Non-Uniform Light Distribution For Direct Lighting Calculation

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Abstract

Most rendering algorithms use Monte Carlo methods for solving the part of direct lighting in rendering equation. And those algorithms usually assume that the lights in the scene are uniform distribution. This paper presents detail of sampling on disc luminaire with non-uniform distribution. We present several non-uniform light distributions on disc luminaire and implement those light distributions in a direct lighting rendering system. Each light distribution causes different shading effect.

1.Introduction

The Photo-realistic image synthesis is a goal of computer graphics. Physically based rendering algorithms use physical rules to simulate the light transport and reflection. In 1986, Kajiya introduced rendering equation [2]. He used Monte Carlo method to solve the illumination problem. Most implementations use Monte Carlo methods to estimate the direct lighting calculation [9][10][11]. And those algorithms generally assume the luminaires are uniform distribution. In fact, there is no perfectly uniform-distributed luminaire. Thus we try to define some non-uniform light distributions with disc luminaire. The light distributions of real-world luminaires are more complicated than the light distribution discussed here, but complicated distribution function is also too difficult to find the sample point. Due to complexity and efficiency considerations, we choose simplify the distribution function to simulate the shading effect. The simplification can speed up the lighting calculation. This paper presents how to generate non-uniform random samples. In Section 2 we describe Monte Carlo method and how to generate random sample on disc with uniform distribution. In Section 3 we present several non-uniform sampling techniques. In Section 4 are our experiment results. In final section we discuss our techniques and future works.

2.Related works

In this section we describe basic Monte Carlo Integration and apply Monte Carlo method [1][5][6][7] to direct lighting calculation. We also present how to how to generate random sample on disc with uniform distribution.

2.1 Monte Carlo method

If we want to find an approximate solution of integral *I*:

$$I = \int_{x \in S} h(x) d\mu(x)$$

There exists a set of random variables, $\{X_1, X_2, \cdots, X_N\} \sim p$. The probability density function (pdf) *p(x)* is defined over *S*. These random variables can be used to approximate *I*. Let $h = f p$ then we can estimate *I* with following equation:

$$I = \int_{X\in S} h(x)d\mu(x) \approx \frac{1}{N}\sum_{i=1}^{N}\frac{h(X_i)}{p(X_i)} = I_s$$

where *h/p* is called primary estimator and I_s is secondary estimator.

$$\begin{aligned} Var(I_S) &= Var\left(\frac{1}{N}\sum_{i=1}^{N}\frac{h(X_i)}{p(X_i)}\right) \\ &= \sum_{i=1}^{N} Var\left(\frac{1}{N}\frac{h(X_i)}{p(X_i)}\right) \\ &= \sum_{i=1}^{N}\frac{1}{N^2} Var\left(\frac{h(X_i)}{p(X_i)}\right) \\ &= \frac{1}{N}\sigma_p^2 \end{aligned}$$

where σ_p is standard deviation of primary estimator, and σ_s is standard deviation of I_S. Thus

$$\sigma_S = \frac{1}{\sqrt{N}}\sigma_p$$

Monte Carlo method is simple, only sampling and point evaluation are required. The standard decreases with the square root of the number of samples N.

2.2 Direct lighting

The separation of direct lighting and indirect lighting can speed up the lighting calculation [9]. The rendering equation shows that outgoing radiance is the sum of the emitted radiance and the radiance from all visible surfaces. The rendering can also be expressed as multiple terms: emitted, direct and indirect radiance.

$$Ls(x,\hat{\omega}) = Le(x,\hat{\omega}) + \int_{\Omega_i} \rho(x,\hat{\omega},\hat{\omega}')L_f(x,\hat{\omega}')\cos\theta d\hat{\omega}'$$

$$L = L_e + L_{direct} + L_{indirect}$$

Then we can express L_{direct} as following equation:

$$L_{direct} = \int_{\forall x'\in\Omega_{x'}} g(x,x')\rho(x,\psi,\psi')L_f(x,\psi')\cos\theta \frac{dA'\cos\theta'}{\|x'-x\|^2}$$

