Optimal Schedulable Region Analysis of FCFS Network Model Supports End-to-end Delay Requirement

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SUMMARY

The modern high-speed integrated service networks have provided multimedia applications. There are diverse traffic characteristics and different quality of service requirements. A scheduling discipline should permit easy admission control. A switch controller should be able to make decision. For a given current set of connections and the description for a new connection, whether it is possible to meet the new connection's performance bounds without degrading the performance of exiting connections. By analysis the schedulable region [1], we can easily do admission control and measure the efficiency of a scheduling discipline. A network structure using FCFS scheduling with homogenous source model has been proposed in [2] and the number of the connection vs. delay bound is solved. But in the real world, the sources are heterogeneous. For example, we can lower the quality of MPEG picture of video conference and lower the data transferred to the network and require same delay bound. In this paper, We extend the previous work of [2] and derived the schedulable region of N classes QOS service. The connections of same class have same (! , ") traffic parameters. By this traffic model, we can fit the requirement of MPEG data and get the same delay. We derived a formula for delay bound requirement and the number of connections of each class. We found that with the delay bound as the QOS metric, the schedule discipline of FCFS meets the optimal allocation policy (OPT)[4]. We also developed an evaluation strategy RG (Relative Gain) with weighting ! to analyze the network utilization. The numerical results of tandem network model, show the proposed FCFS network structure is optimal.

Keywords: deterministic service, scheduling, schedulable region, delay bound, QOS

1. Introduction

The modern high-speed integrated service networks have provided multimedia applications. There are diverse traffic characteristics and different quality of service requirements. A scheduling discipline should permit easy admission control. A switch controller should be able to make decision. For a given current set of connections and the description for a new connection, whether it is possible to meet the new connection's performance bounds without degrading the performance of exiting connections. By analysis the schedulable region [1], we can easily do admission control and measure the efficiency of a scheduling discipline.

To support real-time multimedia applications, a guaranteed service model was proposed in [2], which requires performance bounds that can be promised. In [2], the authors proposed an optimal QOS allocation policy and decide the maximum utilization bound in a deterministic traffic model. They assume the traffic is homogeneous. But for modern application, the requirements are different and so the quantity of the data transferred is different. Above assume is not adequate. In this paper, we deal with N kind of (! ,") traffic and analyze the schedulable region of the proposed network structure. With (! ,") traffic model, we derived a formula for delay bound requirement and the number of connections of each classes. Our deterministic traffic model is (! ,") model that describes the traffic characteristics using a token arrival rate " and a bucket size ! [3]. We adopt the worst case delay bound as the end-to-end and local QOS requirement. The numerical results of tandem model show the correctness and efficiency of the proposed network structure for non-homogenous traffic. The remainder of this paper is organized as follows. In section 2, we propose an evaluation model. In section 3, we derive the schedulable region for the proposed network structure under tandem models. In section 4, we use numerical results to show the correctness of the analysis. In the final section, we present our conclusion.

2. The QOS Criteria for N classes traffic

In this section, we study the schedulable regions of N classes traffic and find out the relationship between delay and number of supportable connections from the single node case to the end-to-end case.

2.1 Single Node

For the simplicity of analysis, we assume the source of traffic for the connection conforms to the leaky bucket with parameter (! $_i$," $_i$). That is, if A(t) is the arrival traffic for the connection (! $_i$," $_i$), then A(t) < ! $_i$ + " $_i$t. Other traffic models can also be used to derive the relationship between delay and number of supportable connections, but it is not apparent. If we have N_i connections of class i sources, then delay bound is d , under the FCFS scheduling algorithm, is given by[6]

$$d = \frac{1}{l}\max_{t\ge 0}\{ \ {}_{i=1}^{N} N_i(\sigma_i + \rho_i t) - lt\} \quad (1)$$

