Combining genetic algorithm and first order Taylor series iterative searching DOA estimation for the CDMA system

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Abstract—**This study considers the problem of estimating the direction-of-arrival (DOA) for code-division multiple access (CDMA) signals. In this situation, DOA estimation is more difficult to solve and becomes a highly nonlinear optimization problem. In this paper, we propose a new application of genetic algorithm (GA) base on first order Taylor series iterative MVDR searching technique for the CDMA system. It has been shown that the iterative searching technique suffers from the local maximum searching problem, causing acquisition errors in DOA estimation. In conjunction with a genetic algorithm (GA) for selecting initial search angle, we present an efficient approach to achieve the advantages of iterative DOA estimation with fast convergence and more accuracy estimate over existing conventional spectral searching methods. Finally, numerical example is presented to illustrate the design procedure and confirm the performance of the proposed method.**

Keywords- adaptive array; CDMA; DOA; genetic algorithm; first order Taylor series; iterative searching.

I. INTRODUCTION

The code division multiple access (CDMA) techniques is a relevant issue in many signal processing areas, including direction detection, and multisensory arrays. In CDMA systems, several independent users are simultaneously active in the same transmission medium and distinguishable at the receiver by different user-specific signature waveforms. The advantages of spread-spectrum for these services include superior operation in multipath environments, flexibility in the allocation of channels, ability to operate asynchronously, increased capacity in burst of fading channels, and ability to share bandwidth with narrow-band communication systems without undue degradation of either system's performance [1-2]. In all these applications, estimation of the direction of arrival (DOA) of the desired signal is required. Array outputs aligned with code-matched filter can make the multiple sources DOA estimation equivalent to that of a single source localization problem in a noisy environment. With the advantage of code-matched filter inherent in the CDMA system, it has been proved that the traditional minimum variance distortionless response (MVDR) algorithm can obtain an unbiased DOA estimation with low mean-square-error (MSE) [3]. It also contributes to solve the limitation that the number of array elements must be more than the number of impinging sources. But for the conventional spectral searching DOA estimators such as MVDR [3] and multiple signal classification (MUSIC) [4] algorithms, their searching complexity and estimating accuracy strictly depend on the number of search grids used during the search. It is time consuming and the required number of search grid is not clear to determine. A major problem of MVDR type algorithms is heavy computational load incurred with spatial spectral search when root finding schemes are not applicable.

Genetic algorithm (GA) is a method for optimizing machine-learning algorithms inspired by the processes of natural selection and genetic evolution [5-6]. The underlying principles of GA have been suggested by Holland in 1962 [7] and the more details about GA could be found in [5]. In addition, a number of experimental studies show that GA's exhibit impressive efficiency in practice. While classical gradient search techniques are more efficient for problems which satisfy tight constraints, GA's consistently outperform both gradient techniques and various forms of random search on more difficult (and more common) problems, such as optimizations involving discontinuous, noisy, high-dimensional, and multimodal objective functions. Recently, the application of GA has been applied to deal with antenna arrays in [8-11]. A typical example of which is GA for the global maximization of likelihood function [8]. This approach can keep the risk of false DOA estimate low, at the expense of a significant increase in the computational burden that may not be always acceptable. Reference [10] uses a GA to optimize a thinned array and a planar array to produce patterns with lowest sidelobe level. An improved GA based on is also applied to array-failure corrections [11].

In this paper, we present an efficient DOA estimation approach for CDMA signals with adaptive search grid at each iteration based on the observed receiving data. First, by employing GA to treat our optimization problem of the desired signal direction angle θ, first coded to be a binary string called a chromosome. In each generation, three basic genetic operators, that is, reproduction, crossover, and mutation, are performed to generate a new population with a constant population size. The chromosomes that remain after the population is reduced by the principle of the survival of the fittest produce a better population candidate solution. Although empirical evidence indicates that GA can sometime find good solution for complex problems. A rigid and comprehensive analysis for GA is difficult. However, the convergence of the proposed GA-based DOA estimator scheme can be guaranteed via the theorem of the schema discussed in [5-6]. The DOA, obtained by the proposed GA-based estimator, converges to the optimal or near optimal solution. Secondly, utilizing a first-order Taylor series approximation to the spatial scanning vector in terms of estimating deviation results in and reduces to a simple one-dimensional optimization problem [12]. Correcting factor is self-tuned and fast adaptively corrected from the iteration previous stage in spite of how well the initial guess is performed. Finally, in conjunction with a GA for selecting initial search angle, we present an efficient approach to achieve the advantages of iterative DOA estimate with fast convergence and more accuracy estimate over existing conventional spectral searching methods.