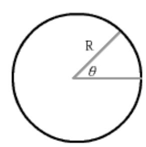
where x is a point on the surface, *x'* is a sampling point on the luminaire. $g(x,x')$ is geometry term，when $g(x,x')$ is 1, it means *x* is visible for *x'*. If $g(x,x')$ is 0, *x* is invisible for *x'*. $\rho(x,\psi,\psi')$ is the Bi-directional Reflection Distribution Function，ψ' and ψ are incoming direction and outgoing direction respectively. $L_f(x,\psi')$ is the radiance contributed to *x* from incoming direction ψ'. θ is the angle between the $-\psi'$ and the surface normal at *x*. θ' is the angle between ψ' and the luminaire normal at *x'*, $\|x-x'\|$ is distance between *x* and *x'*.

we apply *p(x)* = 1/A To solve direct lighting integral, where *A* is the total area of the luminaire. The primary estimator is

$$L_{direct} \approx g(x,x')\rho(x,\psi,\psi')L_f(x,\psi')\cos\theta A\frac{\cos\theta'}{\|x'-x\|^2}$$

2.3 Sampling disc luminaire

To choose a random sample form a disc [3][5][9], first we suppose that its center is at the (0,0), the radius is R.



Thus a point $\boldsymbol{x}(u,v)$ on the disc can be describe as

$$u = r\cdot\cos\theta$$
$$v = r\cdot\sin\theta$$

The area of disc is $\iint dudv = \int_0^{2\pi}\int_0^{R} rdrd\theta = \pi R^2$. So we apply p(*x*)= $\frac{1}{A}$ = $\frac{1}{\pi R^2}$ for sampling disc luminaire. We proceed as follows:

$$F(\theta',r') = \frac{\int_0^{\theta'}\int_0^{r'} rdrd\theta}{2\pi R} = \frac{\theta'\cdot r'}{2\pi R}$$

$$\varepsilon_1 = F(\theta',r'=R) = \frac{\theta'\cdot R}{2\pi R} = \frac{\theta'}{2\pi} \qquad \varepsilon_1,\varepsilon_2 \in [0,1)$$
$$\varepsilon_2 = \frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=R)} = \frac{r'}{R}$$

Thus we can find (θ',r'), where

$$\theta' = 2\pi\varepsilon_1$$
$$r' = R\sqrt{(\varepsilon_2)}$$

To apply the sampling method to a disc luminaire in ray tracing system, we must transform the sample ***x*** to ***x'*** in ray tracing coordinate. Suppose the center of disc is ***c***, and its normal $\vec{N} = \vec{w}$ in its $\vec{u}\vec{v}\vec{w}$ coordinate system. Then the transformation is

$$x' = c + \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \begin{bmatrix} R\sqrt{\varepsilon_2}\cos(2\pi\varepsilon_1) \\ R\sqrt{\varepsilon_2}\sin(2\pi\varepsilon_1) \\ 0 \end{bmatrix}$$

x' is the uniform distribution sample point on the disc.

3.Non-uniform disc luminaire sampling

In previous section we present the uniform disc luminaire sampling. But in the real world the most luminaires are not in non-uniform distribution. In this section we some non-uniform distribution sampling: linearly decreasing, linearly decreasing and then increasing, linearly increasing, linearly increasing and then decreasing, hyperbolically decreasing, and hyperbolically increasing. To reduce variance we design probability density function with importance sampling. We take distribution function of luminaire as a part of probability density function.

3.1 linearly decreasing sampling

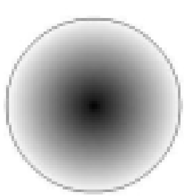
We develop a pdf for linearly decreasing and then linearly increasing:

$$\int_0^{2\pi}\int_0^R |r-t|\cdot rdrd\theta = \int_0^{2\pi}\int_0^t (t-r)\cdot rdrd\theta + \int_0^{2\pi}\int_t^R (r-t)\cdot rdrd\theta$$

$$= 2\pi\cdot(\frac{t^3}{6}) + 2\pi\cdot(\frac{R^3}{3} - \frac{tR^2}{2} + \frac{t^3}{6})$$

where *t* is the turning point.

(a) *t*=R



The darker area means greater intensity.

$$\int_0^{2\pi}\int_0^R |r-R|\cdot rdrd\theta = 2\pi\cdot\frac{R^3}{6}$$

$$F(\theta', r') = \frac{\int_0^{\theta'}\int_0^{r'}(R-r)rdrd\theta}{2\pi\cdot\frac{R^3}{6}} = \frac{\theta'\cdot(\frac{Rr'^2}{2} - \frac{r'^3}{3})}{2\pi\cdot\frac{R^3}{6}}$$

$$\varepsilon_1 = F(\theta', r' = R) = \frac{\int_0^{\theta'}\int_0^{R}(R-r)rdrd\theta}{2\pi\cdot\frac{R^3}{6}} = \frac{\theta'\cdot\frac{R^3}{6}}{2\pi\cdot\frac{R^3}{6}} = \frac{\theta'}{2\pi}$$

$$\varepsilon_2 = \frac{F(\theta' = 2\pi\varepsilon_1, r')}{F(\theta' = 2\pi\varepsilon_1, r' = R)} = \frac{\frac{Rr'^2}{2} - \frac{r'^3}{3}}{\frac{R^3}{6}}$$