Where l is the bandwidth of the output channel. For network stability, we let ${}_{i=1}^{N} N_i\rho_i \le l$, then

$$ {}_{i=1}^{N} N_i\sigma_i + t(\ {}_{i+1}^{N} N_i\rho_i - l) < \ {}_{i=1}^{N} N_i\sigma_i$$

$$\max_{t\geq 0}\{\sum_{i=1}^{N} N_i(\sigma_i+\rho_i t) - lt\} = \sum_{i=1}^{N} N_i\sigma_i$$

And we obtain

$$d = \frac{1}{l}\sum_{i=1}^{N} N_i\sigma_i$$

Given l and ! $_i$ that the sets of N_i are a function of d. That is, given the resources and connections' traffic characteristics, we can find the relationship of N_i and d.

Supposed we currently service N_i connections of class i and give all the extra bandwidth to class k. We can easily get following results:

$$\hat{d} = \frac{1}{l}(\Delta N_k\sigma_k + \sum_{i=1}^{N} N_i\sigma_i)$$

then we can get the extra # N_k connections for some class k is

$$\Delta N_k = \frac{dl - \sum_{i=1}^{N} N_i\sigma_i}{\sigma_k} \qquad (2)$$

and the extra delay over total delay is

$$\frac{\Delta d}{d} = \frac{\hat{d}-d}{d} = \frac{\Delta N_k\sigma_k}{ld} = \frac{\Delta N_k\sigma_k}{\sum_{i=1}^{N} N_i\sigma_i} \qquad (3)$$

2.2 End-to-end Case

In this section, we extend the single node case to the end-to-end case. We assume that there are N classes and k switch nodes. All the sources of each class conform to the (! ," ') traffic model and for the simplicity of the analysis, the propagation delay for each output link is zero. Since the characteristics of the source traffic are modified as the source traffic passes through the network, we add a Delay Jitter (DJ) Regulator[5] for each connection at the switch to reshape the traffic pattern to conform to (! ,") before the traffic enters the FCFS queue. The waiting time at the regulator of switch i does not contribute to the worst case delay at switch I and will not increase the worst case end-to-end delay. The switch architecture is described in figure 1. The end-to-end delay bound under this model is the sum of the delay of FCFS queue of each switch node that the connection goes through. We described it by following theorem and the detailed proof is in [2].

Theorem 1 Consider a connection passing through n switches connected in cascade. Assume that the scheduler of FCFS switch can guarantee that the delay of all the packets on the connection be bounded by $\hat{d}_i$ as long as the connection's input to the scheduler satisfies the given (σ,ρ) specification. If the traffic on the connection obeys the (σ,ρ) specification at the entrance to the first switch. The end-to-end delay $\hat{Q}$ for any packets are bounded by $\sum_{i=1}^{n}\hat{d}_i$ if delay-jitter regulators are used. That is

$$\hat{Q} = \sum_{i=1}^{n}\hat{d}_i \qquad (4)$$

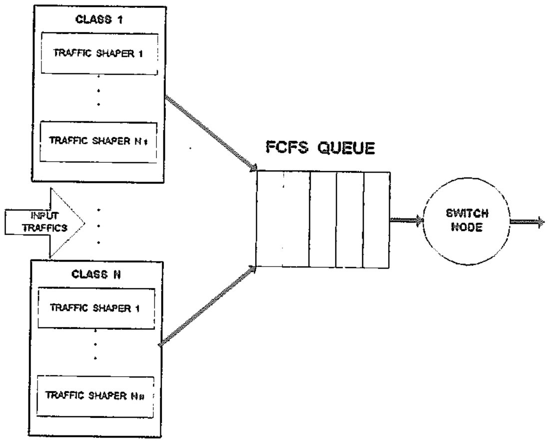

Fig. 1 The Architecture of Switching Node

2.3 Upper Performance Bound for QOS Allocation Policy

The relative gain (RG) will be introduced to evaluate the performance of the QOS allocation policies. The relative gain is defined as shown in the following equation:

$$RG = \frac{\frac{\Delta N}{\hat{N}}}{\frac{\Delta Q}{\hat{Q}}} \qquad (5)$$

where # Q is the superfluous value between the sum of the local worst case QOS and the required QOS of the applications, here the QOS is delay, # N is the additional amount that is admitted for the extra QOS requirement, $\hat{N}$ and $\hat{Q}$ are the number of connection and the QOS for the previous QOS requirement, respectively. For the simplest case, single node, the worst case delay bound is shown as the following equation:

$$\Delta d = \frac{1}{l}\sum_{j=1}^{N}\Delta N_j\sigma_j$$

Where ΔN_j , j=1,2,...N, is the extra number of admitted connections for each class. Since the traffic source is deterministic and the bandwidth is fixed in the node, the extra delay is proportional to the sum of number of superfluous admitted connections multiply the σ of all classes. Furthermore, the RG for the QOS allocation policy is obtained as follows. Let the hop number is k and current number of connections for each class is N_j, then we have

$$\hat{Q} = \sum_{i=1}^{k}\hat{d}_i$$

$$\Delta Q = Q - \hat{Q} = \sum_{i=1}^{k}\Delta d_i$$

$$\hat{d}_i = \frac{\sum_{j=1}^{N} N_j\sigma_j}{l_i}, i = 1,2,\Lambda\ k$$

$$\Delta\hat{d}_i = \frac{\sum_{j=1}^{N}\Delta N_j\sigma_j}{l_i}, i = 1,2,\Lambda\ k$$

Since the higher the σ of a class, the more of loading to the network, a weighting factor is need for each class. Supposed the current number of connection is N_i for class i, and the extra number of connections is $\{\Delta N_{j,i}\}$ (the schedulable region) at switch i. And also the weighting factor is σ_j for class j. Supposed the intersection of the schedulable region of all switches is $\{\Delta N_j\}=\{\Delta N_{j,1}\}\cap\{\Delta N_{j,2}\}\ldots\cap\{\Delta N_{j,k}\}$, j=1,2,...N , then $RG_{w\sigma}$ is obtained as follows:

$$RG_{w\sigma}=\frac{\dfrac{\sum_{j=1}^{N}\Delta N_j\sigma_j}{\sum_{j=1}^{N}\hat{N}_j\sigma_j}}{\dfrac{\sum_{i=1}^{k}\dfrac{\sum_{j=1}^{N}\Delta N_{j,i}\sigma_j}{l_i}}{\sum_{i=1}^{k}\dfrac{\sum_{j=1}^{N}\hat{N}_j\sigma_j}{l_i}}}\le\frac{\sum_{j=1}^{N}\Delta N_j\sigma_j}{\dfrac{\sum_{i=1}^{k}\dfrac{\sum_{j=1}^{N}\Delta N_j\sigma_j}{l_i}}{\sum_{i=1}^{k}\dfrac{1}{l_i}}}=1 \qquad (6)$$

If the $RG_{w\sigma}$ of the QOS allocation policy is equal to 1, the QOS allocation policy is optimal. In the next section, we will show the schedulable region of proposed network meets the optimal allocation (OPT) policy.

3. The Analysis Model

If the worst case end-to-end delay bound for the connection is $\hat{Q}$, that is, the network can guarantee that the delay for the connection will be no worse than $\hat{Q}$. Now if the requirement for end-to-end delay for the connection is Q and Q $\ge\hat{Q}$, the network will allocate Q end-to-end delay to the connection. What is the optimal allocation for this extra amount ? In the following, we focus on this problem. We will show the schedulable region of the proposed FCFS network meets optimal allocation (OPT) under tandem network models. The optimal allocation policy means that the number of supportable connections is maximum, that is, given the network resources and traffic characteristics, the upper bound for the number of supportable connections is the optimal allocation.

