A NEW CODE CONSTRUCTION FOR MULTIRATE TRANSMISSION IN OPTICAL FIBER CDMA NETWORKS

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ABSTRACT

The purpose of this paper is to investigate the multirate transmission in fiber-optic code-division multiple-access (CDMA) networks. In this paper, we show a new construction of optical orthogonal code to implement a multirate optical CDMA system (called as the multirate code system). For comparison, a multirate system where the low rate user sends each symbol twice is implemented and is called as the repeat code system. Theoretical analysis shows that the bit error probability of the multirate code system is smaller than that of the repeat code system, especially when the number of low rate users is larger. Moreover, if there is any low rate user in the system, the multirate code system accommodates more users than the repeat code system when the error probability of system is set below 10^{-9}.

1. INTRODUCTION

In the last few decades, code-division multiple-access (CDMA) technique has been proposed for applications in fiber-optic networks. CDMA technique allows multiple users to share the entire channel and provides asynchronous access to each other. The asynchronous property of CDMA technique is very suitable for LAN's because the traffic in LAN's is typically bursty and asynchronous. In addition, CDMA technique permits multiple users to simultaneously access the channel without waiting time, which results in small queue in the system and less transmission delay.

However, CDMA technique is based upon the assignment of orthogonal codes to the address of each user. Different code sets perform differently. Basically, the orthogonal code sequences need to satisfy two conditions:

(1) Each code sequence can easily be identified from a shifted version of itself.

(2) Each code sequence can easily be distinguished from any shifted version of other sequences in the same code set.

Condition (1) is used for code synchronization. Condition (2) is used to identify the user from a lot of active users. The code sequences used in optical networks are referred to as the optical orthogonal code (OOC). There are four parameters to specify an OOC. They are characterized by a quadruple $(n, w, \lambda_a, \lambda_c)$, where n denotes the sequence length, w the weight (i.e. the number of ones in the sequence), λ_a the maximum value of the out-of-phase autocorrelation function, and λ_c the maximum value of the crosscorrelation function. The main difference in the optical CDMA system when compared to the radio frequency (RF) CDMA system is that the former is a positive system. That is, there are only positive signals, i.e. "1" (light on) and "0" (light off) signals, in its optical fiber delay lines; whereas there are +1 and -1 signals in the RF CDMA system.

Since the code set decides the performance of the optical CDMA systems, there are a lot of papers investigating the code construction and performance of different OOC's [1]-[12]. Most of these papers concentrate on the construction of fixed code length. However, the growing applications of multimedia (voice, data, and image) transmission result in the requirement of multirate fiber-optic networks. For example, a system maybe needs to provide some 32 kbps channels for voice applications and some 64kbps channels for data services. In [13]-[14], Marić *et al.* proposed two different multirate systems. In [13], they introduced the multirate CDMA system where the multirate was achieved by varying the length of OOC sequences. Unfortunately, the system has high error probability for high rate users. In [14], they proposed a different approach where each terminal is given a number of addresses according to its information rate.

In this paper, we propose a new code construction based on any existing OOC's family and a multirate system by using the new constructed OOC sequences.

Meanwhile, we compare the systems using the new code (multirate code systems) with that implementing the multirate function by sending each symbol many times (repeat code systems). The rest of this paper is organized as follows. The code construction based on an existing family is introduced in Section 2. The performance analysis of multirate systems and repeat code systems is presented in Section 3. Some numerical results are discussed in Section 4. Conclusions are drawn in Section 5.

2. CODE CONSTRUCTION

In this section, we first review the definition and some fundamental properties of OOC's. An $(n, w, \lambda_a, \lambda_c)$ optical orthogonal code **C** is a family of (0,1) sequences with length n and weight w, which satisfies the following two properties:

(1)*Autocorrelation Property* :

$$\sum_{i=0}^{n-1} x_i x_{i+\tau} \leqq \lambda_a, \tag{1}$$

for any sequence $X = (x_i) \in \mathbf{C}$ and any integer τ, $0 < \tau < n$.

(2)*Crosscorrelation Property* :

$$\sum_{i=0}^{n-1} x_i x'_{i+\tau} \leqq \lambda_c, \tag{2}$$

for each pair of sequences $X = (x_i)$ and $X' = (x'_i)$, with which $X \neq X' \in \mathbf{C}$, and any integer τ.

Here, we focus on periodical correlations, i.e., the subscripts are reduced to modulo n whenever necessary. Since each sequence X has weight w, the autocorrelation equals w when $\tau = n$ or 0. The numbers λ_a and λ_c are called the autocorrelation and crosscorrelation constraints. The X sequence of an optical orthogonal code **C** is called **C**'s codeword. The size of an optical orthogonal code, denoted by $|\mathbf{C}|$, is the number of codewords in it.

