# A FEASIBLE METHOD TO IMPLEMENT OPTIMUM CIR-BALANCED POWER CONTROL IN CDMA CELLULAR MOBILE SYSTEMS

*, [1]*Jyh-Horng Wen*, [1]*Jeng-Shin Sheu*, [1]*Jyh-Yean Wang*,
[1]*Jyh-Liang Che*n, and [2]*Tai-Kuo Woo*
[1]Institute of Electrical Engineering
National Chung Cheng University
Chia-Yi, Taiwan, ROC
E-mail: wen@ee.ccu.edu.tw
[2]Department of Information Management
National Defense Management College
*Correspondence Addressee

## ABSTRACT

The optimum power control (OPC) with carrier-to-interference ratio (CIR) is obtained by solving eigenvalue problems of link-gain matrices. In this paper, we propose a feasible scheme to implement the CIR balancing power control. A two-level hierarchical power control structure is proposed to carry out the OPC without involving the eigen-decomposition. The shortages of *unbalanced CIR* and *global outage* are two common issues for CIR-balanced power control. To tackle these two problems, a simple linear prediction method and an adaptive on-off strategy are proposed. Furthermore, because of the capacity limitation of wireless communications, a differential pulse code modulation (DPCM) scheme is presented to reduce the number of bits required for the transmission of command words in the two-hierarchy power control structure. Finally, simulation results and conclusions are given.

## 1. INTRODUCTION

For FDMA/TDMA cellular systems, Zander [1] successfully formulated the CIR-balanced power control as an eigenvalue problem of normalized link-gain matrices. In the CDMA cellular system, however, users share the same radio bandwidth, which results in three dimensional link-gain matrices. Link-gain matrices are simplified to two dimensions only when the intracell CIR balancing is assumed. Recently Wu [2] has reformulated the CIR-balanced power control for CDMA systems as an eigenvalue problem based on a very large link-gain matrix, whose size is proportional to the number of total active mobiles in the system.

For the above studies [1-2], the OPCs were realized by solving eigenvalue problems of link-gain matrices. In the implementation of CIR-balanced OPCs in the sense of minimizing outage probability, the stepwise removal algorithm (SRA) [1] and its variants [2, 4-5] were widely used. In real applications, however it is necessary to find a feasible power set to achieve the CIR balancing for all users before link gains change. Therefore, the implementation of SRA-like algorithms would be infeasible, since the eigen-decomposition has dramatically large computational complexity $O(M^3)$ [3], for which the processed matrix has the size of $M \times M$. Therefore the existing SRA-related algorithms for CIR-balanced OPCs are impractical for implementation [1-2, 6].

By benefiting from the power constraints, we will reformulate the CIR-balanced OPC in CDMA cellular systems into an eigenvalue problem based on a novel link-gain matrix, whose size is only proportional to the number of cells. In addition, simulation results in [6] show that the reformulated OPC method has the same global optimum performance as Wu's [2] in spite of the pretty small size of the novel link-gain matrix. For the feasible implementation, a two-level hierarchical power control structure is proposed to carry out the eigen-decomposition required for the CIR-balanced OPC. Although the hierarchical implementation method can feasibly provide the CIR balancing for a whole CDMA system without directly working on the eigen-decomposition, there exist two shortages in this method. One is the *unbalanced CIR* due to both the loop delay of power control and short-term fading. The other one is the *global outage* resulted from the deep fading of some links. To tackle these two problems, a simple linear prediction method and an adaptive on-off strategy are proposed. Furthermore, because of the capacity limitation of wireless communications, a differential pulse code modulation (DPCM) scheme is introduced to reduce the number of bits required for the transmission of command words in the two-hierarchy power control structure.

## 2. SYSTEM MODEL

We consider a downlink CDMA cellular system, consisting of $L$ cells with $N \left(= \sum_{i=1}^{L} N_i\right)$ active mobiles, in which $N_i$ denotes the number of active mobiles in cell $i$. Furthermore, let $\mathrm{B_j}$ denote the base station $j$ (= 1, 2,…, $L$), $\mathrm{M_{ik}}$ the mobile $k$ (= 1, 2,…, $N_i$) in cell $i$, $G_{ik,j}$ the downlink gain from $\mathrm{B_j}$ to $\mathrm{M_{ik}}$, and $P_{ik}$ the downlink power transmitted to $\mathrm{M_{ik}}$ from $\mathrm{B_i}$. We introduce $Q_i$ to denote the total transmitter power of $\mathrm{B_i}$. The value of $Q_j$ is increasing with either $N_i$. Moreover, let $Q_{sys} = \sum_{j=1}^{L} Q_j$ . In what follows, we assume that the transmission quality is dependent only on the CIR ratio, and the receiver noise is not considered. Now, the downlink CIR at mobile $\mathrm{M_{ik}}$ is given by

