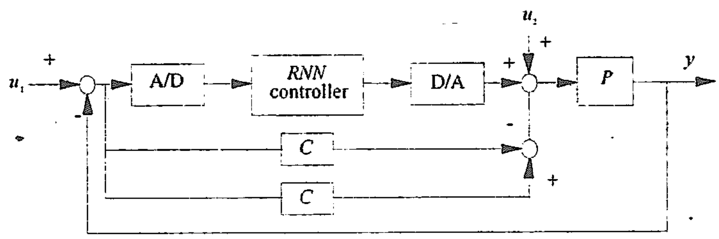

Fig. 4. A equivalent representation of Fig. 3.

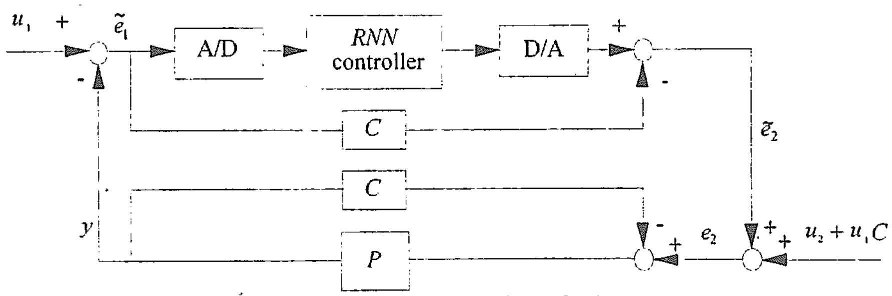

Fig. 5. Loop transformation of Fig. 4.

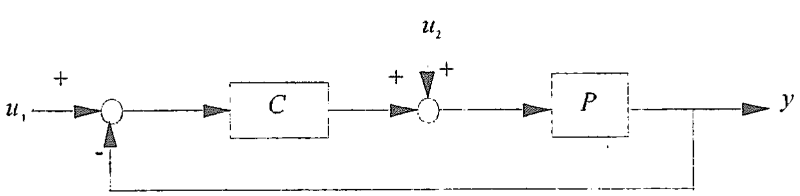

Fig. 6. The general form of output feedback system.