3.1 Tandem Model

In this section. we use a tandem network model to evaluate the schedulable region. Figure 2 describes this model consisting of a single source-destination pair of nodes. The network has n nodes and only a single route. In section 3.2, we derived the number of supportable connections under the proposed network structure. Section 3.2 , we derived the number of supportable connections under OPT policy. By compare the result of above two sections, section 3.3 show the proposed network structure meets optimal policy.

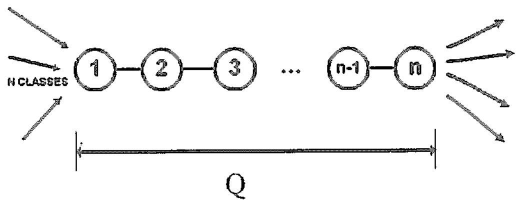

Fig. 2 The Tandem Network Model

3.2 Schedulable region

If we assumed current support number of connections of N_1, N_2, , N_N for class 1,2, N respectably, and the delay requirement is Q, then following equation should meet:

$$\sum_{i=1}^{i=n}\frac{\sum_{j=1}^{N}N_j\sigma_j}{l_i}\le Q$$

All the set of $\{N_j$, j=1,...,N } meet above equation form the schedulable region of the network. We can use the numerical method to get outline of the schedulable region in N dimension. Figure 3 shows the schedulable region of 3 classes is inside the area of four corners N_1, N_2, N_3, and original.

If we assumed current support number of connections of N_1, N_2, , N_N for class 1,2, N respectably, then the extra # N_k for class k that is permitted to enter, is computed as below:

$$\sum_{i=1}^{i=n}\frac{\sum_{j=1}^{N}N_{j,i}\sigma_j+\Delta N_k\sigma_k}{l_i}\le Q$$

$$\sum_{i=1}^{n}\frac{\Delta N_k\sigma_k}{l_i}\le Q-\sum_{i=1}^{i=n}\frac{\sum_{j=1}^{N}N_{j,i}\sigma_j}{l_i}=Q$$

$$\Delta N_k\le\frac{Q}{\sigma_k}\frac{1}{\sum_{i=1}^{n}\frac{1}{l_i}}$$

Let

$$N_f=\frac{Q}{\sigma_k}\frac{1}{\sum_{i=1}^{n}\frac{1}{l_i}} \qquad (7)$$

For network stability condition $N_s\rho_k+\sum_{j=1}^{N}N_j\rho_j<l_i$, then

$$N_s=\min\{\frac{l_i-\sum_{j=1}^{N}N_j\rho_j}{\rho_k},i=1,\ldots,n\}$$

$$N_{extra} = \min\{N_f, N_s\}$$

If the extra number of connections of class k requested is greater than N_{extra}, then the new connection will be rejected. So totally, we can support $N_{extra} + \sum_{j=1}^{N} N_j$ connections. The delay which is allocated to switch i is obtained by adding the delay of each class and is equal to

$$\frac{N_{extra}\sigma_k}{l_i} + \frac{\sum_{j=1}^{N} N_j \sigma_j}{l_i}$$

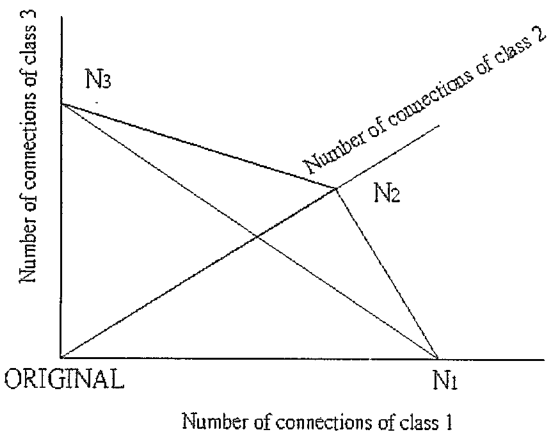

Fig. 3 The schedulable region of 3 classes

3.3 OPT Policy

Assume that the end-to-end delay bound from node 1 to node n is $\hat{Q}$, for the application N classes of (σ,ρ) traffic and the end-to-end delay requirement is Q and $Q > \hat{Q}$. Let # d_i be the amount of increasing delay at switch i, then

$$\hat{d}_1 + \hat{d}_2 + \ldots + \hat{d}_n = \hat{Q}$$

$$\left(\hat{d}_1 + \Delta d_1\right) + \ldots + \left(\hat{d}_n + \Delta d_n\right) = Q$$

