Performance Assessment Measures for Parallel Cascade Control

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Abstract

An analysis of output variances for parallel cascade systems is derived which allows the output variance contributions due to both disturbances and controllers to be established. Following the methodology of the univariate control loop performance, a performance bound is derived based on the minimum variance and the Diophantine decomposition for the parallel cascade control system. It can assess the performance of the overall control scheme. Besides, the achievable performance bound and the corresponding optimal parameters of the PID controller structure computed from the closed loop operating data are also proposed. It can assess the performance of existing control systems. The performance of the proposed method is illustrated through a pilot scaled experiment.

1. Introduction

Cascade control is a multiloop control scheme widely used in chemical process control to improve single-loop control performance for eliminating the disturbances effect in the manipulated variables or the nonlinearity in the final control element (Lee et al., 2002; Shinsky, 1967). One of the typical types is a series cascade control (SCC) structure with two control loops whose inner (or secondary) loop is embedded within an outer (or primary) loop. The secondary controller of the inner loop allows rapid rejection or reduction of the disturbances before the disturbances effects spill over to the primary loop, resulting in little effect on the primary output. On the other hand, due to the physical nature of the processes, the primary and secondary loops are sometimes connected in parallel cascade control (PCC) design instead of series one (Luyben, 1973). This design is related to the characteristics of the process when the manipulated variable and the disturbance affect the primary and the secondary outputs in parallel actions (Semino and Brambilla, 1996). Examples of SCC and PCC processes are described by Luyben (1973). However, the above control research mainly focuses on designing new strategies or giving tuning methods, but it gives little attention to the assessment and maintenance of the installed system. To our best knowledge, there is only one method documented in the literature which makes the assessment of the SCC system based on the minimum variance control (MVC) law (Ko and Edgar, 2000).

Nowadays, increasing complexity of industrial processes results in strong demands for monitoring and measuring the performance of the controllers. If the deterioration of controller performance cannot be identified in time, unwanted variances would prevent the operating processes from achieving their true process capability. If the worse comes to the worst, the malfunction would cause monetary loss or even significant impact on personnel, environment and equipment safety. Furthermore, the regular estimation of the control performance can be used to monitor and evaluate how much potential there is to improve control performance (Hunag and Shah, 1999). In the last decade, many investigations based on the minimum variance as a performance benchmark have been done for the control-loop performance assessment. The stochastic assessment defined by Harris (1989) is basically a procedure used to fit the closed loop data into a time series model with the given process dead time. The performance of the controlled system is evaluated by comparing the difference between the minimum variance of the controlled output and the current variance of the controlled output.

Several authors extended this theory to defining the various performance indices in assessing current control vs. minimum variance control (Desborough and Harris, 1993; Kozub and Garcia, 1993; Thornhill et al., 1999). In this paper, two issues are addressed by PCC structures. The first one is concerned with the development of the minimum variance performance bound for PCC systems. The performance bound can subsequently be used for the performance assessment of the PCC system as a benchmark of performance. The second issue is concerned with the estimation of an achievable minimum variance performance bound of PCC.

The remainder of this paper is structured as follows: The problem of the performance assessment in PCC is first defined in Section 2. The procedures for computing of the achievable PID control performance with the unknown process and disturbance models are discussed in Section 3. The effectiveness of the proposed method and its potential applications are demonstrated through a computer simulation problem and an actual pilot-scaled experimental study in Section 4, followed by concluding remarks in Section 5.

2. Minimum Variance of PCC

The block diagram of a PCC system, consisted of two feedback loops, is shown in Fig. 1. The process, represented by the dashed box, consists of two components $G_{p1}(k)$ and $G_{p2}(k)$. The goal of PCC is to make $y_1(k)$ reach the set point as long as the constraints on $y_2(k)$ are respected. The inner-loop controller, $G_{c2}(k)$, is used to regulate the constrained output $y_2(k)$. $G_{c2}(k)$ is tuned to avoid overshooting of the constrained variable $y_2(k)$. The outer-loop controller, $G_{c1}(k)$, is tuned to regulate the output $y_1(k)$ to its set point. $u_1(k)$ and $u_2(k)$ are the controller outputs of $G_{c1}(k)$ and $G_{c2}(k)$. $G_{w1}(k)$ and $G_{w2}(k)$ are unmeasured disturbances to the primary output and the secondary output respectively. $w_1(k)$ and $w_2(k)$ are the noise sequences. When the primary setpoint is constant, the disturbances, $w_1(k)$ and $w_2(k)$, affect the output variables, $y_1(k)$ and $y_2(k)$, are given by