Let **C** be an $(n, w, \lambda_a, \lambda_c)$ code, we propose a method of constructing another code $\mathbf{C}'$ with $(2n, w, \lambda_a, \lambda_c)$. In addition, the crosscorrelation between any codeword of $\mathbf{C}'$ and that of **C** is also constrained below λ_c in a period of $2n$.

Code Constructed Method

Given an $(n, w, \lambda_a, \lambda_c)$ optical orthogonal code **C** with $|\mathbf{C}|$ codewords, we construct a $(2n, w, \lambda_a, \lambda_c)$ code $\mathbf{C}'$ with $|\mathbf{C}|$ codewords as follows:

(1)Given an $(n, w, \lambda_a, \lambda_c)$ code **C** with $|\mathbf{C}|$ codewords, we first construct a $(2n, 2w, 2w, 2\lambda_c)$ code $\mathbf{C}''$ with the same number of codewords by following the method in [3]. That is, for each codeword X of **C,** we construct a codeword Z of $\mathbf{C}''$ by concatenating 2 copies of X. (Here, the codeword X is considered as a binary n-tuple.) Let x_i and z_i be the value of the ith position in codeword X and Z, respectively. Without loss of generality, we assume that the first position of Z is mapped to the first position of X in the concatenating process. Since the codeword Z is constructed by concatenating 2 copies of X, the length and weight of Z are equal to $2n$ and $2w$, respectively. In addition, both z_i and z_{n+i} must be equal to x_i, where $0 \leqq i \leqq n-1$. Since the code length of code $\mathbf{C}''$ is $2n$, the autocorrelation of any Z of $\mathbf{C}''$can be written as

$$\begin{aligned}\sum_{i=0}^{2n-1} z_i z_{i+\tau} &= \sum_{i=0}^{n-1} z_i z_{i+\tau} + \sum_{i=n}^{2n-1} z_i z_{i+\tau} \\ &= \sum_{i=0}^{n-1} z_i z_{i+\tau} + \sum_{i=0}^{n-1} z_{n+i} z_{n+i+\tau} \\ &= \sum_{i=0}^{n-1} x_i x_{i+\tau} + \sum_{i=0}^{n-1} x_i x_{i+\tau},\end{aligned} \tag{3}$$

for any integer τ, $0 < \tau < 2n$. If $\tau = n$, eqn. (3) becomes

$$\begin{aligned}\sum_{i=0}^{2n-1} z_i z_{i+n} &= \sum_{i=0}^{n-1} x_i x_{i+n} + \sum_{i=0}^{n-1} x_i x_{i+n} \\ &= \sum_{i=0}^{n-1} x_i x_i + \sum_{i=0}^{n-1} x_i x_i \\ &= w + w = 2w.\end{aligned} \tag{4}$$

However, if $\tau \neq n$, eqn. (3) becomes

$$\begin{aligned}\sum_{i=0}^{2n-1} z_i z_{i+\tau} &= \sum_{i=0}^{n-1} x_i x_{i+\tau} + \sum_{i=0}^{n-1} x_i x_{i+\tau} \\ &\leqq \lambda_c + \lambda_c = 2\lambda_c.\end{aligned} \tag{5}$$

Since λ_c is always less than w, the autocorrelation constraint of code $\mathbf{C}''$ is $2w$. The crosscorrelation between any two codewords Z and Z' of code $\mathbf{C}''$ is

$$\begin{aligned}\sum_{i=0}^{2n-1} z_i z'_{i+\tau} &= \sum_{i=0}^{n-1} z_i z'_{i+\tau} + \sum_{i=n}^{2n-1} z_i z'_{i+\tau} \\ &= \sum_{i=0}^{n-1} x_i x'_{i+\tau} + \sum_{i=0}^{n-1} x_i x'_{i+\tau} \\ &\leqq \lambda_c + \lambda_c \\ &= 2\lambda_c,\end{aligned} \tag{6}$$

for $0 < \tau < 2n$. Hence, code $\mathbf{C}''$ is a $(2n, 2w, 2w, 2\lambda_c)$ code.