$$\Gamma_{\mathrm{ik}} = \frac{P_{ik} G_{ik,i}}{\sum_{j=1}^{L} \sum_{m=1}^{N_j} P_{jm} G_{ik,j} - P_{ik} G_{ik,i}} . \tag{1}$$

Rearranging (1) yields

$$P_{ik} = \frac{\Gamma_{\mathrm{ik}}}{1 + \Gamma_{\mathrm{ik}}} C_{ik} , \tag{2}$$

where

$$C_{ik} = \sum_{j=1}^{L} Q_j \left( \frac{G_{ik,j}}{G_{ik,i}} \right) . \tag{3}$$

Notice that the parameter $C_{ik}$ is the *interference index* of the mobile $M_{ik}$. Apparently a large quantity of $C_{ik}$ implies that the corresponding mobile $M_{ik}$ suffers from more interference or experiences a poor transmission channel.

When the CIR is balanced at cell *i*, $\Gamma_{\mathrm{ik}}$ is independent of *k*, that is

$$\Gamma_{\mathrm{ik}} = \Gamma_{\mathrm{i}} , \tag{4}$$

where $\Gamma_{\mathrm{i}}$ is the balancing CIR of cell *i*. In the case of intracell CIR balancing at cell *i*, we take summations on both sides of (2) from $k = 1$ to $N_i$ to obtain

$$Q_i = \frac{\Gamma_{\mathrm{i}}}{1 + \Gamma_{\mathrm{i}}} H_i , \tag{5}$$

where

$$H_i = \sum_{k=1}^{N_i} C_{ik} . \tag{6}$$

Furthermore, observing (2) and (5), we can find that when the CIR is balanced at cell *i*, within cell *i* the transmitter power $P_{ik}$ to mobile $M_{ik}$ is proportional to $C_{ik}$, and when each cell achieves the same CIR, the total transmitter power of $B_i$ is proportional to $H_i$. Symbolically, the power allocations for intracell balancing and intercell balancing are according to

$$P_{ik} \propto C_{ik} , \quad \text{for} \quad k = 1, 2, \ldots, N_i, \tag{7}$$

and

$$Q_i \propto H_i , \quad \text{for} \quad i = 1, 2, \ldots, L, \tag{8}$$

respectively. The name of two-level hierarchy is referred to both intracell balancing within a cell and intercell balancing for the whole system. The term $H_i$ defined in (6) is the index of total interference at cell *i* and can be rewritten as the following product of two vectors:

$$H_i = \left[ \sum_{k=1}^{N_i} \frac{G_{ik,1}}{G_{ik,i}} \quad \sum_{k=1}^{N_i} \frac{G_{ik,2}}{G_{ik,i}} \quad \cdots \quad \sum_{k=1}^{N_i} \frac{G_{ik,L}}{G_{ik,i}} \right] \cdot \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_L \end{bmatrix}$$

$$= \left[ Z_{i1} \quad Z_{i2} \quad \cdots \quad Z_{iL} \right] \cdot \mathbf{\Theta} , \tag{9}$$

where

$$Z_{ij} = \sum_{k=1}^{N_i} \frac{G_{ik,j}}{G_{ik,i}} , \tag{10}$$

and

$$\mathbf{\Theta} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_L \end{bmatrix} . \tag{11}$$

The value $Z_{ij}$ is the sum of $N_i$ link gains from $B_j$ to $M_{ik}$, normalized by the desired link gain between $B_j$ and $M_{ik}$, for $k = 1, 2, \ldots, N_i$.

## 3. A TWO-LEVEL HIERARCHICAL POWER CONTROL STRUCTURE

The OPCs in [1-2] were realized by solving eigenvalue problems. In this section, instead of directly solving the eigenvalue problem, we propose a hierarchical implementation method to carry out the eigen-decomposition. A CDMA cellular system achieves the *global balance*, if

$$\Gamma_{\mathrm{i}} = \Gamma , \quad \text{for} \quad i = 1, 2, \ldots, L, \tag{12}$$

where $\Gamma$ is the global-balanced CIR.