$$y_1 = \frac{(1+G_{P2}G_{c2})G_{w1}}{1+G_{P2}G_{c2}+G_{P1}G_{c2}G_{c1}}w_1 - \frac{G_{P1}G_{c2}G_{w2}}{1+G_{P2}G_{c2}+G_{P1}G_{c2}G_{c1}}w_2$$

$$y_2 = \frac{-G_{P2}G_{c2}G_{c1}G_{w1}}{1+G_{P2}G_{c2}+G_{P1}G_{c2}G_{c1}}w_1 + \frac{(1+G_{P1}G_{c2}G_{c1})G_{w2}}{1+G_{P2}G_{c2}+G_{P1}G_{c2}G_{c1}}w_2 \quad (1)$$

To assess the performance of the PCC system, two different control laws that achieve minimum variance in the primary output only and in both the primary and the secondary outputs are derived when stochastic disturbances occur in both the primary and the secondary loops.

Case 1: Minimum variance of the primary output

Assume G_{p1} and G_{p2} can be represented by $G_{p1}=G_{p1}^{*}z^{-d_1}$ and $G_{p2}=G_{p2}^{*}z^{-d_2}$, where $d_i, i=1,2$ are the time delays and $G_{pi}^{*}, i=1,2$ are the process models without any time delay. Now introduce the identities,

$$G_{w1}=Q_1+R_1q^{-d_1}=Q_3+R_3q^{-d_2}$$

and

$$G_{w2}=Q_2+R_2q^{-d_1}=Q_4+R_4q^{-d_2} \quad (2)$$

Eq. (2) is called Diophantine equations whose solution can be computed manually using long division or a computer by using a recursive algorithm. In Eq. (2), Q_1 and Q_2 are polynomials of degree d_1-1; Q_3 and Q_4 are polynomials of degree d_2-1; $R_i, i=1,\ldots,4$ are proper transfer functions. Substituting the identities in Eq. (2) into Eq. (1), the closed-loop primary output can be classified into cascade-invariant (CI) and cascade dependent (CD) terms.

$$\begin{aligned} y_1(k) &= Q_1w_1 + q^{-d_1}\left[\underbrace{\frac{R_1(1+G_{P2}G_{c2})-Q_1G_{P1}^{*}G_{c2}G_{c1}}{1+G_{P2}G_{c2}+G_{P1}G_{c2}G_{c1}}}_{H_1}w_1(k) + \underbrace{\frac{-Q_2G_{P1}^{*}G_{c2}-R_2G_{P1}^{*}G_{c2}}{1+G_{P2}G_{c2}+G_{P1}G_{c2}G_{c1}}}_{H_2}w_2(k)\right] \\ &= \underbrace{Q_1w_1(k)}_{\text{CI}} + \underbrace{q^{-d_1}\left[H_1w_1(k)+H_2w_2(k)\right]}_{\text{CD}} \end{aligned} \quad (3)$$

If minimum variance controllers are implemented in order to minimize the variance of the primary output, only the CD term of the primary output should be minimized, i.e. solving the simultaneous equations of H_1 and H_2,

$$\underbrace{\begin{bmatrix} -R_1G_{P2} & Q_1G_{P1}^* \\ -\left(Q_2G_{P2}^* + R_2G_{P2}\right) & 0 \end{bmatrix}}_{\mathbf{A1}} \begin{bmatrix} G_{C2} \\ G_{C2}G_{C1} \end{bmatrix} = \underbrace{\begin{bmatrix} R_1 \\ 0 \end{bmatrix}}_{\mathbf{b1}} \tag{4}$$

$$\begin{bmatrix} G_{C2} \\ G_{C2}G_{C1} \end{bmatrix} = \left(\mathbf{A1}^{\mathrm{T}}\mathbf{A1}\right)^{-1}\mathbf{A1}^{\mathrm{T}}\mathbf{b1}$$

Thus, even if the minimum variances controllers are implemented, it is impossible for the controllers to cause zero change after d_1 time delays of y_1. Therefore, the minimum variance or the sum of the invariant portion (CI) and the minimum variant portion (CD) of the primary output variances is

$$\begin{aligned} \sigma^2_{y_1,mv} &= \operatorname{var}\{(Q_1 + e_1)w_1(k) + e_2w_2(k)\} \\ &= trace\left[\left(\sum_{i=0}^{d_1-1} N_{1,i}^T N_{1,i} + \sum_{i=d_1}^{\infty} N_{2,i}^T N_{2,i}\right)\Sigma_w\right] \end{aligned} \tag{5}$$

where e_1 and e_2, which are the minimum variant portion (CD) of output variances from w_1 and w_2, can computed by