(2)Next, we would like to construct code $\mathbf{C}'$ with $(2n, w, \lambda_a, \lambda_c)$ and size $|\mathbf{C}|$ based on code $\mathbf{C}''$. The codewords of $\mathbf{C}'$ and those of $\mathbf{C}''$ are one-to-one mapping. Let Y be a codeword of code $\mathbf{C}'$ and be the correspondent codeword of Z. In addition, let $x_i^{(1)}$, $y_i^{(1)}$

and $z_i^{(1)}$ be the position of the ith one in codeword X, Y and Z, respectively. Without loss of generality, we assume that $x_1^{(1)}$ is equal to $z_1^{(1)}$. Since the codeword Z is constructed by concatenating 2 copies of X, the $z_i^{(1)}$ and $z_{i+w}^{(1)}$ must be equal to $x_i^{(1)}$ and $(x_i^{(1)} + n)$, where $1 \leqq i \leqq w$. For each pair of $z_i^{(1)}$ and $z_{w+i}^{(1)}$, where $1 \leqq i \leqq w$, either the one in position $z_i^{(1)}$ or that in position $z_{w+i}^{(1)}$ will be erased to construct the new codeword Y. Since there are w pairs of ones, only w ones can be reserved. Therefore, the code $\mathbf{C}'$ is a code with code length $2n$ and weight w. The autocorrelation of Y can be written as

$$\sum_{i=0}^{2n-1} y_i y_{i+\tau} = \sum_{i=0}^{n-1} y_i y_{i+\tau} + \sum_{i=n}^{2n-1} y_i y_{i+\tau}, \qquad (7)$$

for any integer τ, $0 < \tau < 2n$. From eqn. (3), we know that if there is a coincide (where a coincide means that two 1's from two codewords or same codeword but different shift versions are in the same position) in the $z_i^{(1)}$th position, there should be another coincide in the $z_{i+w}^{(1)}$th position. Since there is only one of two 1's in position $z_i^{(1)}$ and $z_{i+w}^{(1)}$ reserved when the codeword Y is constructed, at most only one coincide can be kept and the other is left out in the autocorrelation function of Y. Therefore, the sum of eqn. (7) is less than or equal to λ_a when $\tau \neq n$ and equal to 0 when $\tau = n$. Hence, the autocorrelation constraint of code $\mathbf{C}'$ is λ_a. Similarly, based on the same reason, the crosscorrelation constraint of code $\mathbf{C}'$ is λ_c. The crosscorrelation between any codeword X of code $\mathbf{C}$ and any codeword Y, constructed by X' which is different from X, of code $\mathbf{C}'$ in a period of $2n$ is in form of

$$\begin{aligned} \sum_{i=0}^{2n-1} y_i x_{i+\tau} &= \sum_{i=0}^{n-1} y_i x_{i+\tau} + \sum_{i=n}^{2n-1} y_i x_{i+\tau} \\ &= \sum_{i=0}^{n-1} y_i x_{i+\tau} + \sum_{i=0}^{n-1} y_{i+n} x_{i+n+\tau} \\ &= \sum_{i=0}^{n-1} y_i x_{i+\tau} + \sum_{i=0}^{n-1} y_{i+n} x_{i+\tau}. \qquad (8) \end{aligned}$$

for any integer τ, $0 < \tau < 2n$. Based on the same reason in calculating the crosscorrelation constraint of $\mathbf{C}'$, $\sum_{i=0}^{2n-1} y_i x_{i+\tau}$ is equal to or less than λ_c.

3. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the multirate systems based on their error probabilities as a function of the number of different rate users. The first multirate system is referred to as a multirate code system. This system will use the optical orthogonal codes which is based on the construction in section 2. For simplicity, we create only two different lengths of optical orthogonal codes. However, the construction can be expanded to more different lengths. When a new user is added to the multrate code system, it is assigned a codeword of code $\mathbf{C}$ with $(n, w, \lambda_a, \lambda_c)$. All of the users in the system are categorized into two classes according to their bit rates. Users transmitting the high rate information are named as class 1 users and those transmitting the low rate information are termed as class 2 users. The bit rate of class 1 users is twice as that of class 2 users. That is the symbol length of class 2 users is twice of that of class 1 users. Whenever a class 1 user needs to transmit, it uses the assigned codeword X of $\mathbf{C}$ to map its information data bits. On the other hand, a class 2 user should map its information data bits by using the constructed codeword Y of $\mathbf{C}'$, where Y is constructed from the assigned codeword X of the user. Obviously, the code length of class 2 users is also twice as that of class 1 users. For simplicity, we assume that the system is chip-synchronized and λ_a=λ_c=1. Further, it is assumed that there are N_1 class 1 users and N_2 class 2 users in the system. However, the total number of users, $N = N_1 + N_2$, is bounded by the number of codewords of $|\mathbf{C}|$. We also assume that the data sequences of all users are independent with each other and the probabilities of data "1" and data "0" are equal. For comparison, we design the second multirate system. Instead of using different length codeword, the second system implements the multirate function by mapping each symbol two times for class 2 users using the assigned codeword X of $\mathbf{C}$. This system is referred to as a repeat code system.