This rest of this paper is organized as follows. The next section examines the problem descriptions. Section 3 presents proposed estimators to DOA estimation. Simulation example for showing the effectiveness of the proposed estimator is presented in the Section 4. The final part includes conclusions about the proposed method.

II. PROBLEM DESCRIPTION

A. Signal Model

Consider a DOA scenario in a baseband CDMA system with L users. Let the bit duration T_b be equal to the processing gain P times the chip duration T_c . After demodulating and chip sampling, the received signal across a ULA with M elements can be represented as

$$\mathbf{y}(k) = \sum_{l=1}^{L} \mathbf{a}(\theta_l) r_l(k) \mathbf{c}_l^T + \mathbf{n}(k) \tag{1}$$

where $r_l(k) = \sqrt{Q_l} b_l(k)$. Q_l is the received signal power of the lth user and $b_l(k) \in \{-1, 1\}$ is the kth data bit of the lth user spreaded by a pseudo-noise (PN) codeword $\mathbf{c}_p$. $\mathbf{n}(k)$ is the spatially and temporally white complex Gaussian noise with zero mean and variance σ_n^2 . $\mathbf{a}(\theta_l) = [a_1(\theta_l), a_2(\theta_l), \cdots, a_M(\theta_l)]^T$ is the response vector of the lth user signal with direction angle θ_l , where $a_m(\theta) = \exp[-j2\pi d(m-1)\sin\theta / \beta]$ denote the response of the mth sensor array to a signal with unit amplitude arriving from the direction angle θ , $j = \sqrt{-1}$ and β is the wavelength of the signal carrier. For convenience, we will assume that the user of interest is $l = 1$. After passing through the code-matched filter, the despreaded signal at the kth bit interval is given by

$$\begin{aligned} \mathbf{z}(k) &= \mathbf{y}(k)\mathbf{c}_1 \\ &= Pr_1(k)\mathbf{a}(\theta_1) + \sum_{l=2}^{L} r_l(k) q_{l1} \mathbf{a}(\theta_l) + \mathbf{n}_1(k) \end{aligned} \tag{2}$$

where $q_{l1} = \mathbf{c}_l^T \mathbf{c}_1$ and $\mathbf{n}_1(k) = \mathbf{n}(k)\mathbf{c}_1$. The second term of (2) can be viewed as the interference noise [2]. Thus we can put it into the noise term $\mathbf{n}_1(k)$ and the composite vector is replaced by a new nomenclature as $\mathbf{n}_1'(k)$ with zero mean and variance $\sigma_{n_1'}^2$ [2]. Then, (2) can be rewritten as

$$\mathbf{z}(k) = Pr_1(k)\mathbf{a}(\theta_1) + \mathbf{n}_1'(k) . \tag{3}$$

The correlation matrix of $\mathbf{z}(k)$ is obtained by

$$\mathbf{R} = E\{\mathbf{z}(k)\mathbf{z}^H(k)\} = P^2 Q_1 \mathbf{a}(\theta_1)\mathbf{a}(\theta_1)^H + \sigma_{n_1'}^2 \mathbf{I}_M , \tag{4}$$

where $E\{\bullet\}$ and the superscript H denote the expectation and complex conjugate transpose, respectively. $\mathbf{I}_M$ is the identity matrix with size $M \times M$. For finite received signal's samples, the received signal correlation matrix $\mathbf{R}$ is replaced by the estimated sample average $\hat{\mathbf{R}} = (1/N_s)\sum_{k=1}^{N} \mathbf{z}(k)\mathbf{z}^H(k)$ and N_s is the total bits of observation. The eigendecomposition of matrix (4) can be expressed as

$$\mathbf{R} = \sum_{m=1}^{M} \lambda_m \mathbf{e}_m \mathbf{e}_m^H = \lambda_1 \mathbf{e}_1 \mathbf{e}_1^H + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^H , \tag{5}$$