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \mathbf{b1}\left(\mathbf{I} - \mathbf{A1}\left(\mathbf{A1}^{\mathrm{T}}\mathbf{A1}\right)^{-1}\mathbf{A1}^{\mathrm{T}}\right) \tag{6}$$

$N_{1,i}(i = 0,\cdots,d_1-1)$ are the coefficient matrices of the matrix polynomial Q_1; $N_{2,i}(i = d_1,\cdots)$ are the coefficient matrix of the polynomial matrix $[e_1 \quad e_2]$; Σ_w is the variance-covariance matrix of $[w_1 \quad w_2]^T$. The result of Eq. (5) can be used as a benchmark to assess a PCC system when the primary output variance is considered only.

Case 2: Minimum variance of both primary and secondary outputs

The primary output performance assessment concepts are extended to both the primary and the secondary outputs in this subsection. Like the primary output (y_1), according to the Diophantine identities of Eq. (2), the secondary output (y_2) of PCC can be also expressed as the summation of cascade-invariant (CI) and cascade-dependent (CD) terms.

$$\begin{aligned} y_2(k) &= Q_4w_2(k) \\ &+ q^{-d_2}\left[\underbrace{\frac{R_4\left(1 + G_{P1}G_{c2}G_{c1}\right) - Q_4G_{P2}^*G_{c2}}{1 + G_{P2}G_{c2} + G_{P1}G_{c2}G_{c1}}}_{H_3}w_1 - \underbrace{\frac{Q_3G_{P2}^*G_{c2}G_{c1} + R_3G_{P2}G_{c2}G_{c1}}{1 + G_{P2}G_{c2} + G_{P1}G_{c2}G_{c1}}}_{H_4}w_2\right] \\ &= \underbrace{Q_4w_2}_{\mathrm{CI}} + \underbrace{q^{-d_2}\left[H_3w_1(k) + H_4w_2(k)\right]}_{\mathrm{CD}} \end{aligned} \tag{7}$$

The minimum variance of PCC in this case is obtained by solving the simultaneous equations of H_1, H_2, H_3 and H_4.

$$\underbrace{\begin{bmatrix} -R_1G_{P2} & Q_1G_{P1}^* \\ -\left(Q_2G_{P1}^* + R_2G_{P1}\right) & 0 \\ 0 & -\left(Q_3G_{P2}^* + R_3G_{P2}\right) \\ Q_4G_{P2}^* & -R_4G_{P1} \end{bmatrix}}_{\mathbf{A2}} \begin{bmatrix} G_{C2} \\ G_{C2}G_{C1} \end{bmatrix} = \underbrace{\begin{bmatrix} R_1 \\ 0 \\ 0 \\ R_4 \end{bmatrix}}_{\mathbf{b2}}$$

$$\begin{bmatrix} G_{C2} \\ G_{C2}G_{C1} \end{bmatrix} = \left(\mathbf{A2}^{\mathrm{T}}\mathbf{A2}\right)^{-1}\mathbf{A2}^{\mathrm{T}}\mathbf{b2} \tag{8}$$

Therefore, the minimum variance or the sum of the invariant portions (CIs) and the minimum variant portions (CDs) of the primary and the secondary output variances is

$$\begin{aligned} \sigma^2_{y_1,y_2,mv} &= \min\{\operatorname{var}((Q_1 + e_1)w_1 + e_2w_2) \\ &\quad + \operatorname{var}((Q_4 + e_3)w_2 + e_4w_1)\} \\ &= trace\left[\left(\sum_{i=0}^{d_1-1} N_{1,i}^T N_{1,i} + \sum_{i=0}^{d_2-1} M_{1,i}^T M_{1,i} + \sum_{i=d_1}^{\infty} N_{2,i}^T N_{2,i} + \sum_{i=d_2}^{\infty} M_{2,i}^T M_{2,i}\right)\Sigma_w\right] \end{aligned} \tag{9}$$

where e_1, e_2, e_3 and e_4, which are the minimum variant portion (CD) of output variances from w_1 and w_2, can computed by

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \mathbf{b2}\left(\mathbf{I} - \mathbf{A2}\left(\mathbf{A2}^{\mathrm{T}}\mathbf{A2}\right)^{-1}\mathbf{A2}^{\mathrm{T}}\right) \tag{10}$$

where $N_{1,i}(i = 0,\cdots,d_1-1)$ and $M_{1,i}(i = 0,\cdots,d_2-1)$ are the coefficient matrices of the matrix polynomials Q_1 and Q_4 respectively; $N_{2,i}(i = d_1,\cdots)$ and

$M_{2,i}(i=d_2,\cdots)$ are the coefficient matrix of the polynomial matrix $[e_1 \quad e_2]$ and $[e_4 \quad e_3]$. The result of Eq. (9) can be used as a benchmark to assess a PCC system when both the primary and the secondary output variances are considered.