3.1 Performance Analysis of the multirate code system

The simplified structure of the receiver for two different data rates in the multirate code system is shown in Fig.1. The function q(t) is a rate control signal. If the user's bit rate is low, q(t) will be equal to -1 otherwise it is equal to +1. T is one bit duration of class 1 users. As mentioned above, we have assumed that a specific user is assigned a codeword X of $\mathbf{C}$ when it is added to the system. Then, when its bit rate is high, it will use X as its address code. However, if the bit rate is low, it will use the codeword Y with $2n$ length and w weight constructed from the assigned X as its address code. Thus, the user becomes a class 2 user. The codewords X and Y will be termed as class 1 and class 2 codeword in the following paragraph.

The user's error probability could be easily calculated by following Salehi's method [5]. The procedure is:
(1) to derive the probability density function of the interference from a single user,
(2) to obtain the joint probability density function of the total interference, and
(3) to calculate the user's error probability.

However, in this study, since we divide the users into

two classes, there is a slight difference in calculating the error probability. Obviously, the interferences from different types of users will not be the same. Therefore, the procedure becomes:
(1) to get the probability density function of the interference from a class 1 user,
(2) to get the probability density function of the interference from a class 2 user,
(3) to get the joint probability density function of the interference from all users, and
(4) to calculate the user's error probability.
The latter procedure will be used to calculate the bit error probabilities of class 1 and class 2 users.

The bit error probability of class 1 users

Let I_1 and I_2 be the total interference from all of class 1 users and class 2 users, respectively. Since there are N_1 class 1 users, the interference I_1 is the sum of (N_1-1) independent identically-distributed (*iid*) random variables $I^{(1)}$, where $I^{(1)}$ is the interference from a class 1 user. The probability density function of $I^{(1)}$ has already been derived by Salehi *et al.* [4]. It can be presented as

$$P(I^{(1)}= i) = \begin{cases} \dfrac{w^2}{2n} & \text{for } i=1 \\ 1-\dfrac{w^2}{2n} & \text{for } i=0 \\ 0 & \text{elsewhere} \end{cases} . \quad (9)$$

Therefore, the probability density function for I_1, $P_{I_1}^{(class1)}(I_1)$, is the convolution of the probability density functions of (N_1-1) *iid* random variables $I^{(1)}$. Hence, $P_{I_1}^{(class1)}(I_1)$ can be written as

$$P_{I_1}^{(class1)}(I_1=i_1) = \begin{cases} \binom{N_1-1}{i_1}\left(\frac{w^2}{2n}\right)^{i_1}\left(1-\frac{w^2}{2n}\right)^{N_1-1-i_1} & \text{for } 0 \leqq i_1 \leqq N_1-1 \\ 0 & \text{elsewhere} \end{cases} . \quad (10)$$

Similarly, the interference I_2 is the sum of N_2 *iid* random variables $I^{(2)}$, where $I^{(2)}$ is the interference from a class 2 user. The distribution of $I^{(2)}$ is slightly different from that of $I^{(1)}$. For a class 1 codeword and a class 2 codeword, there are $2n$ different phase shifts. Furthermore, in OOC, two codewords with λ_c=1 can only overlap at most one "1" position. Since the crosscorrelation constraint between the codewords of any two users is one in the multirate code system, there are w^2 ways of pairing w 1's positions of class 2 codeword and w 1's positions of class 1 codeword during the $2n$ different phase shifts. Then, the probability that a "1" of a particular class 2 codeword overlapping with one of the "1"s of the desired class 1 codeword is given by $(\frac{1}{2})\frac{w^2}{2n}$, where the factor 1/2 accounts for the probability that the interference is transmitting a data "1". Therefore, the probability density function of $I^{(2)}$ is formulated as

$$P(I^{(2)}= i) = \begin{cases} \dfrac{w^2}{4n} & \text{for } i\text{=1} \\ 1-\dfrac{w^2}{4n} & \text{for } i\text{=0} \\ 0 & \text{elsewhere} \end{cases} , \quad (11)$$

and the $P_{I_2}^{(class1)}(I_2)$ is written as

$$P_{I_2}^{(class1)}(I_2=i_2) = \begin{cases} \binom{N_1-1}{i_1}\left(\frac{w^2}{2n}\right)^{i_1}\left(1-\frac{w^2}{2n}\right)^{N_1-1-i_1} & \text{for } 0 \leqq i_1 \leqq N_1-1 \\ 0 & \text{elsewhere} \end{cases} . \quad (12)$$

The total interference I is the sum of I_1 and I_2. Since I_1 and I_2 are independent with each other, the probability density function for I, $P_I^{(class1)}(I)$, is the convolution of the probability density functions of I_1 and I_2. Hence, $P_I^{(class1)}(I)$ is written in the form of

$$P_I^{(class1)}(I) = P_{I_1}^{(class1)}(I_1) * P_{I_2}^{(class1)}(I_2), \quad (13)$$

where $*$ is the convolution operator.

