Generation of QFT Plant Template for Systems with Affinely Dependent Parametric Uncertainties

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Abstract

It is well known that the boundary of a plant template of a system with affinely dependent parameters and the parameter domain is a box can be obtained by a finite number sets of one-dimensional parameter sweeping since the boundary of the plant template is included in the image of the set of edges of the parameter domain box. In this paper, an efficient algorithm is proposed to reduce the computational burden for generating the boundary of a plant template of the system. The development of the algorithm relies on using the principal point notion to establish a sufficient condition for testing if the image of a point on an edge of the parameter domain box lies in the interior of the plant template. On the basis of the sufficient condition, the algorithm is developed for identifying, from an edge of the parameter domain box, the set of parameter points whose image lies in the interior of the plant template. The computational burden of the one-dimensional parameter sweeping procedure thus can be obviously reduced by omitting the identified set of parameter points from the edges. A numerical example is included to illustrate the efficiency of the algorithm.

1. Introduction

Quantitative feedback theory (QFT) [1]-[3] is a powerful design technique for robust feedback control systems with plant uncertainties. In QFT, all plant uncertainties are represented in terms of plant templates on a Nichols chart or Nyquist plot. The term plant template is used to denote the collection of frequency responses of an uncertain system at a fixed frequency for all possible uncertainties. The generation of plant templates plays a key role in applying QFT to design robust feedback control systems.

For a plant with parametric uncertainties, the parameter gridding method may be the simplest way to generate the plant template. However, this brute-force method has several disadvantages. First, most of the points obtained using the grid method are useless interior points of the plant template. Second, the computational burden is formidably heavy when the number of uncertain parameters is large. Third, critical points of the template may be missed. In recent years, several efficient algorithms [4]-[13] have been developed for generating plant templates for transfer functions with special forms of parameter dependencies. For an interval plant, i.e., the coefficients of the transfer function are defined on intervals of the real axis, Bartlett *et al.* [7] showed that the plant template boundary is covered by the value sets of 32 one-parameter segment plants. The plant template of an interval plant generated by this characterization contains a lot of interior points. For an exact computation of the template boundary of an interval plant, Brown and Petersen [8], Karamancioğlu and Dzhafarov [12], and Karamancioğlu *et al.* [14] proposed criteria for eliminating the generation of interior points of the template. With the fact that the value sets of the numerator and denominator of an interval plant at a fixed frequency are two independent rectangles, Bailey *et al.* [5] proposed a phase-angle sweeping technique for computing the plant template boundary. Based on a geometric interpretation of the plant template of an interval plant, Hwang and Chen [11] applied a modified Cohen-Sutherland algorithm along with a pivoting procedure to trace the plant template boundary.

For a linear-polytopic uncertain system, i.e., the uncertain parameters in the numerator and denominator of a transfer function are affinely dependent and the parameter domain is a box, Bailey and Hui [4] and Tan and Atherton [15] utilized the geometrical characteristics of the value set of a polynomial family with affinely dependent parameters to develop algorithms for generating the plant template. Those two algorithm are suitable for the case that the uncertain parameters in the numerator and those in the denominator are independent. For the case where the coefficients of the numerator and denominator of a linear-polytopic uncertain system are correlated to each other, Chen and Hwang [9] applied a zero inclusion test algorithm along with a pivoting procedure to trace out the approximate template boundaries. Shen *et al.* [13] improved the algorithm developed by Bailey and Hui [4] to generate the plant template boundary. Bartlett [6] and Fu [10] showed that the plant template boundary is included in the images of the set of edges of the parameter domain box. With the elegant result proposed by Bartlett [6] and Fu [10], the plant template boundary of a linear-polytopic uncertain system can be exactly obtained by a finite number sets of one-dimensional parameter sweeping. However, this approach has the shortcoming that it produces a lot of interior points of the plant template. Owing to the combinatoric explosion of the number of edges of the parameter domain box, it can lead to heavy computational burden when the number of uncertain parameters is large. The purpose of this paper is to present an efficient algorithm for identifying the set of points on an edge of the parameter domain box of a linear-polytopic uncertain system whose image lies in the interior of the plant template. By omitting the identified set of points in the one-dimensional parameter sweeping procedure, the computational burden thus can be obviously reduced.