3. Achievable Minimum Variance of PCC

In reality, the process and the disturbance models are rarely available. In this case, it is important to identify the models using the process operating data. A two-stage method is the subset which applies an identification to closed-loop feedback data (Van Den Hof and Schrama, 1993). It is extended for identification of the PCC process and the disturbance models, separately. To obtain this, the external input is intermittently introduced into the controlled system. The sensitivity function and the process models for the primary and the secondary loops are separately identified by a two-stage identification method. Based on the identified closed loop transfer functions, the proper disturbance models for the primary and the secondary loop can be estimated. After the models are obtained, the achievable minimal variance performance bound of PCC can be computed to assess the current status of the current controller performance. Even though the models are estimated, the performance bound benchmark based on the minimum variance control may not be realistic for general applications, because it considers only the absolute lower performance bound. According to the required minimum variance of the primary output only or of both the primary and the secondary outputs objective functions, the achievable performance bound of PCC based on the specific controllers can be obtained by solving the following optimization problem for a given process and disturbance models:

$$\min_{G_{c1},G_{c2}} \sigma_{y_1}^2 \tag{11}$$

and

$$\min_{G_{c1},G_{c2}} \left(\lambda_1\sigma_{y_1}^2+\lambda_2\sigma_{y_2}^2\right) \tag{12}$$

It is apparent that the variance of the output y_1 (or y_2) is the function of the controller parameters G_{c1} and G_{c2}. Note that in Eq (12), it is a standard technique for multi-objective optimization to minimize a positively weighted sum of the objectives. For easy explanations, $\lambda_1=1$ and $\lambda_2=1$ are used here. Due to the space limitation, the entire modeling methodology is described in our previous work. For a detailed coverage, see (Huang, 2004)

With the identified process and disturbance models, the closed-loop outputs of Eq. (1) under the current control action can be computed as a moving average of a series of an identically distributed random disturbance, $w_1(k)$ and $w_2(k)$,

$$\begin{aligned}
y_1(k) &= \Big(\underbrace{1+\varphi_{1,1}z^{-1}+\cdots+\varphi_{1,d_1}z^{-d_1}}_{\text{CI}} + \underbrace{\varphi_{1,d_1+1}z^{-(d_1+1)}+\cdots}_{\text{CD}}\Big)w_1(k) \\
&\quad + \Big(\underbrace{\varphi_{2,d_1+1}z^{-(d_1+1)}+\cdots}_{\text{CD}}\Big)w_2(k) \\
y_2(k) &= \Big(\underbrace{\phi_{1,d_2+1}z^{-(d_2+1)}+\cdots}_{\text{CD}}\Big)w_1(k) \\
&\quad + \Big(\underbrace{1+\phi_{2,1}z^{-1}+\cdots+\phi_{2,d_2}z^{-d_2}}_{\text{CI}} + \underbrace{\phi_{2,d_2+1}z^{-(d_2+1)}+\cdots}_{\text{CD}}\Big)w_2(k)
\end{aligned} \tag{13}$$

Here the outputs of Eq. (13) contain two parts. Each part can be grouped into two terms: cascade invariant and cascade dependent. The closed-loop output variance is calculated by the sum of squares of the impulse response coefficients. Thus, the achievable minimum variance of the cascade control can be estimated by minimizing the following equation,

$$\begin{aligned}
&\min_{G_{c1},G_{c2}} E\left[\hat{\sigma}_{y_1}^2\right] \\
&= trace\left[\left(\sum_{i=0}^{d_1-1} N_{a1,i}^T N_{a1,i} + \sum_{i=d_1}^{\infty} N_{a2,i}^T N_{a2,i}\right)\Sigma_w\right]
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
&\min_{G_{c1},G_{c2}} E\left[\hat{\sigma}_{y_1}^2+\hat{\sigma}_{y_1}^2\right] \\
&= trace\left[\left(\sum_{i=0}^{d_1-1} N_{a1,i}^T N_{a1,i} + \sum_{i=0}^{d_2-1} M_{a1,i}^T M_{a1,i} \right.\right. \\
&\quad \left.\left. + \sum_{i=d_1}^{\infty} N_{a2,i}^T N_{a2,i} + + \sum_{i=d_2}^{\infty} M_{a2,i}^T M_{a2,i}\right)\Sigma_w\right]
\end{aligned} \tag{15}$$