The bit error probability P_{E1} of class 1 users is defined as

$$\begin{aligned} P_{E1} &= \Pr(R \geqq Th|b=0)\Pr(b=0) \\ &\quad +\Pr(R < Th|b=1)\Pr(b=1), \end{aligned} \quad (14)$$

where Th, R, and b denote the threshold, the output of the desired user's integrator at time T and the data sent by the desired user. For $0 \leqq Th \leqq w$, $\Pr(R < Th|b=1)$ is equal to $\Pr(w-Th+I<0) = \Pr(\delta+I<0)$, where $\delta = w-Th \geqq 0$ and I is the total interference. Since both δ and I are greater than or equal to 0, $\Pr(\delta+I<0) = 0$ and the second term of eqn.(**??**) is zero. In other words, the probability of error when $b=1$ is zero. However, when data "0" is sent, the bit error could occur. The probability of occurring these error is the first term of eqn.(**??**), i.e.,

$$\begin{aligned} P_{E1} &= \Pr(R \geqq Th|b=0)\Pr(b=0) \\ &= \frac{1}{2}\textstyle\sum_{i=Th}^{N-1} P_I^{(class1)}(i). \end{aligned} \quad (15)$$

The bit error probability of class 2 users

Similarly, the bit error probability of class 2 users can be derived by the method used in the bit error probability of class 1 users. Although the desired user becomes a class 2 user, the probability density functions of $I^{(1)}$ and $I^{(2)}$ are the same as those when the desired user is a class 1 user. However, the numbers of interfering users from class 1 and class 2 become N_1 and (N_2-1), respectively. Hence, $P_{I_1}^{(class2)}(I_1)$ and $P_{I2}^{(class2)}(I2)$ are listed as follows

$$P_{I_1}^{(class2)}(I_1=i_1) = \begin{cases} \binom{N_1}{i_1}\left(\frac{w^2}{2n}\right)^{i_1}\left(1-\frac{w^2}{2n}\right)^{N_1-i_1} & \text{for } 0 \leqq i_1 \leqq N_1 \\ 0 & \text{elsewhere} \end{cases} , \quad (16)$$

and

$$P_{I_2}^{(class2)}(I_2 = i_2) = \begin{cases} \binom{N_2-1}{i_2}\left(\frac{w^2}{4n}\right)^{i_2}\left(1-\frac{w^2}{4n}\right)^{N_2-i_2-1} & \text{for } 0 \leqq i_2 \leqq N_2-1 \\ 0 & \text{elsewhere} \end{cases} \quad (17)$$

As a result, $P_I^{(class2)}(I)$ is written as

$$P_I^{(class2)}(I) = P_{I_1}^{(class2)}(I_1) * P_{I_2}^{(class2)}(I_2), \quad (18)$$

and the bit error probability of class 2 user is

$$P_{E2} = \frac{1}{2}\sum_{i=Th}^{N-1} P_I^{(class2)}(i). \quad (19)$$

The average bit error probability of the multirate code system P_E is

$$P_E = \frac{N_1R_1P_{E1} + N_2R_2P_{E2}}{N_1R_1 + N_2R_2},$$

where R_i is the data rate of class i. Since $R_1 = 2R_2$, P_E can be written as

$$P_E = \frac{2N_1P_{E1} + N_2P_{E2}}{2N_1 + N_2}. \quad (20)$$

3.2 Performance Analysis of the repeat code system

For comparison, we analyze the performance of repeat code systems where each symbol of class 2 users is sent twice. Each of both class users uses the assigned codeword as its address code. The structure of the receiver is shown in Fig.2. The q(t) is still the rate control signal and has the same function as that in multirate code systems.

The bit error probability of class 1 users

Using the same notations as those of performance analysis of the multirate code system, we calculate the error probability of class 1 users. First, we derive the probability density function of $I^{(1)}$. Since the interference $I^{(1)}$ from a class 1 user is the same as that in the multirate code system, the probability density function of $I^{(1)}$ is the same as eqn.(9). Hence, the probability density function for I_1, $P_{I_1}^{(class1)}(I_1 = i_1)$, can be written as

$$P_{I_1}^{(class1)}(I_1 = i_1) = \begin{cases} \binom{N_1-1}{i_1}\left(\frac{w^2}{2n}\right)^{i_1}\left(1-\frac{w^2}{2n}\right)^{N_1-1-i_1} & \text{for } 0 \leqq i_1 \leqq N_1-1 \\ 0 & \text{elsewhere} \end{cases} \quad (21)$$

