利用區間值乏晰集合作雙向近似推理之新方法
A NEW METHOD FOR BIDIRECTIONAL APPROXIMATE REASONING USING INTERVAL-VALUED FUZZY SETS

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摘要

在本論文中，我們提出利用區間值乏晰集合作雙向近似推理的新方法，其中乏晰生產規則被用來表示知識，而且出現在乏晰生產規則中之乏晰詞是以區間值乏晰集合表示。本論文所提之方法比在〔1〕中所提之方法更具彈性，而且本論文中所提之方法比在〔11〕中所提之單輸入-單輸出近似推理方法可更快速的被執行及更具有彈性。

關鍵詞：雙向近似推理，乏晰生產規則，區間值乏晰集合，知識庫，規則式系統

Abstract

In this paper, we propose a new method for bidirectional approximate reasoning using interval-valued fuzzy sets. The proposed method is more flexible than the one presented in [1] due to the fact that it allows the fuzzy terms appearing in the fuzzy production rules of a rule -based system to be represented by interval-valued fuzzy sets rather than general fuzzy sets. Furthermore, the proposed method can be executed much faster and more flexible than the single-input-single-output approximate reasoning method presented in [11].

Keywords: Bidirectional approximate reasoning, fuzzy production rule, interval-valued fuzzy set, knowledge base, rule-based system

1. Introduction

It is obvious that much knowledge in the knowledge base of a rule-based system is fuzzy and imprecise. Therefore, a powerful rule-based system must have the capability to deal with approximate (fuzzy) reasoning [1]-[7], [9]-[12]. The following single-input-single-outpout (SISO) approximate reasoning scheme has been discussed by many researchers:

$$
\begin{array}{l}
R_1 : \text{IF X is } A_1 \text{ THEN Y is } B_1 \\
R_2 : \text{IF X is } A_2 \text{ THEN Y is } B_2 \\
\quad\vdots \\
R_p : \text{IF X is } A_p \text{ THEN Y is } B_p \\
\text{Fact} : \text{X is } A_0 \\
\hline
\text{Consequence} : \text{Y is } B_0
\end{array} \tag{1}
$$

where R_i are fuzzy production rules [13], $1 \leq i \leq p$; X and Y are linguistic variables [16], A_0, A_1, A_2, ..., A_p, B_1, B_2, ..., and B_p are fuzzy terms, such as "very small", "large", etc. A linguistic variable is a variable whose values are fuzzy terms. For example, let "speed" be a linguistic variable, its values may be fuzzy terms, such as "slow", "moderate", "fast", "very slow", "more or less fast", etc. The fuzzy terms are usually represented by fuzzy sets [15].

In [1], Bien et al. presented an inference network for bidirectional approximate reasoing based on fuzzy sets; if a fuzzy input is given for the inference network, then the network renders a reasonable fuzzy output after performing approximate reasoning based on an equality measure, and conversely, for a given fuzzy output, the network can yield its corresponding reasonable fuzzy input after performing approximate reasoning. In [14], Turksen proposed the definitions of interval valued fuzzy sets for the representation of combined concepts based on normal forms. In [11], Gorzalczany presented a method of inference in approximate reasoning based on interval-valued fuzzy sets. In [12], Gorzalczany further presented some basic properties of the interval-valued fuzzy inference method described in [11].

In this paper, we extend the works of [1] and [11] to develop a new method for bidirectional approximate reasoning based on interval-valued fuzzy sets. The proposed method is more flexible than the one presented in [1] due to the fact that it allows the fuzzy terms appearing in the fuzzy production rules of a rule-based system to be represented by interval-valued fuzzy sets rather than general fuzzy sets. Furthermore, because the proposed method requires only simple arithmetic operations and because it allows bidirectional approximate reasoning, it can be executed much faster and more flexible than the single-input-single-output approximate reasoning scheme presented in [11].

The rest of this paper is organized as follows. In Section 2, we briefly review some basic definitions of interval-valued fuzzy sets from [11] and [12]. In Section 3, a method for measuring the degree of similarity between interval-valued fuzzy sets is presented. In Section 4, we present a method for bidirectional approximate reasoning using interval-valued fuzzy sets. The conclusions are discussed in Section 5.