where

$$N_{a1,i}=\begin{bmatrix}\varphi_{1,i}\\0\end{bmatrix},N_{a2,i}=\begin{bmatrix}\varphi_{1,i}\\\varphi_{2,i}\end{bmatrix},\quad \varphi_{1,0}=1$$
$$M_{a1,i}=\begin{bmatrix}\phi_{2,i}\\0\end{bmatrix},M_{a2,i}=\begin{bmatrix}\phi_{1,i}\\\phi_{2,i}\end{bmatrix}\quad \phi_{2,0}=1 \tag{16}$$

The PID-achievable performance bound is given by minimizing the variance calculated from Eq. (14) or Eq. (15) with those optimal tuning PID parameters.

$$G_{C1}\left(z^{-1}\right)=-\frac{k_1+k_2z^{-1}+k_3z^{-2}}{1-z^{-1}}$$
$$G_{C2}\left(z^{-1}\right)=-\frac{k_4+k_5z^{-1}+k_6z^{-2}}{1-z^{-1}} \tag{17}$$

Astrom (1970) has presented a procedure for the evaluation of the output variance with complex integral in the complex plane, but the evaluation is not easy. Here the gradient method is adopted to find out the optimal parameters resulting achievable performance.

4. Illustration Examples

In this section, two examples, including a simulation process with the given system models and a pilot scaled level-to-flow cascade experimental process without any prior knowledge of process models, are discussed to demonstrate the wide applicability of the proposed estimation and the assessment of PID-achievable performance bound with the closed loop operating data.

Example 1: Simulation Process with the Given Models

A process output transfer functions in a PCC system related to the controller outputs ($u_1(k)$ and $u_2(k)$) and the unmeasurable disturbances ($w_1(k)$ and $w_2(k)$) are,

$$y_1(k)=\frac{1}{1-0.8q^{-1}}u_2(k-5)+\frac{1}{1-0.7q^{-1}}w_1(k)$$
$$y_2(k)=\frac{1}{1-0.4q^{-1}}u_2(k-3)+\frac{1}{1-0.5q^{-1}}w_2(k) \tag{18}$$

where the disturbances $w_1(k)$ and $w_2(k)$ have the variance covariance matrix $\Sigma_w=\begin{bmatrix}0.6863 & 0.3231\\0.3231 & 0.6785\end{bmatrix}$. To obtain the minimum variance performance bound, the Diophantine decompositions of G_{w1} and G_{w2} are

$$G_{w1}=\frac{1}{1-0.7q^{-1}}=Q_1+R_1q^{-4}=Q_3+R_3q^{-2}$$
$$G_{w2}=\frac{1}{1-0.5q^{-1}}=Q_2+R_2q^{-4}=Q_4+R_4q^{-2} \tag{19}$$

where

$$Q_1=1+0.7q^{-1}+0.49q^{-2}+0.343q^{-3}+0.2401q^{-4}$$
$$Q_2=1+0.5q^{-1}+0.25q^{-2}+0.125q^{-3}+0.0625q^{-4}$$
$$Q_3=1+0.7q^{-1}+0.49q^{-2}$$
$$Q_4=1+0.5q^{-1}+0.25q^{-2}$$
$$R_1=\frac{0.168}{1-0.7q^{-1}}$$
$$R_2=\frac{0.0312}{1-0.5q^{-1}}$$
$$R_3=\frac{0.343}{1-0.7q^{-1}}$$
$$R_4=\frac{0.125}{1-0.5q^{-1}} \tag{20}$$

Thus, from Eq. (5), the minimum variance of the PCC system for minimizing only the variance of the primary output without considering the variance of the secondary output is

$$\sigma^2_{y1,mv}=\mathrm{var}\{(Q_1)w_1+e_1w_1+e_2w_2\}$$
$$=trace\left[\left(\sum_{i=0}^{d_1-1}N_{1i}^TN_{1i}\right)\Sigma_w\right]+trace\left[\left(\sum_{i=d_1}^{\infty}N_{2i}^TN_{2i}\right).\Sigma_w\right]$$
$$=1.3735 \tag{21}$$