The distribution of interference I_2 is derived after $I^{(2)}$ is known. Although the class 2 user sends each bit twice, the probability density function of $I^{(2)}$ is the same as that of $I^{(1)}$ because each data bit is independent with others. Therefore, the $P_{I_2}^{(class1)}(I_2)$ is written as

$$P_{I_2}^{(class1)}(I_2 = i_2) = \begin{cases} \binom{N_2}{i_2}\left(\frac{w^2}{2n}\right)^{i_2}\left(1-\frac{w^2}{2n}\right)^{N_2-i_2} & \text{for } 0 \leqq i_2 \leqq N_2 \\ 0 & \text{elsewhere} \end{cases}, \quad (22)$$

and the probability density function for I, $P_I^{(class1)}(I)$, is written in the form of

$$P_I^{(class1)}(I) = P_{I_1}^{(class1)}(I_1) * P_{I_2}^{(class1)}(I_2). \quad (23)$$

The bit error probability of class 1 users P_{e1} is

$$P_{e1} = \frac{1}{2}\sum_{i=Th}^{N-1} P_I^{(class1)}(i). \quad (24)$$

The bit error probability of class 2 user

The bit error probability of class 2 user will be derived after both $I^{(1)}$ and $I^{(2)}$ are calculated. We first calculate the distribution of $I^{(1)}$. Since class 2 users send each bit twice, we consider that the codeword of a class 2 user has $2n$ length and $2w$ weight (i.e. the codeword Z in section 2). The probability for a particular "1" of the codeword of a class 1 user overlapping with one of the "1"s belonging to the codeword of the desired class 2 user (denoted by p_{12}) is given by $(\frac{1}{2})\frac{2w^2}{2n} = \frac{w^2}{2n}$. However, during a $2n$ period, a class 1 user will send two bits. If both bits being sent during $2n$ period are "1"s, then the class 1 user will contribute two units of interference to the desired class 2 user. The probability of the second data bit of class 1 user being "1" conditioned on the first bit being "1" (denoted by $p(b_2 = 1|b_1 = 1)$) is 1/2. Hence, the probability of $I^{(1)} = 2$ when both data bits of the class 1 interferer are "1"s is given by $p_{12}p(b_2 = 1|b_1 = 1)$ $(=(\frac{1}{2})\frac{w^2}{2n})$. If only one of the two bits is "1", then the desired class 2 user receives only one unit of interference. Therefore, the probability for $I^{(1)}$=1 is equal to $\frac{w^2}{2n}(\frac{1}{2})+(\frac{1}{2})\frac{w^2}{2n}$. As a result, the distribution of $I^{(1)}$ is in the form of

$$P(I^{(1)}= i) = \begin{cases} \dfrac{w^2}{4n} & \text{if } i = 2 \\ \dfrac{w^2}{2n} & \text{if } i = 1 \\ 1-\dfrac{3w^2}{4n} & \text{if } i = 0 \\ 0 & \text{elsewhere} \end{cases}. \quad (25)$$

The distribution of $I^{(2)}$ can be derived by counting the number of phase shifts which cause a particular amount of interference between the desired class 2 user and other class 2 users. Since each bit of the desired user is interfered by two bits from other class 2 users, there are four possible interference patterns from any other class 2 user as shown in Fig.3. In addition, each pattern will have $4n$ phase shift versions with the codeword of the desired user. Since the codeword of the desired user has $2w$ wight and the pattern has $4w$ wight, there are $8w^2$ pairs of coincides. However, because each coincide will accompany with another one, for pattern 1, there are only $4w^2$ possible phase shift versions which have two "1"s overlapping with two "1"s of the codeword of the desired user during $2n$ code length. Therefore, the probability that there are two coincides for pattern 1 is $(\frac{1}{4})(\frac{4w^2}{4n})$, where the factor 1/4 accounts for the probability that pattern 1 takes place. Because there are two possible amounts of interference when pattern 2 and 3 occur, it is difficult to directly count the number of possible phase shift versions which interfere the desired user. For deriving the distribution of interference when pattern 2 and 3 occur, we combine the "1" parts of the two patterns to become pattern 1. We know that pattern 1 has two $4w^2$ possible phase shift versions which have two coincides with the codeword of the desired user. However, in the $4w^2$ phase shift versions, there are only $2w^2$ phase shift versions where both coincides are caused by one single pattern. In the other $2w^2$ phase shift versions, one of the two coincides is from the pattern 2 and the other is from pattern 3. Therefore, the probabilities that there is one coincide from pattern 2 and 3 are the same and are equal to $(\frac{1}{4})(\frac{2w^2}{4n})$. In addition, the total probability that there are two coincides for pattern 2 and pattern 3 is $(\frac{1}{4})(\frac{2w^2}{4n})$. Hence, the distribution of $I^{(2)}$ is in form of

$$P(I^{(2)}= i) = \begin{cases} \dfrac{3w^2}{8n} & \text{if } i = 2 \\ \dfrac{w^2}{4n} & \text{if } i = 1 \\ 1-\dfrac{5w^2}{8n} & \text{if } i = 0 \\ 0 & \text{elsewhere} \end{cases} . \tag{26}$$