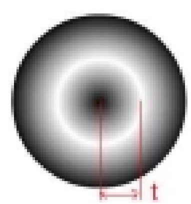
Then

$$\theta' = 2\pi\varepsilon_1$$

$$2r'^3 - 3Rr'^2 + R^3\varepsilon_2 = 0$$

Because *r'* doesn't have analytical solution, we use numeric method to find the solution of *r'*.

(b) 0≦*t*≦R



In this case we assume t = R/2.

$$\int_0^{2\pi}\int_0^R \left|r - \frac{R}{2}\right|\cdot rdrd\theta = \int_0^{2\pi}\int_0^{\frac{R}{2}} (\frac{R}{2} - r)\cdot rdrd\theta + \int_0^{2\pi}\int_{\frac{R}{2}}^{R} (r - \frac{R}{2})\cdot rdrd\theta$$

$$= \frac{\pi R^3}{24} + \frac{5\pi R^3}{24}$$

There is an integral of absolute value, we separate the integral into two integral to find the solution of each integral. Before calculate the sampling point we must generate another random ε_3 variable to determine what integral we should use.

$$\frac{\frac{\pi R^3}{24}}{\frac{\pi R^3}{24}+\frac{5\pi R^3}{24}}=\frac{1}{6}$$

If ε_3 is less then 1/6 then the pdf $=\int_0^{2\pi}\int_0^{\frac{R}{2}}(\frac{R}{2}-r)\cdot rdrd\theta=\frac{\pi R^3}{24}$

$$F(\theta',r')=\frac{\int_0^{\theta'}\int_0^{r'}(\frac{R}{2}-r)rdrd\theta}{\frac{\pi R^3}{24}}=\frac{\theta'\cdot(\frac{Rr'^2}{4}-\frac{r'^3}{3})}{\frac{\pi R^3}{24}}$$

$$\varepsilon_1=F(\theta',r'=\frac{R}{2})=\frac{\int_0^{\theta'}\int_0^{\frac{R}{2}}(\frac{R}{2}-r')r'drd\theta}{\frac{\pi R^3}{24}}=\frac{\theta'}{2\pi}$$

$$\varepsilon_2=\frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=\frac{R}{2})}=\frac{\frac{3Rr'^2-4r'^3}{12}}{\frac{R^3}{48}}$$

then

$\theta'=2\pi\varepsilon_1$

$16r'^3-12Rr'^2+R^3\varepsilon_2=0$

If ε_3 greater then 1/6, then

pdf $=\int_0^{2\pi}\int_{\frac{R}{2}}^{R}(r-\frac{R}{2})\cdot rdrd\theta=\frac{5\pi R^3}{24}$

$$F(\theta',r')=\frac{\int_0^{\theta'}\int_{\frac{R}{2}}^{r'}(r-\frac{R}{2})rdrd\theta}{\frac{5\pi R^3}{24}}=\frac{\theta'\cdot(4r'^3-3Rr'^2+\frac{R^3}{4})}{\frac{5\pi R^3}{24}}$$

$$\varepsilon_1=F(\theta',r'=R)=\frac{\int_0^{\theta'}\int_{\frac{R}{2}}^{r'}(r-\frac{R}{2})rdrd\theta}{\frac{5\pi R^3}{24}}=\frac{\theta'}{2\pi}$$

$$\varepsilon_2=\frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=\frac{R}{2})}=\frac{4r'^3-3Rr'^2+\frac{R^3}{4}}{\frac{5R^3}{4}}$$

then

$\theta'=2\pi\varepsilon_1$

$16r'^3-12Rr'^2+R^3-5R^3\varepsilon_2=0$

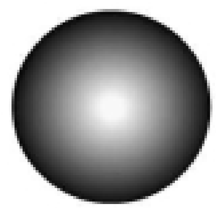
3.2 linearly increasing sampling

First we develop a pdf for linearly increasing and then decreasing:

$$\int^{\pi}\int r\ drd\theta+\int^{\pi}\int^{R}(R-r)\cdot rdrd\theta=2\pi(\frac{t^3}{3})+2\pi(\frac{R^3}{6}-\frac{Rt}{2}+\frac{t^3}{3})$$

where t is the turning point. We present two cases as following:

(a) t = R



Intensity increases linearly form the center of disc, the darker area means stronger intensity.