When the CIR is balanced at cell *i*, according to (7), within cell *i* the transmitter power to the mobile $M_{ik}$ is given by

$$P_{ik} = Q_i \frac{C_{ik}}{\sum_{m=1}^{N_i} C_{im}} , \quad k = 1, 2, \ldots, N_i . \tag{13}$$

Then substituting (13) into (1) yields the balanced CIR $\Gamma_{\mathrm{i}}$ at cell *i* in the following

$$\Gamma_{\mathrm{i}} = \frac{Q_i}{\sum_{m=1}^{N_i} C_{im} - Q_i} . \tag{14}$$

Notice that the value of total *interference index* at cell *i*, $\sum_{m=1}^{N_i} C_{im}$, is not identical for all $i = 1, 2, \ldots, L$. Therefore, to achieve the *global balance* for the whole system, the total transmitter power of each base station should not be the same.

Proposition 1 If the total transmitter power of $B_i$, $i = 1, 2, \ldots, L$, is allocated according to

$$Q_i = Q_{sys} \frac{H_i}{\sum_{j=1}^{L} H_j} , \tag{15}$$

then a CDMA cellular system can achieve the *global balance*, and the global-balanced CIR $\Gamma$ and the corresponding power vector **P** are

$$\Gamma = \frac{1}{\left( H_i \middle/ Q_i \right) - 1} , \quad i = 1, 2, \ldots, L , \tag{16}$$

and

$$\mathbf{P} = \frac{Q_i}{H_i} \left[ C_{11}\ C_{12} \ldots C_{1N_1}\ C_{21}\ C_{22} \ldots C_{2N_2} \ldots C_{LN_L} \right]^{\mathrm{T}} , \tag{17}$$

where

$$P_{ik} = \frac{Q_i}{H_i} C_{ik} \tag{18}$$

is the downlink power transmitted to $M_{ik}$.

Proof According to (8), when a CDMA cellular system achieves the *global balance*, the total transmitter power located for base station $B_i$ is given by

$$Q_i = Q_{sys} \frac{H_i}{\sum_{j=1}^{L} H_j} ,$$

for $i = 1, 2, \ldots, L$. Substituting the above equation into (14) yields the balanced CIR $\Gamma_{\mathrm{i}}$ at cell *i* as follows.

$$\Gamma_{\mathrm{i}} = \frac{Q_{sys}}{\sum_{j=1}^{L} H_j - Q_{sys}} , \quad i = 1, 2, \ldots, L.$$

We can find that the above CIR is independent of *i*. It follows that the CDMA cellular system achieves the *global balance*. Taking advantage of (15) again, we can rearrange the above $\Gamma_i$ to get the global-balanced CIR $\Gamma$ in the following

$$\Gamma = \frac{1}{\left( H_i \big/ Q_i \right) - 1}, \quad i = 1, 2, \ldots, L.$$

Therefore, it is proved that the *global balance* is achieved when the allocation of total transmitter power is according to (15). From (13) it is straightforward that the corresponding transmitter power to mobile $M_{ik}$ in cell $i$ is $P_{ik} = \frac{Q_i}{H_i} C_{ik}$, $k = 1, 2, \ldots, N_i$, where $Q_i$ is given by (15). Therefore the corresponding power vector, achieving the global-balanced CIR $\Gamma$, is given by (17). ■

To implement the global-balanced power control described above, we assume that each base station in a system is connected to a system control center (SCC) by a fixed network, and that the round-trip transmission delay between base station and SCC is negligible. The main task of SCC is to calculate the total power $Q_i$ for each base station $B_i$ according to (15) and executes the adaptive on-off algorithm proposed in the next section. Next we describe the proposed two-level hierarchical structure, shown in Fig. 1, for the global-balanced downlink power control in detail. As for the uplink, a similar procedure can be followed.