2. Main results

Consider the transfer function family

$$G(s;\mathbf{q}) = \frac{N(s;\mathbf{q})}{D(s;\mathbf{q})} = \frac{N_0(s) + \sum_{l=1}^{m} q_l N_l(s)}{D_0(s) + \sum_{l=1}^{m} q_l D_l(s)},\ \mathbf{q} \in \mathbf{B} \tag{1}$$

where $N_l(s)$ and $D_l(s)$, $l = 0, \ldots, m$ are polynomials in s and $\mathbf{q} = (q_1, \ldots, q_m)^T$, where the superscript T denotes the transpose of a vector, is an m-dimensional real vector taking values from the m-dimensional box

$$\mathbf{B} = \{\mathbf{q} : q_l \in [q_l^-, q_l^+], l = 1, \ldots, m\} \tag{2}$$

The m-dimensional box $\mathbf{B}$ has 2^m vertices and $m2^{m-1}$ edges. Let the m-digit binary number representation of integer $i \in \{0, 1, \ldots, 2^m - 1\}$ be

$$i = i_1 2^0 + i_2 2^1 + i_3 2^2 + \cdots + i_m 2^{m-1} \tag{3}$$

where $i_l = 0$ or 1 for $l = 1, \ldots, m$. Then, a vertex and an edge of the m-dimensional box $\mathbf{B}$ can be respectively represented as

$$\mathbf{V}_i := \Big\{(v_{i,1}, \ldots, v_{i,m})^T : v_{i,l} = \begin{cases} q_l^-, & \text{if } i_l = 0 \\ q_l^+, & \text{if } i_l = 1 \end{cases}, l = 1, \ldots, m\Big\} \tag{4}$$

and

$$\begin{aligned}\mathbf{E}_{i,k,\ell} := \{&\mathbf{q} : \mathbf{q} = \lambda \mathbf{V}_i + (1-\lambda)\mathbf{V}_k, \lambda \in [0,1], \\ &\ell \in \{1, \ldots, m,\}, i_\ell \neq k_\ell, q_\ell \in [q_\ell^-, q_\ell^+], \\ &i_l = k_l \text{ for all } l \in \mathbf{L}\}\end{aligned} \tag{5}$$

where

$$\mathbf{L} := \{1, \ldots, m\} \setminus \{\ell\} \tag{6}$$

In the QFT terminology, the set of the mapped values $G(j\omega;\mathbf{q})$ for a fixed frequency ω and all $\mathbf{q} \in \mathbf{B}$, i.e.,

$$G(j\omega;\mathbf{B})=\{G(j\omega;\mathbf{q}):\mathbf{q}\in\mathbf{B}\},\ j=\sqrt{-1} \quad (7)$$

is referred to as a plant template. To guarantee that the plant template $G(j\omega;\mathbf{B})$ is bounded, we assume that $D(j\omega;\mathbf{q})\neq 0$ for all $\mathbf{q}\in\mathbf{B}$. In applying QFT to design robust control systems, only the boundarys of the plant templates at different frequencies are required. The boundary of the plant template $G(j\omega;\mathbf{B})$ can be characterized by the following theorem

Theorem 1 [6],[10]: Let ω be a fixed frequency and suppose that the denominator of $G(j\omega;\mathbf{q})$ does not vanish for all $\mathbf{q}\in\mathbf{B}$, i.e., $D(j\omega;\mathbf{q})\neq 0,\mathbf{q}\in\mathbf{B}$. Then,

$$\partial G(j\omega;\mathbf{B})\subset G(j\omega;\mathbf{E}(\mathbf{B})) \quad (8)$$

where ∂ and $\mathbf{E}(\mathbf{B})$ denote the boundary and the set of the edges of $\mathbf{B}$, respectively.

From theorem 1, the boundary of the plant template $G(j\omega;\mathbf{B})$ can be found from the images of the $m2^{m-1}$ edges of the box $\mathbf{B}$ under the transfer function mapping $G(j\omega;\mathbf{q})$. However, this approach to generating the plant template boundary $\partial G(j\omega;\mathbf{B})$ often wastes time in calculating some interior points of the plant template $G(j\omega;\mathbf{B})$ since the image of a point on an edge of the box $\mathbf{B}$ may lie in the interior of the plant template. Therefore, in order to reduce the computational burden for generating the plant template boundary $\partial G(j\omega;\mathbf{B})$, it is desired to develop an efficient method to identify the set of points lying on the edges of the box $\mathbf{B}$ whose image does not contribute to the boundary of the plant template $G(j\omega;\mathbf{B})$. To this end, the principal point notion introduced by Kogan and Leizarowitz [16] and Polyak and Kogan [17] is used. A point $\mathbf{q}\in\mathbf{B}$ is a principal point associated with a mapping $h:\mathbf{B}\to\mathbf{C}$, where $\mathbf{C}$ denotes the complex plane, if there exists a nonzero complex number g such that