According to the optimization of Eq. (14), the corresponding PID controller parameters resulting in achievable minimum variance can be evaluated ($\min\hat{\sigma}^2_{y1}=1.4827$) and the corresponding controllers are

$$G_{C1}=\frac{0.5769-0.5735z^{-1}}{1-z^{-1}}$$
$$G_{C2}=\frac{0.0656-0.1038z^{-1}+0.0383^{Z^{-2}}}{1-z^{-1}} \tag{22}$$

If the minimum variance of the PCC system for minimizing both the variances of the primary output and the secondary output is considered, from Eq. (9), it can be computed

$$\sigma^2_{y_1,y_2,mv} = \mathrm{var}((Q_1)w_1 + e_1w_1 + e_2w_2) + \mathrm{var}((Q_4)w_2 + e_3w_2 + e_4w_1)$$
$$= trace\left[\left(\sum_{i=0}^{d_1-1} N_{1i}^T N_{1i}\right)\Sigma_w\right] + trace\left[\left(\sum_{i=d_1}^{\infty} N_{2i}^T N_{2i}\right)\Sigma_w\right] + trace\left[\left(\sum_{i=0}^{d_2-1} M_{1i}^T M_{1i}\right)\Sigma_w\right] + trace\left[\left(\sum_{i=d_2}^{\infty} M_{2i}^T M_{2i}\right)\Sigma_w\right] \quad (23)$$
$$= 2.5045$$

From Eq. (15), the corresponding PID controller parameters resulting in achievable minimum variance can be evaluated ($\min\left(\hat{\sigma}^2_{y_1} + \hat{\sigma}^2_{y_2}\right) = 2.6958$) and the corresponding controllers are

$$G_{C1} = \frac{2.2975 - 2.7632z^{-1} + 0.4657^{z^{-2}}}{1 - z^{-1}}, \quad G_{C2} = \frac{0.0438 - 0.0693z^{-1} + 0.0255^{z^{-2}}}{1 - z^{-1}} \quad (24)$$

Example 2: Level Tank Controlled Process

A pilot level in three gravity-drained tanks shown in Fig. 2 is studied. The primary loop is to maintain the level in Tank 1 and the secondary loop is to reject the effect of disturbance (w_2) in manipulated variable (f_i). f_i regulates the level of Tank 1 and Tank 2 in a parallel way. Also, there is another unmeasured disturbance (w_1) in the primary process. The objective is to maintain the two levels at the desired set points and to assess the current control performance in the presence of feed flow rate disturbances. The tanks are equipped with differential-pressure to current (DP/I) transducers to provide continuous measurements of the levels. The computer is connected to a PCI-1710 analog/digital I/O expansion card from Advantech. The expansion board uses a 12-bit converter; therefore, the digital signals are 12-bit. The analog signals from the measured levels are amplified and conditioned EDM35 (4-20mA/0-5volts) modules. The data acquisition software and the PID controller algorithm are MATLAB of MathWorks, Inc.

Fig. 3 shows 800 measurements of the primary and the secondary outputs as deviation variables from its set point and steady-state value when the primary and secondary controllers are

$$G_{C1} = \frac{7.4198 - 7.1368z^{-1}}{1 - z^{-1}}, \quad G_{C2} = \frac{6.2958 - 6.2958z^{-1}}{1 - z^{-1}} \quad (25)$$

Under the current operation, the primary output variance ($\sigma^2_{h_1}$) is 4.6797 and the overall outputs variance ($\sigma^2_{h_1} + \sigma^2_{h_2}$) is 8.9695.

To assess and evaluate the current control performance with PID feedback control loops, using the two-stage method, the system is excited by the extra flow rate fed into Tank 1. Both the extra flow rate and the corresponding change of two levels are measured, a total of 1100 sets of the data are generated. From the data, the dead times of the primary and the secondary processes calculated by cross-correlation analysis are 25 and 20 respectively. According to the identification procedure, the sensitivity function with the finite impulse response model is evaluated by the process input and the external input. Using the estimated sensitivity function, the filtered outputs can be calculated. Then the FOPDT process transfer functions of the primary and secondary loops are

$$G_{P1} = \frac{0.00498z^{-1}}{1 - 0.9322z^{-1}} z^{-25}, \quad G_{P2} = \frac{0.0048z^{-1}}{1 - 0.9369z^{-1}} z^{-20} \quad (26)$$

With the identified process models and known controllers, the disturbance models can be evaluated as

$$G_{w1} = \frac{0.7259}{1 - 0.9841z^{-1}}, \quad G_{w2} = \frac{0.8706}{1 - 0.9897z^{-1}} \quad (27)$$