*The Two-level Hierarchical Downlink Power Control:*

1. During the control period $T_p$, each mobile $M_{ik}$ in a system measures its respective *interference index* $C_{ik}$ according to (3). Then $M_{ik}$ sends it as a command word to its home base station $B_i$.
2. In the $B_i$, all the parameters $C_{im}$'s received from the $N_i$ active mobiles in cell $i$ are sent to the SCC. When the SCC receives parameters $C_{im}$'s ($m$= 1, 2,…, $N_i$), it computed the parameter $H_i$ according to (6) and allocates the total transmitter power for each base station according to (15). Then, the SCC informs all the base stations what the total downlink transmitter power they should have.
3. The base station $B_i$ transmits power computed by (17) to each mobile in cell $i$. ■

## 4. LINEAR PREDICTION METHOD AND ADAPTIVE ON-OFF GLOBAL-BALANCED STRATEGY

Although the two-level hierarchical power control structure proposed in previous section is feasible and can provide the *global balance* for a whole CDMA system, there are two shortages in this method should be specified. In the first place, the system will achieve an *unbalanced CIR* due to the short-term fading. Secondly, the system will suffer *global outage* when one or more links suffer deep fading.

### 4.1 Linear Prediction Method

To settle the *unbalanced CIR*, a simple linear prediction method is introduced to predict the future variation of current measurement of *interference index* and then compensate it in time. At $n$th control period, for mobile $M_{ik}$ its base station $B_i$ uses two recent $C_{ik}^{(n)}$ and $C_{ik}^{(n-1)}$ to predict the *interference index* $C_{ik}^{(n+1)}$ at $(n+1)$th control period according to

$$\hat{C}_{ik}^{(n+1)} = \left( C_{ik}^{(n)} - C_{ik}^{(n-1)} \right) + C_{ik}^{(n)}, \tag{19}$$

where $\hat{C}_{ik}^{(n+1)}$ is the prediction of $C_{ik}^{(n+1)}$. Then, incorporating (18) with (19), the base station $B_i$ allocates the $(n+1)$th transmitter power to $M_{ik}$, according to

$$P_{ik} = Q_i \frac{\hat{C}_{ik}^{(n+1)}}{\sum_{m=1}^{N_i} \hat{C}_{im}^{(n+1)}}, \tag{20}$$

where $Q_i$ is calculated according to

$$Q_i = Q_{sys} \frac{\hat{H}_i^{(n+1)}}{\sum_{j=1}^{L} \hat{H}_j^{(n+1)}}, \tag{21}$$

and

$$\hat{H}_j^{(n+1)} = \sum_{k=1}^{N_j} \hat{C}_{jk}^{(n+1)}. \tag{22}$$

We name this power control method "Prediction-balanced" power control method.

### 4.2 Adaptive On-off Global-balanced Strategy

To solve the *global outage*, we propose the adaptive on-off global-balanced strategy. As implied by the name, the on-off strategy treats the poor connections with "*off* " policy, that is, at each removal a weaken power is allocated to the removed connection for one control period. For normal connections, the "*on*" policy would allow the system to allocate their required power for them. In what follows, we will incorporate the prediction-balanced method into the following adaptive on-off global-balanced strategy.

*On-off Algorithm:*

1. At $n$th control period, compute the target global-balanced CIR according to (16) using (22). If the target CIR is equal to or larger than a predefined threshold $\gamma$, then go to step 3.
2. The SCC searches the maximum prediction of *interference index* $\hat{C}_{ik}^{(n+1)}$ and reassigns its value as 0.0025, which is approximately equivalent to the time average of $C_{ik} \big/ 1000$ in our simulation. Then return to step 1.
3. Each base station allocates the transmitter power for mobiles connected with it via (20). ■

## 5. THE DIFFERENTIAL PULSE CODE MODULATION (DPCM) SCHEME

For the capacity limitation of wireless communications, in this section a differential pulse code modulation (DPCM) scheme is proposed to reduce the number of bits required for the transmission to command words of $C_{ik}$'s. The DPCM scheme for downlink power control is shown in Fig. 2. At mobile side, by the linear-prediction method each mobile $M_{ik}$ measures the prediction of $C_{ik}^{(n)}$ at the $n$th control period, denoted by $\hat{C}_{ik}^{(n)}$, and then executes the following operation:

$$e_{ik}^{(n)} = \hat{C}_{ik}^{(n)} - \tilde{C}_{ik}^{(n)}, \quad n = 1,2,\ldots, \tag{23}$$

where $\tilde{C}_{ik}^{(n)}$ is obtained from the recursive form

$$\tilde{C}_{ik}^{(n+1)} = \tilde{C}_{ik}^{(n)} + \tilde{e}_{ik}^{(n)}, \tag{24}$$