2. Interval-Valued Fuzzy Sets

In [15], Zadah proposed the theory of fuzzy sets. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. A fuzzy set A of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, is a set of ordered pairs, $\{(u_1, f_A(u_1)), (u_2, f_A(u_2)), \ldots, (u_n, f_A(u_n))\}$, where f_A is the membership function of A, $f_A : U \rightarrow [0, 1]$, and $f_A(u_i)$ indicates the degree of membership of u_i in A. In [11] and [12], Gorzalczany has presented interval-valued fuzzy inference methods based on interval-valued fuzzy sets. If a fuzzy set is represented by an interval-valued membership function, then it is called an interval-valued fuzzy set. An interval-valued fuzzy set A of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, can be represented by

$$A = \{(u_1, [a_{11}, a_{12}]), (u_2, [a_{21}, a_{22}]), \ldots, (u_n, [a_{n1}, a_{n2}])\}, \quad (2)$$

where interval $[a_{i1}, a_{i2}]$ indicates that the grade of membership of u_i in the interval-valued fuzzy set A is between a_{i1} and a_{i2}, where $0 \le a_{i1} \le a_{i2} \le 1$ and $1 \le i \le n$.

Let A and B be two interval-valued fuzzy sets,

$$A = \{(u_1, [a_{11}, a_{12}]), (u_2, [a_{21}, a_{22}]), \ldots, (u_n, [a_{n1}, a_{n2}])\} = \{(u_i, [a_{i1}, a_{i2}]) \mid 1 \le i \le n\}, \quad (3)$$

$$B = \{(u_1, [b_{11}, b_{12}]), (u_2, [b_{21}, b_{22}]), \ldots, (u_n, [b_{n1}, b_{n2}])\} = \{(u_i, [b_{i1}, b_{i2}]) \mid 1 \le i \le n\}. \quad (4)$$

The interval-valued fuzzy sets A and B are called equal (i.e., A = B) if and only if $\forall i$, $[a_{i1}, a_{i2}] = [b_{i1}, b_{i2}]$ (i.e., $a_{i1} = b_{i1}$ and $a_{i2} = b_{i2}$), where $1 \le i \le n$. The union, intersection, and complement operations of the interval-valued fuzzy sets are defined as follows.

$$A \cup B = \{(u_i, [c_{i1}, c_{i2}]) \mid c_{i1} = \mathrm{Max}(a_{i1}, b_{i1}), c_{i2} = \mathrm{Max}(a_{i2}, b_{i2}), \text{and } 1 \le i \le n\}, \quad (5)$$

$$A \cap B = \{(u_i, [d_{i1}, d_{i2}]) \mid d_{i1} = \mathrm{Min}(a_{i1}, b_{i1}), d_{i2} = \mathrm{Min}(a_{i2}, b_{i2}), \text{and } 1 \le i \le n\}, \quad (6)$$

$$\overline{A} = \{(u_i, [x_{i1}, x_{i2}]) \mid x_{i1} = 1 - a_{i2}, x_{i2} = 1 - a_{i1}, \text{and } 1 \le i \le n\}. \quad (7)$$

3. Similarity Measures

In [17], Zwick et al. have made a comparative analysis of measures of similarity among fuzzy concepts. A method for measuring the distance between two real intervals is also presented in [17]. Let X and Y be two real intervals contained in $[\beta_1, \beta_2]$, where $X = [x_1, x_2]$ and $Y = [y_1, y_2]$. The distance between the intervals X and Y can be calculated as follows:

$$\Delta(X, Y) = \frac{|x_1 - y_1| + |x_2 - y_2|}{2(\beta_2 - \beta_1)}. \quad (8)$$

It is obvious that if X and Y are identical intervals (i.e., X = Y), then $\Delta(X, Y) = 0$. In [8], we have developed a similarity measure based on [17] to measure the degree of similarity between interval-valued fuzzy sets summarized as follows:

Case 1 : If $X \in [0, 1]$ and Y is a real interval $[y_1, y_2]$ in [0, 1], then

$$S(X, Y) = 1 - \frac{|X - y_1| + |X - y_2|}{2}, \quad (9)$$

where $S(X, Y) \in [0, 1]$. The larger the value of S(X, Y), the higher the similarity between X and Y.