For switch i,

$$\hat{d}_i + \Delta d_i = \frac{\sum_{j=1}^{N} N_{j,i}\sigma_j}{l_i}$$

$$\Delta d_i = \frac{\sum_{j=1}^{N} N_{j,i}\sigma_j}{l_i} - \hat{d}_i$$

Since

$$\sum_{i=1}^{n} \Delta d_i < Q - \hat{Q},$$

then

$$\sum_{i=1}^{i=n}\left(\frac{\sum_{j=1}^{N} N_{j,i}\sigma_j}{l_i} - \hat{d}_i\right) < Q - \hat{Q}$$

$$\sum_{i=1}^{i=n}\frac{\sum_{j=1}^{N} N_{j,i}\sigma_j}{l_i} < Q$$

If we assumed current support number of connections of N_1, N_2, … N_N for class 1,2,… N respectably, then the extra # N_k for some k, is computed as below:

$$\sum_{i=1}^{i=n}\frac{\Delta N_k\sigma_k + \sum_{j=1}^{N} N_j\sigma_j}{l_i} \le Q$$

$$\sum_{i=1}^{n}\frac{\Delta N_k\sigma_k}{l_i} \le Q - \sum_{i=1}^{i=n}\frac{\sum_{j=1}^{N} N_j\sigma_j}{l_i} = Q$$

$$\Delta N_k \le \frac{Q}{\sigma_k}\frac{1}{\sum_{i=1}^{n}\frac{1}{l_i}}$$

Let

$$N_f = \frac{Q}{\sigma_k}\frac{1}{\sum_{i=1}^{n}\frac{1}{l_i}} \qquad (8)$$

For network stability condition $N_s\rho_k + \sum_{j=1}^{N} N_j\rho_j < l_i$, then

$$N_s = \min\{\frac{l_i - \sum_{j=1}^{N} N_j\rho_j}{\rho_k}, i = 1, \ldots, n\}$$

We have the number of supportable connections under OPT allocation policy, N_{OPT}, is given by

$$N_{OPT} = \min\{N_f, N_s\}$$

Now, we will use the N_{NOPT} be the maximum extra number of connections of class k to enter the network. If the extra request number of connections is greater than N_{OPT}, the new connections will be rejected. So totally, we can support $N_{OPT} + \sum_{j=1}^{N} N_j$ connections. The delay is allocated to switch i is obtained by adding the delay of each class and is equal to

$$\frac{N_{OPT}\sigma_k}{l_i} + \frac{\sum_{j=1}^{N} N_j\sigma_j}{l_i}$$

3.4 Comparison of the schedulable of FCFS and OPT

The results of eqn (7) and (8) are same. We conclude that FCFS meet OPT policy under tandem model and use all the extra bandwidth of nodes. The extra number of connection we permitted to enter the network is N_{OPT}. We explain this result as example 1.

Example 1:

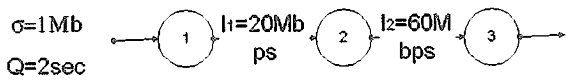

Fig 4 The network structure for example 1

The network have two switch nodes and one destination node as described in figure 4. The bandwidth between node 1 and node 2, node 2 and node 3 are l_1=20Mbps and l2=60Mbps, respectively. The characteristic of input traffics are σ=1 and Q=2, we get the number of connections is N_{OPT}=2/(0.5(1/20+1/60))=60. The delay of node 1 and node 2 are d_1=60*0.5/20=1.5 and d_2=60*0.5/60=0.5, respectively. The end-to-end delay is $\hat{Q}$=2=Q. Therefore, the maximum number of connections that is permit to enter the network is 60 for the request delay 2 sec.