in which $\tilde{C}_{ik}^{(1)} = 0$ and $\tilde{e}_{ik}^{(n)}$ is the *M*-level quantization quantity of the differential error $e_{ik}^{(n)}$. In addition, at the *n*th control period, the quantization quantity $\tilde{e}_{ik}^{(n)}$ is determined according to the following *M*-level quantizer

$$\tilde{e}_{ik}^{(n)} = \begin{cases} (M/2)\Delta, & \text{if } e_{ik}^{(n)} \in [M\Delta, \infty) \\ (k+1)\Delta, & \text{if } e_{ik}^{(n)} \in [k\Delta, (k+1)\Delta) \\ (-M/2)\Delta, & \text{if } e_{ik}^{(n)} \in (-\infty, -M\Delta], \end{cases} \tag{25}$$

where $\Delta$ is a predefined step size of the *M*-level quantizer. Then the quantization quantity $\tilde{e}_{ik}^{(n)}$ of the mobile $M_{ik}$ is encoded by a decimal-to-binary operator

$$E_{ik} = \mathrm{DecToBin}\left(\tilde{e}_{ik}^{(n)}\right), \tag{26}$$

where the encoded word $E_{ik}$ has length of $\alpha = \log_2^M$ bits. Now the mobile $M_{ik}$ sends $E_{ik}$ as a command word to base station $B_i$. At base station side, $E_{ik}$'s are decoded, and the power allocated to $M_{ik}$ at *n*th power control period is computed as follows:

$$p_{ik}^{(n)} = Q_i \frac{\hat{C}_{ik}^{(n)}}{\sum_{m=1}^{N_i} \hat{C}_{im}^{(n)}}, \tag{27}$$

where $Q_i$ and the modified $\hat{C}_{ik}^{(n)}$'s, if any, are returned from SCC after executing the adaptive on-off algorithm.

## 6. NUMERICAL RESULTS

In our simulations, consider the downlink power control in a CDMA cellular radio system, which is composed of 19 cells. The link gain $G(t)$ is modeled as

$$G(t) = L(t)S(t), \tag{28}$$

where $L(t)$ is the long-term fading and $S(t)$ is the short-term fading. For the performance comparison, we present simulation results to compare the performance of the two-level hierarchical global-balanced power control with that of the power control scheme proposed by Chen [7]. The outage probability, which is defined as $P_{out}$ = Prob[received CIR < protection ratio], is used as our performance criterion. The protection ratio $\gamma$ is considered to be –13.5 (dB).

Fig. 3 compares both two-level hierarchical and Chen's power control schemes in terms of outage probability versus system load. We find that the outage probability of the proposed scheme is significantly better than Chen's scheme especially in the heavy load. Fig. 4 gives the comparison of two different traffic models. One is the "fixed load", which means that the number of mobiles in each cell is identical and fixed, and the other is the "dynamic load", in which the number of mobiles in cell *i* is a random variable uniformly distributed over [$N_i$ – 5, $N_i$ + 5] in each simulation cycle. We can observe that the two-level hierarchical scheme performs almost equally well in both traffic loads. However, Chen's scheme is susceptible to the variation of traffic load. This is because of that the two-level hierarchical scheme adaptively adjusts the total transmitter power of each base station according to the variation of *interference indices* due to the short-term fading. Fig. 5 shows the performance in terms of the outage probability versus the system load for two-level hierarchical power control with and without the adaptive on-off algorithm. The performance is indeed improved while the adaptive on-off algorithm is employed especially in heavy load. Fig. 6, finally, presents the outage probability of proposed global-balanced power control with without DPCM scheme. We find that the curves of 7 bits, infinite bits, and DPCM 3 bits are almost overlap. This means that DPCM 3 bits can provide the same performance for the command words of $C_{ik}$'s transmitted as the infinite bits; meanwhile, compared with the 7 bits, reduce more than half amount of bits required for the transmission to command words. Even the DPCM 2 bits, we also observe that it is much better than 4 bits especially in the heavy load.