Case 2 : If X and Y are both real interval in [0, 1], where $X = [x_1, x_2]$ and $Y = [y_1, y_2]$, then

$$S(X,Y) = 1 - \frac{|x_1 - y_1| + |x_2 - y_2|}{2}, \quad (10)$$

where $S(X, Y) \in [0, 1]$. The larger the value of $S(X, Y)$, the higher the similarity between X and Y.

In the following, we introduce a method for measuring the degree of similarity between interval-valued fuzzy sets [8]. Let A and B be two interval-valued fuzzy sets of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, where

$$A = \{(u_1, [a_{11}, a_{12}]), (u_2, [a_{21}, a_{22}]), \ldots, (u_n, [a_{n1}, a_{n2}])\}$$
$$B = \{(u_1, [b_{11}, b_{12}]), (u_2, [b_{21}, b_{22}]), \ldots, (u_n, [b_{n1}, b_{n2}])\},$$

where $0 \le a_{i1} \le a_{i2} \le 1$, $0 \le b_{i1} \le b_{i2} \le 1$, and $1 \le i \le n$. Then, based on the matrix representation method, the interval-valued fuzzy sets A and B can be represented by the matrix $\overline{A}$ and $\overline{B}$, respectively, where

$$\overline{A} = <[a_{11}, a_{12}], [a_{21}, a_{22}], \ldots, [a_{n1}, a_{n2}]>$$
$$\overline{B} = <[b_{11}, b_{12}], [b_{21}, b_{22}], \ldots, [b_{n1}, b_{n2}]>.$$

Based on formula (10), the degree of similarity between the interval-valued fuzzy sets A and B can be measured by the function T,

$$T(\overline{A}, \overline{B}) = \frac{\sum_{i=1}^{n} S([a_{i1}, a_{i2}], [b_{i1}, b_{i2}])}{n}, \quad (11)$$

where $T(\overline{A}, \overline{B}) \in [0, 1]$. The larger the value of $T(\overline{A}, \overline{B})$, the higher the similarity between the interval-valued fuzzy sets A and B. It is obvious that if A and B are identical interval-valued fuzzy sets (i.e., A = B), then $T(\overline{A}, \overline{B}) = 1$.

4. Bidirectional Approximate Reasoning Using Interval-Valued Fuzzy Sets

Let's consider the following generalized modus ponens (GMP):

Rule : IF X is A THEN Y is B
Fact : X is A^* (12)

Conclusion : Y is B^*

where X and Y are linguistic variables, A^* and A are interval-valued fuzzy sets of the universe of discourse U, $U = \{u_1, u_2, \ldots, u_n\}$, and B^* and B are interval-valued fuzzy sets of the universe of discourse V, $V = \{v_1, v_2, \ldots, v_m\}$. Assume that the interval-valued fuzzy sets A^*, A, and B have the following forms :

$$A^* = \{(u_1, [x_{11}, x_{12}]), (u_2, [x_{21}, x_{22}]), \ldots, (u_n, [x_{n1}, x_{n2}])\},$$
$$A = \{(u_1, [y_{11}, y_{12}]), (u_2, [y_{21}, y_{22}]), \ldots, (u_n, [y_{n1}, y_{n2}])\},$$
$$B = \{(v_1, [z_{11}, z_{12}]), (v_2, [z_{21}, z_{22}]), \ldots, (v_m, [z_{m1}, z_{m2}])\},$$

where $0 \le x_{i1} \le x_{i2} \le 1$, $0 \le y_{i1} \le y_{i2} \le 1$, $1 \le i \le n$, $0 \le z_{j1} \le z_{j2} \le 1$, and $1 \le j \le m$. Let $\overline{A^*}$ and $\overline{A}$ be the matrix representation of the interval-valued fuzzy sets A^* and A, respectively, where

$$\overline{A^*} = <[x_{11}, x_{12}], [x_{21}, x_{22}], \ldots, [x_{n1}, x_{n2}]>,$$
$$\overline{A} = <[y_{11}, y_{12}], [y_{21}, y_{22}], \ldots, [y_{n1}, y_{n2}]>.$$