where $\lambda_1 \ge \lambda_2 = \lambda_3 = \cdots = \lambda_M = \sigma_{n_1'}^2$ are the eigenvalues of $\mathbf{R}$ and $\mathbf{e}_m$ denotes the eigenvector associated with λ_m for $m = 1, 2, \cdots, M$. Moreover, $\mathbf{e}_1$ and $\mathbf{E}_n = [\mathbf{e}_2, \cdots, \mathbf{e}_M]$ are orthogonal and span the signal and noise subspace corresponding to $\mathbf{R}$, respectively. $\mathbf{\Lambda}_n = \sigma_{n_1'}^2 \mathbf{I}_{M-1}$ is the noise eigenvalue matrix. Furthermore, $\mathbf{e}_1$ spans the same signal subspace as that spanned by $\mathbf{a}(\theta_1)$. Thus, we have $\mathbf{E}_n^H \mathbf{a}(\theta_1) = \mathbf{0}$ and $\mathbf{a}^H(\theta_1)\mathbf{E}_n = \mathbf{0}$.

B. MVDR Estimators

The MVDR estimator is a beamforming-based estimator which estimates the DOA of the desired signal source by way of spatial searching the peak of the power spectrum defined as [3]

$$J(\theta) = \max_{\theta} \frac{1}{|\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)|}, \tag{6}$$

where $\mathbf{a}(\theta) = [a_1(\theta), a_2(\theta), \cdots, a_M(\theta)]^T$ is the spatial scanning vector and $\theta \in [-90^\circ, 90^\circ]$ varies within the whole searching space. It is noted that the DOA estimation is to finding the maximum of the spectrum cost function of estimator as well as to find the minimum of their denominators.

In general, $J(\theta)$ is a very highly nonlinear function of the probability θ. There are may exist many local maximum. It is very difficult to find the global maximum of $J(\theta)$ in (6) by the conventional searching methods. GAs are optimization and machine learning algorithms, initially inspired from the processes of natural selection and evolution genetics. GA exhibits fast initial convergence, but its performance deteriorates as it approaches the desired global extreme. Therefore, in this paper, combining GA and first order Taylor series iterative searching estimator will be employed to specify the signal direction angle θ in (6) to achieve weak dependence on initial parameters, fast convergence, and high accuracy [13].

III. The Proposed DOA Estimators

A. The GA-based Estimator

In this subsection, we describe the GA-based estimator, which is a stochastic optimization algorithm [5-6]. The GA is composed of three operators: (1) reproduction, (2) crossover, and (3) mutation [5-6]. For the GA-based DOA estimator, the cost function is defined as

$$K(\theta) = J(\theta). \tag{7}$$

Our objective is to search θ to achieve the maximum of (7). Then the cost function $K(\theta)$ takes a chromosome and returns a value. The value of the cost is then mapped into a fitness value $Fit(\theta)$ so as to fit into the GA. The fitness value is a reward based on the performance of the possible solution represented by the string, or it can be thought of as how well a signal direction angle θ can be tuned according to the string to actually maximum the cost function. The better the solution encoded by a string (chromosome), the higher the fitness. To maximum $K(\theta)$ is equivalent to getting a maximum fitness value in genetic searching process, a chromosome that has a higher $K(\theta)$ should be assigned a larger fitness value. Then, GA tries to generate better offsprings to improve the fitness. Therefore, the objective function in (6) has to be changed to the maximization of fitness to be used in the simulated roulette wheel as follows [6]

$$Fit(\theta) = \begin{cases} K_{\min} + K(\theta); \text{ if } K(\theta) \geq K_{\min} \\ 0; \text{ otherwise} \end{cases} \tag{8}$$

where $K_{\min}$ is the prescribed the smallest values. Then the GA tries to generate better offspring to improve the fitness. Using the three operations.

B. The First Order Taylor Series Iterative Searching Estimator

The performance of the abovementioned spectral searching estimators are governed by the scanning grid size and the number of search grids while implementing the high-resolution DOA estimation. It is time consuming and the search grid is not clear. A fast and simple approach which can effectively speed up the MVDR searching process is proposed [3] in the presence of pointing error. In this paper, it can be applied to iteratively search for the correct DOA by maximizing the spectrum with respect to the DOA estimating deviation. A first-order Taylor series approximation to the spatial scanning vector in terms of the estimating deviation results in and reduces to a simple one-dimensional optimization problem. The correcting factor is self-tuned and fast adaptively corrected from the iteration previous stage regardless of the accuracy of the initial guess. Then it contributes to enhance the convergence rate of the conventional spectrum-search estimator.