## 7. CONCLUSIONS

In this paper, we proposed a two-level hierarchical power control structure, which is feasible to carry out the eigenvalue problem of a CIR-balanced OPC. It has been also proved that the two-level hierarchical scheme can provide *global balance* and has optimum performance if the perfect measurement of *interference indices* is assumed. Moreover, the simple linear prediction method and adaptive on-off strategy were proposed to tackle both *unbalanced CIR* and *global outage*, respectively. Furthermore, the adaptive on-off algorithm can in the meanwhile provide the soft capacity, that is, the performance of a removed connection will be temporally degraded for one control period, and only gradually, not abruptly. Simulation results showed that the hierarchical power control method can provide better performance, especially in heavy load, and is insensitive to the variation of traffic load. This is because of that the two-level hierarchical structure adaptively adjusts the total transmitter power for each base station to compensate the variation of total interference in each cell. The DPCM scheme is also shown to be able to reduce more than half amount of bits required for the transmission of command words. Therefore, the implementation cost of our proposed scheme is greatly reduced while DPCM scheme is applied. Moreover from the simulation results, the performance is indeed improved while the adaptive on-off algorithm is applied to power control scheme especially in heavy load.

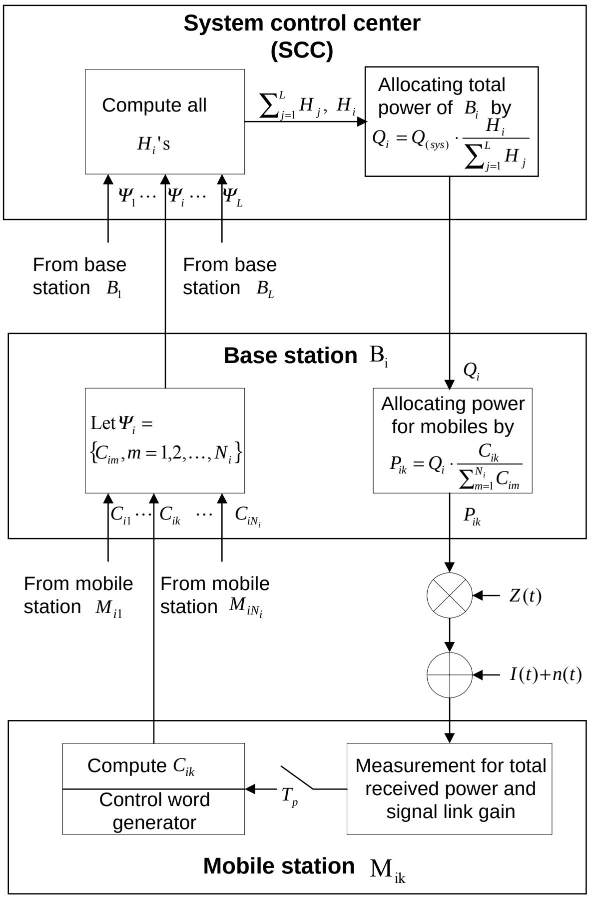

Figure 1. The block diagram of two-level hierarchical downlink power control.

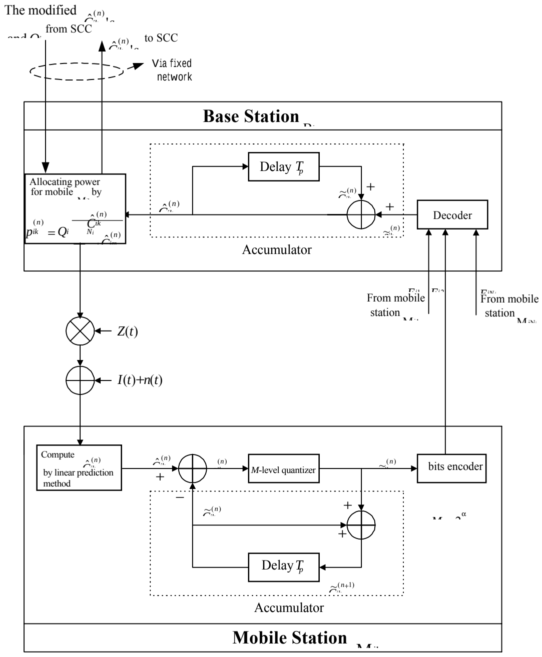

Figure 2. The block diagram of DPCM scheme.