$$\mathrm{Im}\{\frac{h_\nu(\mathbf{q})}{g}\}\begin{cases}=0, & q_\nu^-<q_\nu<q_\nu^+\\ \geq 0, & q_\nu=q_\nu^-\\ \leq 0, & q_\nu=q_\nu^+\end{cases} \quad (9)$$

where $\mathrm{Im}\{\cdot\}$ denotes the complex part of the indicated quantity and $h_\nu(\mathbf{q})=\partial h(\mathbf{q})/\partial q_\nu,\nu=1,\ldots,m$. The role of principal points plays in the characterization of the boundary of the value set $h(\mathbf{B})$ is stated in the following theorem [16], [17]:

Theorem 2: If $\eta\in\partial h(\mathbf{B})$, then every $\mathbf{q}\in\mathbf{B}$ such that $\eta=h(\mathbf{q})$ is a principal point.

Theorem 2 indicates that the boundary of a value set is included in the set of images of the principal points associated with the mapping function. Therefore, the boundary of the plant template $G(j\omega;\mathbf{B})$ is included in the set of images of the principal points lying on the edges of the box $\mathbf{B}$ since theorem 1 indicates that the boundary of the plant template $G(j\omega;\mathbf{B})$ is covered in the images of the edges of $\mathbf{B}$. In other words, the points on the edges which are not principal points can be omitted in the plant template generation procedure. According to the definition of principal point and theorem 1 and theorem 2, we have the following sufficient condition for testing if the image of a point on an edge of the domain box lies in the interior of the plant template.

Theorem 3: The image of a point $\mathbf{q}$ on the edge $\mathbf{E}_{i,k,\ell}$ of the box $\mathbf{B}$ associated with the mapping $h(\mathbf{q})=G(j\omega;\mathbf{q})$ lies in the interior of the plant template $G(j\omega;\mathbf{B})$ if the following two conditions hold:

(i) $h_\ell(\mathbf{q})\neq 0$
(ii) The following two values

$$\mathrm{Im}\{\frac{h_\nu(\mathbf{q})}{h_\ell(\mathbf{q})}\}\text{ for each }\nu\in\mathbf{I}_0(i)\setminus\{\ell\} \quad (10a)$$

$$-\ \mathrm{Im}\{\frac{h_\mu(\mathbf{q})}{h_\ell(\mathbf{q})}\}\text{ for each }\mu\in\mathbf{I}_1(i)\setminus\{\ell\} \quad (10b)$$

do not have the same sign, where

$$\mathbf{I}_0(i)=\{l:i_l=0\text{ for all }l\in\{1,\ldots,m\}\} \quad (11a)$$

$$\mathbf{I}_1(i)=\{l:i_l=1\text{ for all }l\in\{1,\ldots,m\}\} \quad (11b)$$

By fairly straightforward derivation, it is found that

$$\mathrm{Im}\{\frac{h_\eta(\mathbf{q})}{h_\ell(\mathbf{q})}\}=c_{1,\eta}q_\ell+c_{0,\eta},\ \eta\in\mathbf{L} \quad (12)$$

where

$$c_{1,\eta}=\mathrm{Im}\left\{\frac{N_\eta(j\omega)D_\ell(j\omega)-N_\ell(j\omega)D_\eta(j\omega)}{\Delta_\ell(\mathbf{q})}\right\} \quad (13a)$$

$$c_{0,\eta} = \mathrm{Im}\left\{\frac{\Delta_\eta(\mathbf{q})}{\Delta_\ell(\mathbf{q})}\right\} \tag{13b}$$

and

$$\Delta_l(\mathbf{q}) = N_l(j\omega)D_0(j\omega) - N_0(j\omega)D_l(j\omega) + \sum_{\substack{k=1\\k\neq\ell}}^{m} q_k\left[N_l(j\omega)D_k(j\omega) - N_k(j\omega)D_l(j\omega)\right], \quad l \in \{1,\ldots,m\} \tag{14}$$

It is noted that for fixed q_l, $l = 1,\ldots,\ell-1,\ell+1,\ldots,m$, the function in (12) is a first order polynomial in q_ℓ. Therefore, we have the following theorem:

Theorem 4: The maximum number of sign change of the following function is one:

$$f_{\ell,\eta}(q_\ell) := \mathrm{Im}\{\frac{h_\eta(\mathbf{q})}{h_\ell(\mathbf{q})}\}\bigg|_{q_l=v_{i,l},\, l=1,\ldots,\ell-1,\ell+1,\ldots,m}, \quad q_\ell \in \mathbf{R},\, \eta \in \mathbf{L} \tag{15}$$

where $\mathbf{R}$ denotes the set of real numbers.