PDF: $\int_0^{2\pi}\int_0^{R}r^2drd\theta=\frac{2\pi R^3}{3}$

$$F(\theta',r')=\frac{\int_0^{\theta'}\int_0^{r'}r^2drd\theta}{\frac{2\pi R^3}{3}}=\frac{\theta'\cdot\frac{r'^3}{3}}{\frac{2\pi R^3}{3}}$$

$$\varepsilon_1=F(\theta',r'=R)=\frac{\int_0^{\theta'}\int_0^{R}r^2drd\theta}{\frac{2\pi R^3}{3}}=\frac{\theta'}{2\pi}$$

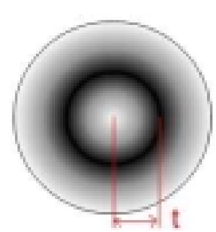
$$\varepsilon_2=\frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=R)}=\frac{r'^3}{R^3}$$

then

$\theta'=2\pi\varepsilon_1$

$r'=R\cdot(\varepsilon_2)^{\frac{1}{3}}$

(b) 0≦t≦R



In this case we assume t = R/2

PDF:

$$\int^{\pi}\int^{R/}r\ drd\theta+\int^{\pi}\int_{R/}^{R}(R-r)\cdot rdrd\theta=2\pi(\frac{R^3}{24})+2\pi(\frac{2R^3}{24})$$

There is an integral of absolute value, we separate the integral into two integral to find the solutions of each integral. Before calculate the sampling point we must generate another random ε_3 variable to determine what integral we should use.

$$\frac{2\pi(\frac{R^3}{24})}{2\pi(\frac{R^3}{24})+2\pi(\frac{2R^3}{24})}=\frac{1}{3}$$

If ε_3 is less then 1/6 then the

$$\text{pdf}=\int_0^{2\pi}\int_0^{\frac{R}{2}}r^2drd\theta=\frac{2\pi R^3}{24}$$

$$F(\theta',r')=\frac{\int_0^{\theta'}\int_0^{r'}r^2drd\theta}{\frac{2\pi R^3}{24}}=\frac{\theta'\cdot\frac{r'^3}{3}}{\frac{2\pi R^3}{24}}=\frac{\theta'\cdot r'^3}{4\pi R^3}$$

$$\varepsilon_1=F(\theta',r'=\frac{R}{2})=\frac{\theta'\cdot\frac{R^3}{8}}{4\pi R^3}$$

$$\varepsilon_2=\frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=\frac{R}{2})}=\frac{r'^3}{\frac{R^3}{8}}$$

then

$$\theta'=2\pi\varepsilon_1$$
$$r'=\frac{R}{2}\cdot(\varepsilon_2)^{\frac{1}{3}}$$

If ε_3 greater then 1/3 then pdf ：

$$\int_0^{2\pi}\int_{R/2}^{R}(R-r)\cdot rdrd\theta=2\pi(\frac{2R^3}{24})$$

$$F(\theta',r')=\frac{\int_0^{\theta'}\int_{\frac{R}{2}}^{r'}(R-r)\cdot rdrd\theta}{\frac{2\pi\cdot 2R^3}{24}}=\frac{\theta'(\frac{Rr'^2}{2}-\frac{r'^3}{3}-\frac{R^3}{12})}{\frac{2\pi\cdot 2R^3}{24}}$$

$$\varepsilon_1=F(\theta',r'=R)=\frac{\frac{R^3}{2}-\frac{R^3}{3}-\frac{R^3}{12}}{\frac{2\pi\cdot 2R^3}{24}}$$

$$\varepsilon_2=\frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=R)}=\frac{\frac{Rr'^2}{2}-\frac{r'^3}{3}-\frac{R^3}{12}}{\frac{R^3}{12}}$$

and

$$\theta'=2\pi\varepsilon_1$$
$$4r'^3-6Rr'^2+R^3+R^3\varepsilon_2=0$$

3.3 hyperbolically increasing

We define the distribution is increasing form the center of the disc, and the pdf is