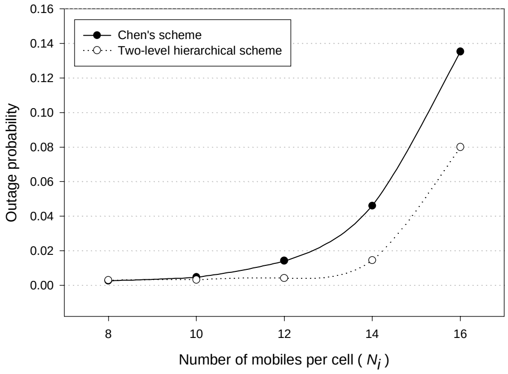

Figure 3 Outage probability comparison between the two-level hierarchical and Chen's power control schemes. (Number of bits of command word $\alpha$=7 with step size $\Delta$ = 0.2.)

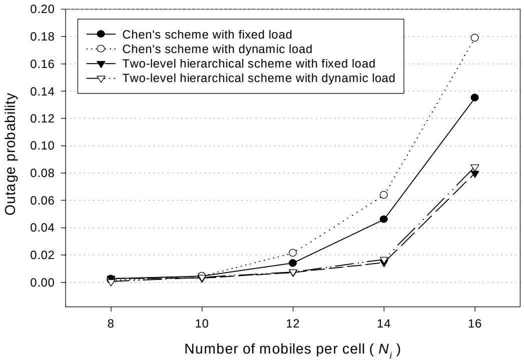

Figure 4 Outage probability comparison between the two-level hierarchical and Chen's power control schemes with different traffic models. (Number of bits of command word $\alpha$=7 with step size $\Delta$ = 0.2.)

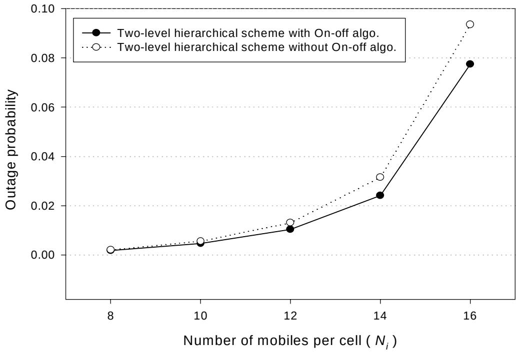

Figure 5 The outage probability of the two-level hierarchical power control with and without the adaptive on-off algorithm. (Number of bits of command word $\alpha$=7 with step size $\Delta$ = 0.2.)

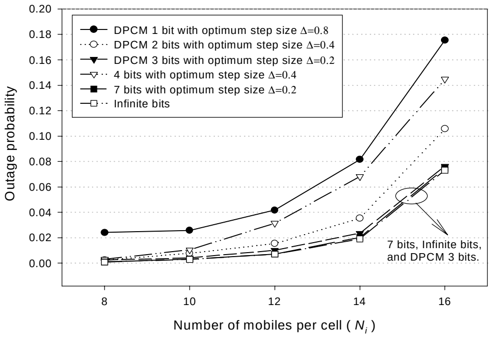

Figure 6 The outage probability of proposed global-balanced power control with the number of bits of command word: {4,7, Infinite} bits and DPCM {1,2,3} bits.

## REFERENCES

[1] J. Zander, “Performance of optimum transmitter power control in cellular radio systems,” *IEEE Trans. Veh. Technol*., vol. 41, pp. 57-62, Feb. 1992.

[2] Qiang Wu, “Performance of optimum transmitter power control in CDMA cellular mobile systems,” *IEEE Trans. Veh. Technol*., vol. 48, pp. 571-575, Mar. 1999.

[3] Gene H. Golub and Charles F. Van Loan, *Matrix Computations*, 1996, Johns Hopkins.

[4] T. H. Lee, J. C. Lin, and Y. T. Su, “Downlink power control algorithms for cellular radio systems,” *IEEE Trans. Veh. Technol*., vol. 44, pp. 89-94, 1995.

[5] T. H. Lee, J. C. Lin, and Y. T. Su, “Power control algorithm for cellular radio systems,” *Elect. Lett*., vol. 30, pp.195-197, 1994.

[6] J. H. Wen, J. S. Sheu, and J. L. Chen, “An Optimum Power Control Method for CDMA Cellular Mobile Systems,” submitted to *IEICE Trans. on Com*., June 2000.

[7] C. J. Chang and F.C Ren, “Down-link power control in DS/CDMA cellular mobile radio network” *Proceedings of 3<sup>rd</sup> IEEE International Conference on Universal Personal Communications.*, pp. 89-93, San Diego, U.S.A., Jul. 1994.