Then, based on the matching function T, the degree of similarity between the fuzzy sets A^* and A can be measured, where

$$T(\overline{A^*}, \overline{A}) = \frac{\sum_{i=1}^{n} S([x_{i1}, x_{i2}], [y_{i1}, y_{i2}])}{n}. \quad (13)$$

Let $T(\overline{A^*}, \overline{A}) = k$, where $k \in [0, 1]$. The deduced consequence of the rule is "Y is B^*", where the membership function of the interval-valued fuzzy set B^* is as follows:

$$B^* = \{(v_1, [w_{11}, w_{12}]), (v_2, [w_{21}, w_{22}]), \ldots, (v_m, [w_{m1}, w_{m2}])\}, \quad (14)$$

where $w_{i1} = k * z_{i1}$, $w_{i2} = k * z_{i2}$, and $1 \le i \le m$.

It is obvious that if A^* and A are identical interval-valued fuzzy sets (i.e., $A^* = A$), then $T(\overline{A^*}, \overline{A}) = 1$ and B^* is equal to B.

Let's consider the following single-input-single-output approximate reasoning scheme:

R_1 : IF X is A_1 THEN Y is B_1
R_2 : IF X is A_2 THEN Y is B_2
⋮ (15)
R_p : IF X is A_p THEN Y is B_p
Fact : X is A_0

Consequence : Y is B_0

where A_0, A_1, A_2, ..., and A_p are interval-valued fuzzy sets of the universe of discourse U, $U = \{u_1, u_2, ..., u_n\}$, and B_0, B_1, B_2, ..., and B_p are interval-valued fuzzy sets of the universe of discourse V, $V = \{v_1, v_2, ..., v_m\}$. Assume that

$$A_i = \{(u_1, [x_{i1}, x_{i1}^*]), (u_2, [x_{i2}, x_{i2}^*]), ..., (u_n, [x_{in}, x_{in}^*])\},$$
$$B_j = \{(v_1, [y_{j1}, y_{j1}^*]), (v_2, [y_{j2}, y_{j2}^*]), ..., (v_m, [y_{jm}, y_{jm}^*])\},$$

where $0 \le i \le p$ and $1 \le j \le p$. By using the matrix representation method, the interval-valued fuzzy set A_i can be represented by the matrix $\overline{A_i}$, $0 \le i \le p$, where

$$\overline{A_0} = <[x_{01}, x_{01}^*], [x_{02}, x_{02}^*], ..., [x_{0n}, x_{0n}^*]>$$
$$\overline{A_1} = <[x_{11}, x_{11}^*], [x_{12}, x_{12}^*], ..., [x_{1n}, x_{1n}^*]>$$
$$\overline{A_2} = <[x_{21}, x_{21}^*], [x_{22}, x_{22}^*], ..., [x_{2n}, x_{2n}^*]>$$
$$\vdots$$
$$\overline{A_p} = <[x_{p1}, x_{p1}^*], [x_{p2}, x_{p2}^*], ..., [x_{pn}, x_{pn}^*]>.$$

Based on the previous discussions, we can get the following results:

$T(\overline{A_0}, \overline{A_1}) = k_1 \Rightarrow$ the deduced consequence of rule R_1 is "Y is B_1^*", where

$$B_1^* = \{(v_1, [k_1 * y_{11}, k_1 * y_{11}^*]), (v_2, [k_1 * y_{12}, k_1 * y_{12}^*]), ..., (v_m, [k_1 * y_{1m}, k_1 * y_{1m}^*])\},$$