$$\int_0^{2\pi}\int_0^{R}\frac{1}{|R-t|}\cdot rdrd\theta=2\pi[2R-R\cdot\ln(R)]$$

$$F(\theta',r')=\frac{\int_0^{\theta'}\int_0^{r'}\frac{1}{|R-t|}\cdot rdrd\theta}{2\pi[2R-R\cdot\ln(R)]}$$
$$=R\cdot\ln(R-r')+r'-R\cdot\ln(R)\cdot\frac{\theta'}{2\pi[2R-R\cdot\ln(R)]}$$

$$\varepsilon_1=F(\theta',r'=R)=\frac{\theta'}{2\pi[2R-R\cdot\ln(R)]}\cdot R\cdot\ln(0)+R\cdot\ln(R)$$

$$\varepsilon_2=\frac{F(\theta'=2\pi\varepsilon_1,r')}{F(\theta'=2\pi\varepsilon_1,r'=\frac{R}{2})}=\frac{R\ln(R-r')+r'-R\ln(R)}{2R-R\ln(R)}$$

then

$$\theta'=2\pi\varepsilon_1$$
$$R\cdot\ln(R-r')+r'-R\cdot\ln(R)+[R\cdot\ln(R)-2R]\cdot\varepsilon_2=0$$

Because r' doesn't have analytic solution, we must use numeric method to find out the solution. Note that when R=1 then ln(R) = $-\infty$. It will cause error while sampling. Thus R can't be 1. When implementing this sampling method R should greater than 1.

3.4 hyperbolically decreasing

This case is that the intensity decreases hyperbolically form the center of the disc. The pdf = $\int_0^{2\pi}\int_0^{R}\frac{1}{r}\cdot rdrd\theta = 2\pi R$

$$F(\theta',r') = \frac{\int_0^{\theta'}\int_0^{r'}\frac{1}{r}\cdot rdrd\theta}{2\pi R} = \frac{\theta'\cdot r'}{2\pi R}$$

$$\varepsilon_1 = F(\theta', r' = R) = \frac{\theta'\cdot R}{2\pi R}$$

$$\varepsilon_2 = \frac{F(\theta' = 2\pi\varepsilon_1, r')}{F(\theta' = 2\pi\varepsilon_1, r' = R)} = \frac{r'}{R}$$

then

$$\theta' = 2\pi\varepsilon_1$$
$$r' = R\varepsilon_2$$

The sample points on disc luminaire can be estimated by following equation:

$$x = r'\cdot\cos\theta'$$
$$y = r'\cdot\sin\theta'$$

These sample points can be used to solve the render equation. Because some above distribution functions don't have analytic solution, we must numerically solve the equation.

4.Results

In this section we verify our sampling methods and also implement the disc luminaire in a rendering system, *Rayshade*. Each distribution case has three figures. First figure shows the 5000 points on a disc with their own distribution function. In secondary figure we divide the radius of the disc into ten sections and create ten concentric circles. X-axis is *i*th concentric circle range. Y-axis is the number of sampling points in unit area of each concentric circle. There are forty thousand sampling points in this figure totally. Third figure is the direct lighting image that is rendered by Rayshade. And there are 400 samples on luminaire and 1 sample per pixel.

While Comparing figure 1.3 with figure 2.3 we can find the shadows of tow red torus are clearly different. Figure 1.3 is uniform distribution and its shadow of red torus is blurry and the shadow is almost a circle, not a ring. Because figure 1.3 is uniform distribution and the luminaire is bigger than the torus. Almost entire shadow is penumbra. But In figure 2.3 the shadow of torus is almost a ring. Because the intensity of the luminaire is more centralized. It causes shadow like the point luminaire, but unlike point luminaire causes sharp shadow. In Figure 3.3 the torus has two shadows. One is produced by the luminous intensity gathered round the center of luminaire. The other is produced by the luminous intensity round the edge of disc luminaire. In figure 4.3 the shadow of the torus is a little like figure 1.3, but more blurry. This intensity of disc luminaire is distributed round the edge. In figure 5.3 the intensity is distribution round concentric circle with the half of radius of the disc luminaire. Its shadow is like the figure 2.3's but a little blurry and bigger. The shadow of red torus in figure 6.3 is more sharp and smaller than figure 4.3. The shadow of red torus in figure 7.3 the more sharp than figure 2.3. Because the luminous intensity in figure 6.3 is more centralized than figure 4.3. And figure 7.3 is similar condition. In all experiments we find that if the intensity is more centralized, the variance of the image will be lower. It means converge rate of Monte Carlo method will be higher.