From (6), the MVDR spectrum cost function, we know that finding out the maximum is equivalent to finding out the minimum of the denominator (null spectrum) of this cost function, i.e.,

$$\min_{\mathbf{a}} \mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta). \tag{9}$$

In a small region $\Delta\theta$ about the assumed initial angle-of-arrival θ_0, $\mathbf{a}(\theta)$ can be approximated by a first order Taylor series [14]

$$\mathbf{a}(\theta) = \mathbf{a}(\theta_0 + \Delta\theta) \cong \mathbf{a}(\theta_0) + \Delta\theta\, \mathbf{a}'(\theta_0), \tag{10}$$

where $\mathbf{a}'(\theta_0) = \frac{d}{d\theta}\mathbf{a}(\theta)|_{\theta=\theta_0}$. Substituting (10) into (9) results in

$$\min_{\Delta\theta} \; [\mathbf{a}(\theta_0) + \Delta\theta\, \mathbf{a}'(\theta_0)]^H \mathbf{R}^{-1} [\mathbf{a}(\theta_0) + \Delta\theta\, \mathbf{a}'(\theta_0)], \tag{11}$$

which is a simple one-dimensional optimization problem. And, it can be easily shown that the optimum $\Delta\theta_o$ is given by

$$\Delta\theta_o = -\frac{\mathrm{Re}[\mathbf{a}^H(\theta_0)\mathbf{R}^{-1}\mathbf{a}'(\theta_0)]}{[\mathbf{a}'(\theta_0)]^H \mathbf{R}^{-1}\mathbf{a}'(\theta_0)}, \tag{12}$$

where Re[•] denotes the real part of the complex quantity. The optimum spatial response vector can then be found by substituting (12) into (10). If the estimating deviation $\Delta\theta$ is very small, the spatial response vector obtained from (12) is equivalent to the real response vector with actual desired signal's DOA. However, if the estimating deviation is quite large, then we require an iterative algorithm that updates the estimating deviation toward to the actual value.

However, we note that the iteration MVDR (IMVDR) estimator also suffers from the local minimum searching problem when the assumed initial angle-of-arrival (AOA) estimates are near to the interferers' AOAs, especially the processing gain is not sufficiently large. And, the local minimum searching problem leads to a degradation of the effectiveness of the IMVDR estimator. To deal with this problem, we present a GA/IMVDR estimator in the next subsection.

C. Proposed Estimator

In order to associate the initial fast convergence and weak dependence on initial parameters of GA with the high accuracy of first order Taylor series iterative searching, we combined both methods. The combined GA and first order Taylor series iterative searching estimator always starts the search procedure as a pure-GA and ends as a pure first order Taylor series iterative searching. The transition from GA to first order Taylor series iterative searching occurs when the following condition is satisfied. The fittest individual remains the same for L_g generations [15]. This condition is satisfied whenever the algorithm converges to an intermediate solution. The solution thus far constitutes a good initial guess to first order Taylor series iterative searching estimator.

Based on the above analysis, the design procedure of proposed direction angle θ estimation is divided into the following steps.

a) Given the received data vector $\mathbf{y}(k)$*, and generate a random population* θ *of T chromosomes.*

b) Compute the corresponding fitness value Fit from (8).

c) Use the GA operators (reproduction, crossover, and mutation) to produce chromosomes of next generation.

d) Repeat the procedure from step (b) to step (d) until the fittest individual remains the same for L_g *generations.*

e) Given the best chromosome, θ *are the initial values of first order Taylor series iterative searching estimator.*

f) Compute the $\Delta\theta$ *from (12).*

g) If $\Delta\theta > \varepsilon$ *, then renew the* $\theta = \theta + \Delta\theta$ *and repeat the procedure from step (f) to step (g) until* $\Delta\theta \le \varepsilon$ *, the suitable* θ *is obtained.*

IV. SIMULATION RESULTS

In this section, we present several simulation examples to illustrate the DOA estimate performance of the proposed estimator for CDMA signals. Comparison results with other estimators, including the MVDR [3], and GA-based. All CDMA signals were generated with BPSK modulation and PN codes of processing gain 27. The additive background noise is assumed to be spatially and temporally white complex Gaussian distribution with zero-mean and unit variance. The deviation precision is $\varepsilon = 0.001^\circ$ and searching grid size for the spectrum searching MVDR [3] is set at 0.001° . The parameters of GA-based estimator are set as $T = 50$, $p_m = 0.02$, $p_c = 0.95$, and $L_g = 5$, where T is size of population, p_c is the crossover rate and p_m is the mutation rate [6]. The simulation results are obtained by averaging 50 independent Monte Carlo runs.