$T(\overline{A_0}, \overline{A_2}) = k_2 \Rightarrow$ the deduced consequence of rule R_2 is "Y is B_2^*", where

$$B_2^* = \{(v_1, [k_2 * y_{21}, k_2 * y_{21}^*]), (v_2, [k_2 * y_{22}, k_2 * y_{22}^*]), ..., (v_m, [k_2 * y_{2m}, k_2 * y_{2m}^*])\},$$

$$\vdots$$

$T(\overline{A_0}, \overline{A_p}) = k_p \Rightarrow$ the deduced consequence of rule R_p is "Y is B_p^*", where

$$B_p^* = \{(v_1, [k_p * y_{p1}, k_p * y_{p1}^*]), (v_2, [k_p * y_{p2}, k_p * y_{p2}^*]), ..., (v_m, [k_p * y_{pm}, k_p * y_{pm}^*])\}.$$

where $k_i \in [0, 1]$ and $1 \le i \le p$, and the deduced consequence of the SISO approximate reasoning scheme is "Y is B_0", where

$$B_0 = B_1^* \cup B_2^* \cup \ ... \ \cup B_p^*, \tag{16}$$

and "$\cup$" is the union operator of the interval-valued fuzzy sets. That is,

$$B_0 = \{(v_1, [z_1, z_1^*]), (v_2, [z_2, z_2^*]), ..., (v_m, [z_m, z_m^*])\}, \tag{17}$$

where

$$\begin{aligned} z_1 &= Max(k_1 * y_{11}, k_2 * y_{21}, ..., k_p * y_{p1}) \\ z_1^* &= Max(k_1 * y_{11}^*, k_2 * y_{21}^*, ..., k_p * y_{p1}^*) \\ z_2 &= Max(k_1 * y_{12}, k_2 * y_{22}, ..., k_p * y_{p2}) \\ z_2^* &= Max(k_1 * y_{12}^*, k_2 * y_{22}^*, ..., k_p * y_{p2}^*) \\ &\vdots \\ z_m &= Max(k_1 * y_{1m}, k_2 * y_{2m}, ..., k_p * y_{pm}) \\ z_m^* &= Max(k_1 * y_{1m}^*, k_2 * y_{2m}^*, ..., k_p * y_{pm}^*), \end{aligned} \tag{18}$$

$0 \le z_i \le z_i^* \le 1$, and $1 \le i \le m$. If k_i is the largest value among the values k_1, k_2, ..., and k_p, then the interval-valued fuzzy set B_0 is the most similar to the interval-valued fuzzy set B_i, where $1 \le i \le p$.

Conversely, let's consider the following SISO approximate reasoning scheme:

$$\begin{array}{ll} R_1 : & \text{IF X is } A_1 \text{ THEN Y is } B_1 \\ R_2 : & \text{IF X is } A_2 \text{ THEN Y is } B_2 \\ & \vdots \\ R_p : & \text{IF X is } A_p \text{ THEN Y is } B_p \\ \text{Fact} : & \text{Y is } B_0 \\ \hline \text{Consequence} : & \text{X is } A_0 \end{array} \tag{19}$$

where

$$A_i = \{(u_1, [x_{i1}, x_{i1}^*]), (u_2, [x_{i2}, x_{i2}^*]), ..., (u_n, [x_{in}, x_{in}^*])\},$$
$$B_j = \{(v_1, [y_{j1}, y_{j1}^*]), (v_2, [y_{j2}, y_{j2}^*]), ..., (v_m, [y_{jm}, y_{jm}^*])\},$$

$1 \le i \le p$ and $0 \le j \le p$. By using the matrix representation method, the interval-valued fuzzy set B_j can be represented by the matrix $\overline{B_j}$, $0 \le j \le p$, where

$$\overline{B_0} = <[y_{01}, y_{01}^*], [y_{02}, y_{02}^*], ..., [y_{0m}, y_{0m}^*]>$$
$$\overline{B_1} = <[y_{11}, y_{11}^*], [y_{12}, y_{12}^*], ..., [y_{1m}, y_{1m}^*]>$$
$$\overline{B_2} = <[y_{21}, y_{21}^*], [y_{22}, y_{22}^*], ..., [y_{2m}, y_{2m}^*]>$$
$$\vdots$$
$$\overline{B_p} = <[y_{p1}, y_{p1}^*], [y_{p2}, y_{p2}^*], ..., [y_{pm}, y_{pm}^*]>.$$