5.Conclusion

In this paper we present some non-uniform light distribution function for direct lighting calculation. Each distribution causes different shading effect. They also have different converge rate. To simplify the complexity of integrals we choose the constant probability density function for disc luminaire. This may cause higher variance, but increasing sampling numbers will reduce the variance. We also simplify the light distribution function. It may be inappropriate, but it also produces vivid shadow. We can take it as a special case of light distribution.

Future work should include more sophisticated ways to develop the light distribution function and construct better probability density function to produce lower-variance image.

6.Reference

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(1) Uniform distribution

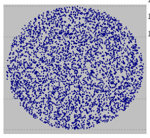

Figure 1.1

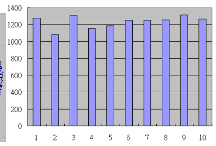

Figure 1.2

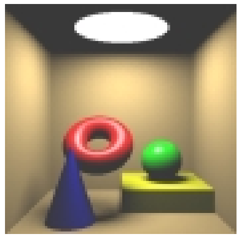

Figure 1.3

(2) Linearly decreasing form center (t=R)

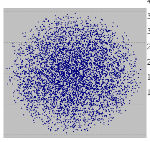

Figure 2.1

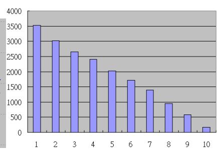

Figure 2.2

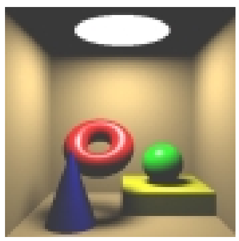

Figure 2.3

(3) Linearly decreasing and then increasing (t=R/2)

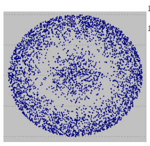

Figure 3.1

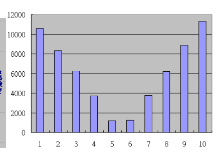

Figure 3.2

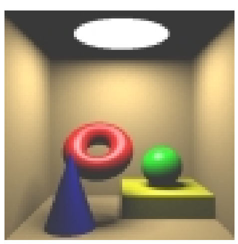

Figure 3.3

(4) Linear increase form center(t=R)

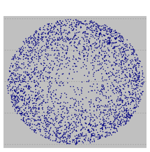

Figure 4.1

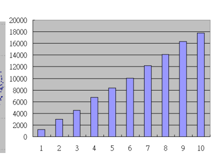

Figure 4.2

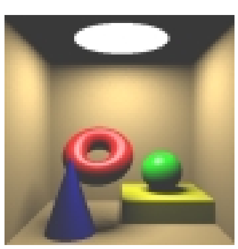

Figure 4.3

(5) Linearly increasing then decreasing (t=R/2)

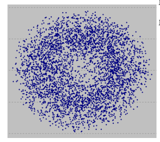

Figure 5.1

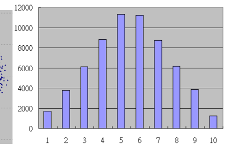

Figure 5.2

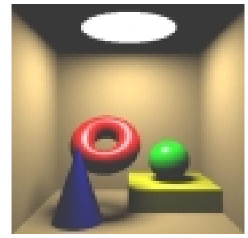

Figure 5.3

(6) hyperbolically increasing form center（R=100）

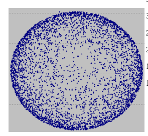

Figure 6.1

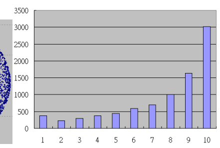

Figure 6.2

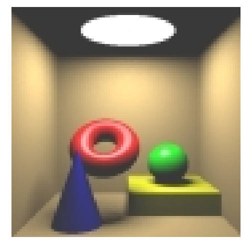

Figure 6.3

(7) hyperbolic decreasing form center

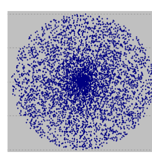

Figure 7.1

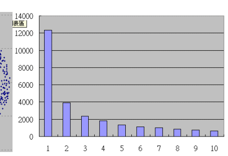

Figure 7.2

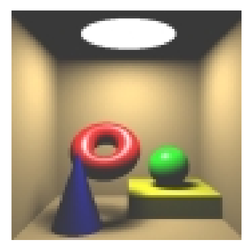

Figure 7.3