Therefore, $P_{I_1}^{(class2)}(I_1)$ and $P_{I_2}^{(class2)}(I_2)$ are the convolution of the probability density functions of N_1 *iid* random variables $I^{(1)}$ and (N_2-1) *iid* random variables $I^{(2)}$, respectively. The probability density function of the total interference I, $P_I^{(class2)}(I)$, is the convolution of the probability density functions of I_1 and I_2. The bit error probability of class 2 users is

$$P_{e2} = \frac{1}{2}\sum_{i=2Th}^{2(N-1)} P_I^{(class2)}(i). \tag{27}$$

The total bit error probability of repeat code systems P_e is

$$P_e = \frac{N_1R_1P_{e1} + N_2R_2P_{e2}}{N_1R_1 + N_2R_2}.$$

Since $R_1 = 2R_2$, P_e can be written as

$$P_e = \frac{2N_1P_{e1} + N_2P_{e2}}{2N_1 + N_2}. \tag{28}$$

4. NUMBERICAL RESULTS

In this section, the performances of systems using the proposed OOC's are compared with those of systems using the repeat code.

Fig.4 shows the bit error probability as a function of the number of class 2 users. The bit error probability is numerically calculated with n=1000, w=5, and the total number of users in the system is fixed to 49. In Fig.4, P_{e1} keeps a fixed value no matter what the number of class 2 users are. P_{e2} gets larger as the number of class 2 users increases (i.e. the number of class 1 users decreases). However, P_{e2} is always lower than P_{e1}. This is reasonable because the repeat code inherits the time diversity property. Moreover, Fig.4 shows that P_{E1} and P_{E2} get smaller as the number of class 2 users increases. As expected, the system using the multirate code performs better than that using the repeat code, especially when the number of class 2 users is large.

The bit error probabilities of systems as a function of the number of class 2 users when n is fixed to 1000 and the total number of users in the system is fixed to 13 is shown in Fig.5. The result of Fig.5 shows that the bit error probabilities of both codes get smaller as w increases. However, if any class 2 user is in the system, P_E is lower than P_e when the weights are the same in both systems. Moreover, the difference between P_E and P_e becomes larger as the number of class 2 users increases.

Fig.6 shows the total number of users simultaneously accommodated by the systems as a function of the number of class 2 users when the bit error probability of systems is below 10^{-9}. As the number of class 2 users increases, the total number of users accommodated by the systems also increases. However, there is an exception. When w is equal to nine, the total number of users accommodated by the systems keeps 13 regardless of the number of class 2 users because the number of codewords of OOC with (1000,9,1,1) is bounded by 13. Although the bit error probability of systems is much lower than 10^{-9} when the total number of users is 13, there is no codeword for new users. This figure also reveals that when the weights are the same, the multirate code system could accommodate more users than the repeat code system if any class 2 user exists in the system.

Fig.7 shows the bit error probabilities of systems as a function of the number of class 2 users when w is fixed to eight and the total number of users in the system is fixed to 10. The result of Fig.7 reveals that the bit error probabilities of both systems decrease as n increases. If there is any class 2 user in the system, P_E would be lower than P_e when the code lengths are the same in both systems. Furthermore, the difference between P_E and P_e becomes larger as the number of class 2 users increases.

Fig.8 shows the total number of users accommodated by the system as a function of the number of class 2 users when the bit error probability of systems is below 10^{-9} and w is fixed to six. This figure reveals three facts. First, as the number of class 2 users increases, the total number of users accommodated by both systems increases. Second, as code length gets longer, the total number of users accommodated by both systems increases, too. Third, the multirate code system could accommodate more users than the repeat code system if there is any class 2 user in the system.

5. CONCLUSION

In this paper, we have shown a new construction of OOC to implement a multirate CDMA fiber-optic network. The performances of the multirate code system and repeat code system are investigated respectively. Theoretical analysis showed that the former performs better than the latter, especially when the number of class 2 users is great. Furthermore, the multirate code system could accommodate more users than the repeat code system for the bit error probability of system under 10^{-9} if any class 2 user exists in the system.