Based on the previous discussions, we can get the following results:

$T(\overline{B_0},\overline{B_1}) = s_1 \Rightarrow$ the deduced consequence of rule R_1 is "X is A_1^*", where

$A_1^* = \{(u_1, [s_1 * x_{11}, s_1 * x_{11}^*]), (u_2, [s_1 * x_{12}, s_1 * x_{12}^*]), \ldots, (u_n, [s_1 * x_{1n}, s_1 * x_{1n}^*])\}$,

$T(\overline{B_0},\overline{B_2}) = s_2 \Rightarrow$ the deduced consequence of rule R_2 is "X is A_2^*", where

$A_2^* = \{(u_1, [s_2 * x_{21}, s_2 * x_{21}^*]), (u_2, [s_2 * x_{22}, s_2 * x_{22}^*]), \ldots, (u_n, [s_2 * x_{2n}, s_2 * x_{2n}^*])\}$,

$\vdots$

$T(\overline{B_0},\overline{B_p}) = s_p \Rightarrow$ the deduced consequence of rule R_p is "X is A_p^*", where

$A_p^* = \{(u_1, [s_p * x_{p1}, s_p * x_{p1}^*]), (u_2, [s_p * x_{p2}, s_p * x_{p2}^*]), \ldots, (u_n, [s_p * x_{pn}, s_p * x_{pn}^*])\}$,

where $s_i \in [0, 1]$ and $1 \le i \le p$, and the deduced consequence of the SISO approximate reasoning scheme is "X is A_0", where

$$A_0 = A_1^* \cup A_2^* \cup \ldots \cup A_p^*, \tag{20}$$

and "$\cup$" is the union operator of the interval-valued fuzzy sets. That is,

$$A_0 = \{(u_1, [w_1, w_1^*]), (u_2, [w_2, w_2^*]), \ldots, (u_n, [w_n, w_n^*])\}, \tag{21}$$

where

$$\begin{aligned} w_1 &= \mathrm{Max}(s_1 * x_{11}, s_2 * x_{21}, \ldots, s_p * x_{p1}) \\ w_1^* &= \mathrm{Max}(s_1 * x_{11}^*, s_2 * x_{21}^*, \ldots, s_p * x_{p1}^*) \\ w_2 &= \mathrm{Max}(s_1 * x_{12}, s_2 * x_{22}, \ldots, s_p * x_{p2}) \\ w_2^* &= \mathrm{Max}(s_1 * x_{12}^*, s_2 * x_{22}^*, \ldots, s_p * x_{p2}^*) \\ &\vdots \\ w_n &= \mathrm{Max}(s_1 * x_{1n}, s_2 * x_{2n}, \ldots, s_p * x_{pn}) \\ w_n^* &= \mathrm{Max}(s_1 * x_{1n}^*, s_2 * x_{2n}^*, \ldots, s_p * x_{pn}^*), \end{aligned} \tag{22}$$

$0 \le w_i \le w_i^* \le 1$, and $1 \le i \le n$. If s_i is the largest value among the values $s_1, s_2, \ldots,$ and s_p, then the interval-valued fuzzy set A_0 is the most similar to the interval-valued fuzzy set A_i, where $1 \le i \le p$.

5. Conclusions

In this paper, we have presented a new method for bidirectional approximate reasoning using interval-valued fuzzy sets. The proposed method is more flexible than the one presented in [1] due to the fact that it allows the fuzzy terms appearing in the fuzzy production rules of a rule-based system to be represented by interval-valued fuzzy sets rather than general fuzzy sets. Furthermore, because the proposed method requires only simple arithmetic operations and because it allows bidirectional approximate reasoning, it can be executed much faster and more flexible than the single-input-single-output approximate reasoning method presented in [11].

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