The estimated variance-covariance matrix is $\Sigma_w = \begin{bmatrix} 0.1676 & 0.1642 \\ 0.1642 & 0.7187 \end{bmatrix}$. Thus, the estimated minimum variances of the primary and the secondary outputs can be calculated from Eq. (9),

$$\sigma^2_{h_1,h_2,mv} = \mathrm{var}((Q_1)w_1 + e_1w_1 + e_2w_2) + \mathrm{var}((Q_4)w_2 + e_3w_2 + e_4w_1) = 4.0851 \quad (28)$$

For practical applications, the next achievable minimum variance computed by optimizing Eq. (15) is 5.1455, which is significantly reduced when comparing the initial operating condition, but a little larger than the minimum variance. The transfer functions for these two controllers are

$$G_{C1} = \frac{0.5825 - 0.555z^{-1}}{1 - z^{-1}} \qquad (29)$$

$$G_{C2} = \frac{0.6530 - 0.6530z^{-1}}{1 - z^{-1}}$$

The aforementioned performance assessment of the cascade system is based on the design of the PCC structure; however, once the control system under the design of a series cascade structure is mistreated, the resulting performance benchmark is also considered here. The calculation procedures are similar to Ko and Edgar's work (2000), but the estimated minimum variances of both the primary and the secondary outputs are considered. The minimum variance can be obtained as 22.7737. If the achievable PID-based minimum variance is considered, the corresponding two controllers are $G_{C1} = \frac{36.76 - 63.34z^{-1} + 27.38z^{-2}}{1 - z^{-1}}$ and $G_{C2} = \frac{16.03 - 32.06z^{-1} + 16.03z^{-2}}{1 - z^{-1}}$. Applying the controllers onto the level tank system, the sum of the both output variances is 26.0272, which is larger than the one under PCC design (5.1455). It is interesting to note that the improper control design in the cascade system will lead to unsatisfactory output performance. Furthermore, the output variance of the redesigned condition is worse than that of the initial condition. Fig. 4 plots the controlled results of the level tank system for different controller tunings. It indicates the output variance is satisfactory only if the system, under the correct control design (PCC), can achieve the minimum variance.

5. Conclusions

In this work, the evaluation of the PCC loop performance assessment is developed. The proposed method provides a way to monitor the control-loop performance of the PCC processes by taking the controlled outputs into account in calculating both minimum variance and the process variance terms. In a SCC system, the process variance terms can be equal to zero when the minimum variance cascade controllers are implemented. Unlike the SCC system, the minimum variance controllers for the PCC system can only minimize the process variance terms but it is impossible to have the zero values. To evaluate the minimum bound, a least squares estimator is developed based on the Diophantine decompositions of the disturbance models. The minimum variance performance can also be estimated from the optimization of the minimum PID variance controllers based on the pre-identified process and disturbance models. The proposed performance method has been illustrated through a simulation example and demonstrated by a pilot scaled experimental application.

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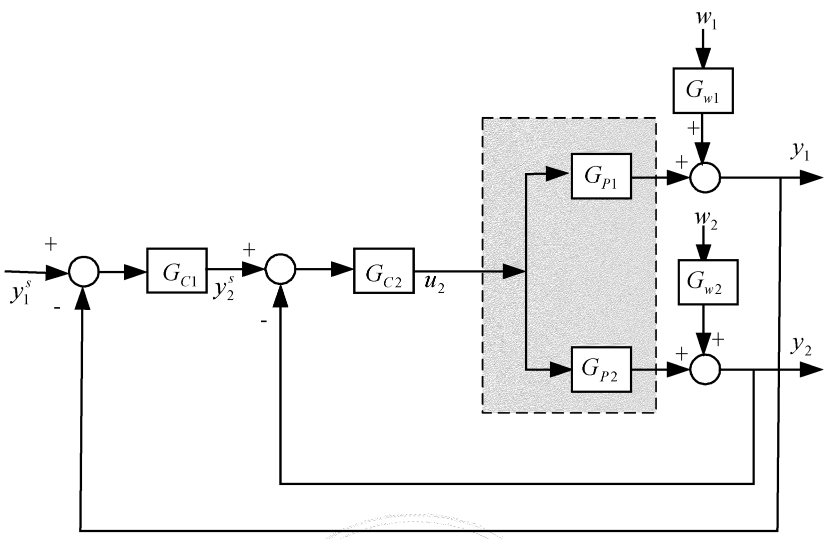