4. Numerical Results

To evaluate the efficiency and optimality of the proposed structure, a series of numerical examples are presented in this section.
Two performance measure indexes are adopted to decide the efficiency and optimality of the propose structure. One is the network utilization, in terms of the number of admissible connections, which is used to represent the efficiency of the proposed scheme. Moreover, other index is the relative gain (RG) with weighting to evaluate the optimality of the network. In the sequential subsections, we run the numerical computation to get the schedulable region in a tandem network model. We varied the hop numbers and traffic load to observe the relationship between the performance index and the end-to-end QOS requirement.

Experiment 1 : Two hop Tandem model:(! $_1$=1,! $_2$=3)
The bandwidths between node 1 and node 2, node 2 and node 3, were l1=5000 unit and l2 =3000 units, respectively. Figure 5 shows the variation in the schedulable region of network over end-to-end delay requirement. Figure 6 shows the variation in the relative gain with weighting over the end-to-end delay requirement. The $RG_{w\sigma}$ (RG weighting to !) of the proposed network structure usually approaches one. It appears that the efficiency and optimality of the proposed network structure is proved in the topology above. The price charged for each connection is proportional to the weighting factor is a good chose.

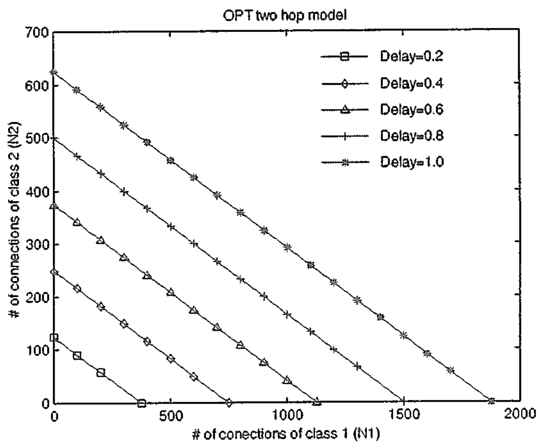

Fig. 5 The Schedulable Region of proposed network model with 2 Hops

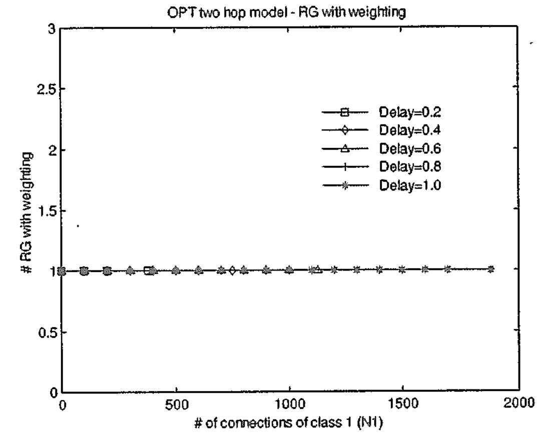

Fig. 6 The RG with Weighting of proposed network model with 2 Hops

Experiment 2 : Five hops (h=5) Tandem model:(! $_1$=1,! $_2$=3)
The bandwidth between each node was 5000 units except for l3, which ranged from 3000 to 4000 units. With l3 equal to 3000, Figures 7 and 8 show the schedulable region and relative gain with weighting over the end-to-end delay requirement in the five hops tandem network model, respectively. For l3 equal to 4000, Figures 9 and 10 show the schedulable region and relative gain with weighting over the end-to-end delay in the five hops tandem network model, respectively. The change in the hop number and bandwidth in each node did not affect the fact that the schedulable region of the network is optimal in the Tandem model.