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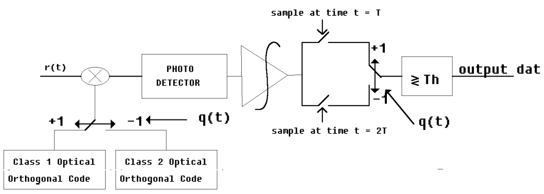

Fig.1 The simplified structure of the multirate code receiver

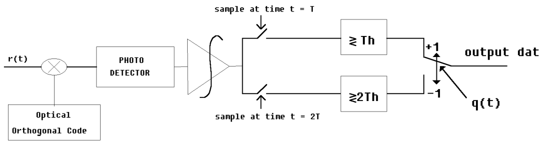

Fig.2 The simplified structure of the repeat code receiver

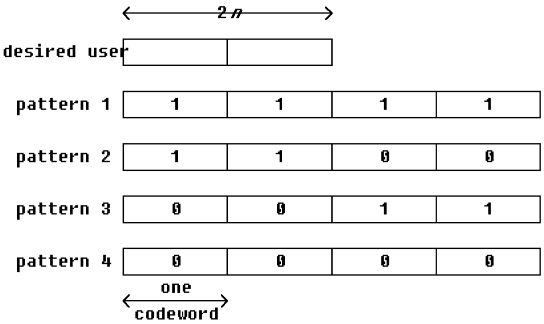

Fig. 3 The possible interference patterns from class 2 users in a repeat code system, when the desired user is a class 2 user.

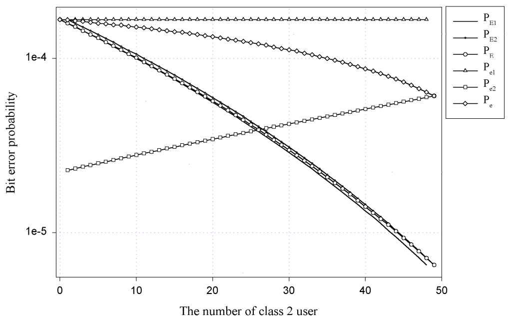

Fig.4 The bit error probability as a function of number of class 2 users when n=1000, w=5 and the number of total users in the system is fixed to 49.

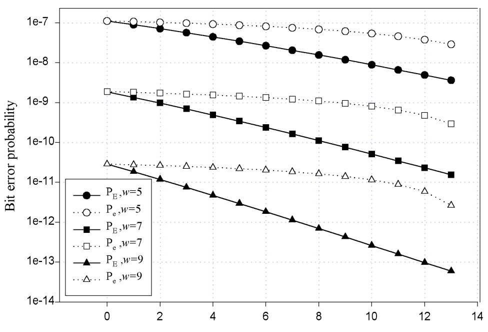

Fig.5 The bit error probability as a function of number of class 2 users when n is fixed to 1000 and the number of total users in the system is fixed to 13.

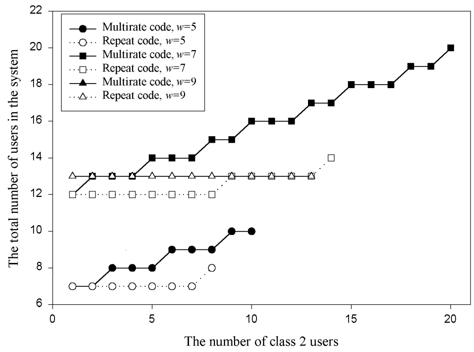

Fig.6 The total number of users accommodated by the system as a function of number of class 2 users when the bit error probability is below 10^{-9} and n is fixed to 1000.

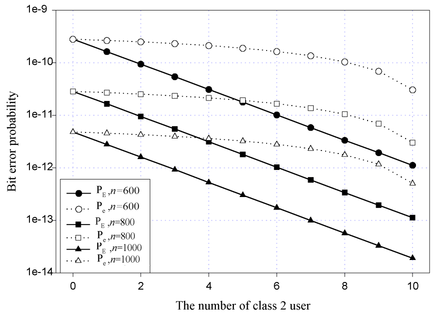

Fig.7 The bit error probability as a function of number of class 2 users when w is fixed to 8 and the number of total users in the system is fixed to 10.

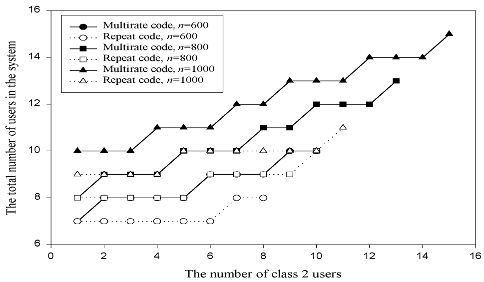

Fig.8 The total number of users accommodated by the system as a function of number of class 2 users when the bit error probability is below 10^{-9} and w is fixed to 6.