Fig. 1: A parallel cascade control system

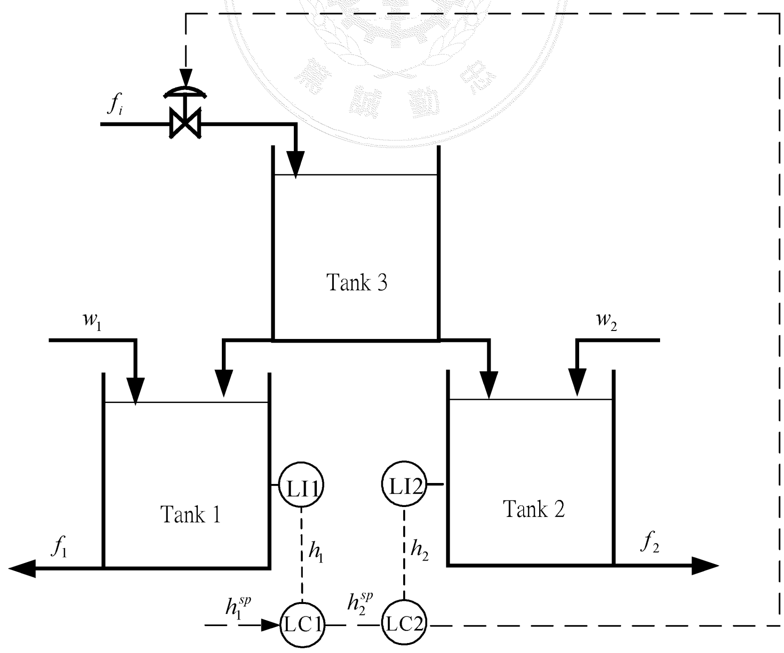


Fig. 2: Level tank control system with PCC structure

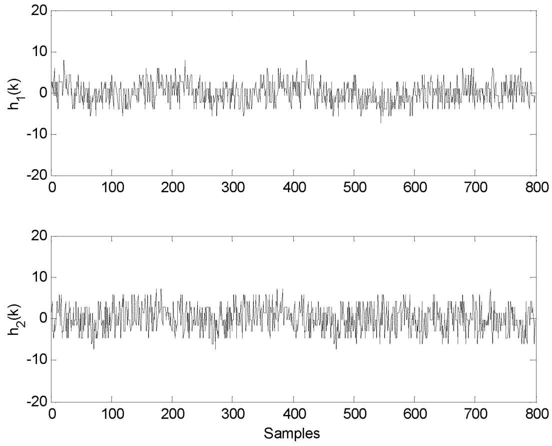
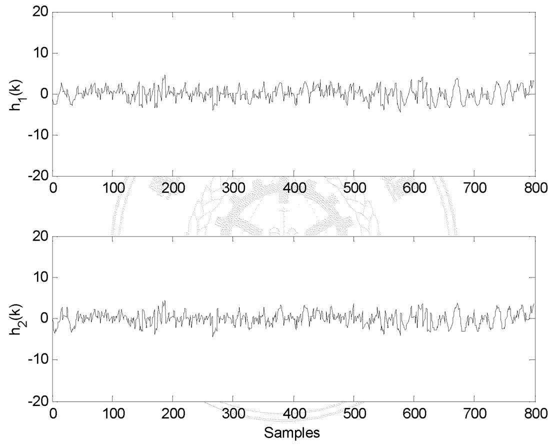


Fig. 3: Controlled output behavior with the initial non-optimal PID controllers in Example 2

(a)



(b)

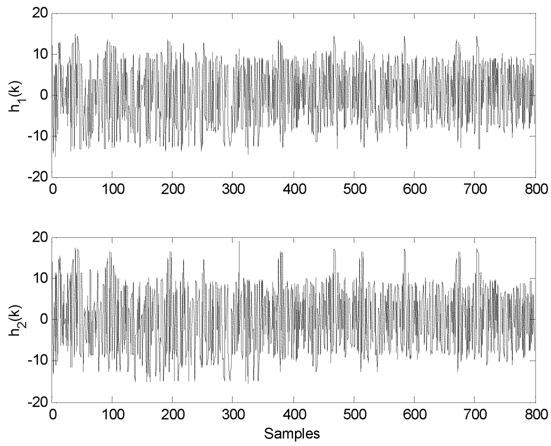


Fig. 4: Controlled output behavior in Example 2 with the achievable PID controllers based on (a) PCC structure; (b) SCC structure