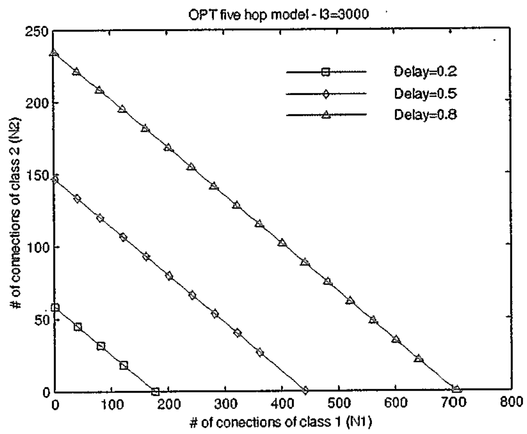

Fig. 7 The Schedulable Region of OPT Allocation Policy with 5 Hops(l_3=3000)

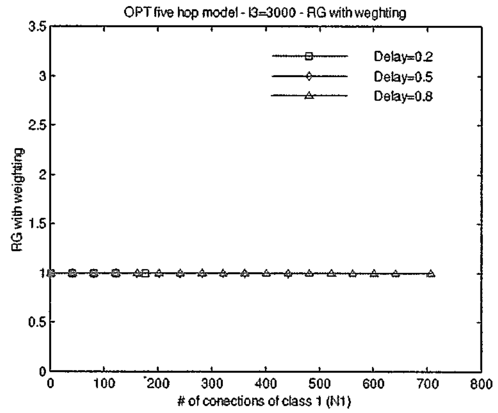

Fig. 8 The RG with Weighting of proposed network model with 5 Hops (l_3=3000)

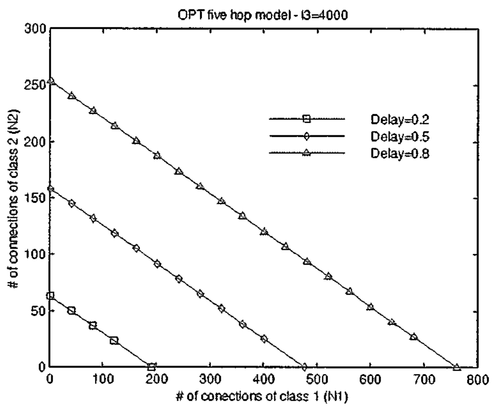

Fig. 9 The Schedulable Region of proposed network model with 5 Hops(l_3=4000)

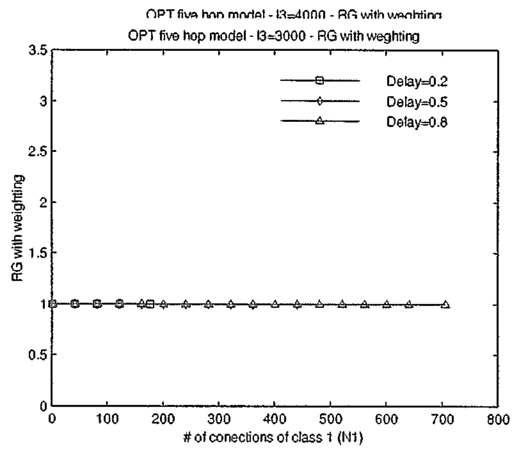

Fig. 10 The RG with Weighting of proposed network model with 5 Hops(l_3=4000)

5. Conclusion

We have proposed a strategy to evaluate the allocation of the end-to-end delay requirement. In the real world, the source traffic can be heterogeneous. We fit this requirement by assuming that there is N classes of homogeneous sources. The sources of same class conformed to the same (! ,") model. We add a Delay Jitter regulator for each connection at the switching node to prevent traffic distortion while the traffic for the connection passed through the network. The results show the relationship of the delay to the number of supportable connections at a single node and in end-to-end cases. The analysis model derived the number of supportable connections for the proposed network model. The numerical results present this structure meets the optimal allocation policy. In this research, we use deterministic guaranteed service. However this typically results in an underutilized network. Currently, we are going to use probabilistic performance bound to improve the utilization of network.

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