Example: Consider active CDMA signals $L = 5$ and a 8-element ULA with half-wavelength. The impinging angles of all four interferers with equal power 10 dB are $[-20^\circ,\ -10^\circ,\ 10^\circ,\ 20^\circ]$. The desired user is impinging on the array with directional angle of $\theta_1 = 5^\circ$. Table 1 gives the DOA estimates (estimation mean value and standard deviation error) and search number of MVDR[3], GA-based, and proposed estimator with different initial values. From Table 1, the proposed estimator is high accuracy and fast convergence with weak dependence on initial parameters. Fig. 1 shows that the RMSE of DOA estimation versus different signal-to-noise ratio (SNR) of the desired user. In this figure, we can see that the performance of the proposed estimator has been improved. The histogram of the DOA estimation using the first order Taylor iterative searching estimator with initial random angles of uniform distribution in the interval of $(-90^\circ,\ 90^\circ)$ under SNR = 25 dB is presented in Fig. 1. It demonstrates that the DOA estimation of the first order Taylor iterative searching estimator is vulnerable to the effect by the initial value. And, it is obvious that the DOA estimation value interval of (0, 10) is only about 16.41% .

V. CONCLUSION

This paper has presented an efficient approach for DOA estimation with iterative searching, which reduces the required search grids for the conventional spectral searching estimators. With the code-matched filter, the MAIs after code-decorrelation appear as noises for CDMA signals. However, the first order Taylor iterative searching estimator is gradient search approach to create efficient DOA estimation. It seems to converge more rapidly than the MVDR and GA estimators, but its performance heavily depends on initial value. The GA is parallel search method that weak depend on initial value, but its performance deteriorates as it approaches the desired global extreme. For

CDMA bearing estimation, we combine the selected features form GA and IMVDR to achieve weak dependence on initial values, high accuracy, and fast convergence. Computer simulations have demonstrated the effectiveness of the proposed estimator under lower SNR and larger MAI environments.

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TABLE I. DOA ESTIMATION AND NUMBER OF SEARCHES USING THE MVDR ESTIMATOR, GA-BASED ESTIMATOR, AND PROPOSED ESTIMATOR.

	Estimators	SNR	Number of Searches		Estimation Value	
			Iterations	Generations	mean	standard deviation
Example	MVDR	*5dB*	180001		5.0070	0.0315
		25dB	180001		4.9999	0.0132
	Proposed method	*5dB*	44	15	4.9881	0.0282
		25dB	42	14	4.9973	0.0130
	GA-based	*5dB*		30	4.8601	0.2626
		25dB		30	4.8314	0.2652

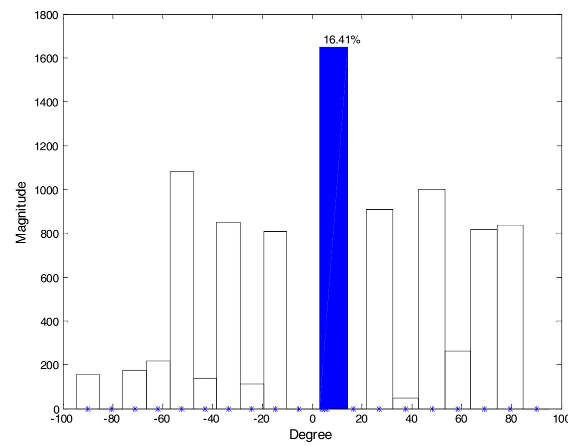

Figure 1. The histogram of DOA estimation using the first order Taylor iterative searching estimator with initial random angles of uniform distribution in the interval of $(-90^\circ, 90^\circ)$ under SNR = 25 dB.