Theorem 4 reveals that for each $\eta \in \mathbf{L}$, once the solution to the equation $f_{\ell,\eta}(q_\ell)$ denoted by $q_{\ell,\eta}$, and the sign of the value of $f_{\ell,\eta}(\tilde{q}_\ell)$ with $\tilde{q}_\ell \neq q_{\ell,\eta}$ is obtained, the sign of $f_{\ell,\eta}$ at any q_ℓ can be determined without having to compute the value of $f_{\ell,\eta}(q_\ell)$.

On the basis of theorem 3 and theorem 4, an efficient algorithm is developed for identifying the set of points on the edge $\mathbf{E}_{i,k,\ell}$ of the parameter domain box $\mathbf{B}$ whose image lies in the interior of the plant template $G(j\omega;\mathbf{B})$. In the algorithm, the set of points identified by the algorithm is denoted by $\Xi_{i,k,\ell}$. The function $sgn\{x\}$ works as

$$sgn\{x\} = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases} \tag{16}$$

The algorithm reads as follows:

Algorithm 1

Step 1. Set $\Lambda := \emptyset$ and $\mathbf{L} := \{1,\ldots,m\} \setminus \{\ell\}$.

Step 2. Compute $\Delta_\ell(\mathbf{V}_i)$. If $\Delta_\ell(\mathbf{V}_i)$ vanishes, go to step 17.

Step 3. Find the solution $q_{\ell,\eta}$ to $f_{\ell,\eta}(q_\ell) = 0$ for each $\eta \in \mathbf{L}$. If $c_{1,\eta} = 0$ and $c_{0,\eta} \neq 0$, set $q_{\ell,\eta} := q_\ell^+ + \epsilon$, $\epsilon > 0$. If $c_{1,\eta} = c_{0,\eta} = 0$, set $\mathbf{L} := \mathbf{L} \setminus \{\eta\}$.

Step 4. Denote by x_ξ, $\xi = 2,\ldots,n-1$ with $x_2 < \cdots < x_{n-1}$ the numbers obtained in step 3 that lie in the open interval (q_ℓ^-, q_ℓ^+). Set $x_1 := q_\ell^-$ and $x_n := q_\ell^+$.

Step 5. Set

$$\mathbf{L}_0 := \{l : q_{\ell,l} \notin [q_\ell^-, q_\ell^+] \text{ for all } l \in \mathbf{L}\} \tag{17a}$$

$$\mathbf{L}_\rho := \{l : q_{\ell,l} = x_\rho \text{ for all } l \in \mathbf{L}\}, \rho = 1,\ldots,n \tag{17b}$$

Step 6. For each $\eta \in \mathbf{L}$, set

$$\sigma_\eta := \begin{cases} sgn\{f_{\ell,\eta}(x^*)\}, & \text{if } \eta \in \mathbf{I}_0(i) \\ sgn\{-f_{\ell,\eta}(x^*)\}, & \text{if } \eta \in \mathbf{I}_1(i) \end{cases} \tag{18}$$

where $x^* = (x_1 + x_2)/2$.

Step 7. If $\sigma_l \neq \sigma_k$ for some $l,k \in \mathbf{L}_0$, set $\Lambda := [q_\ell^-, q_\ell^+]$ and go to step 17.

Step 8. Set

$$\sigma_0 := \sigma_\eta \text{ for some } \eta \in \mathbf{L}_0 \tag{19a}$$

$$\rho := 0 \tag{19b}$$

$$\tilde{\mathbf{L}} := \mathbf{L} \setminus \mathbf{L}_0 \tag{19c}$$

Step 9. Set $\rho := \rho + 1$.

Step 10. If $\sigma_\eta \neq \sigma_0$ for some $\eta \in \tilde{\mathbf{L}}$, set $\Lambda := \Lambda \cup (x_\rho, x_{\rho+1})$.

Step 11. Set $\tilde{\mathbf{L}} := \tilde{\mathbf{L}} \setminus \mathbf{L}_\rho$.

Step 12. If $\sigma_\eta \neq \sigma_0$ for some $\eta \in \tilde{\mathbf{L}}$, set $\Lambda := \Lambda \cup x_\rho$.

Step 13. If $\sigma_\eta \neq \sigma_0$ for some $\eta \in \mathbf{L}_\rho$, set $\Lambda := \Lambda \cup [x_{\rho+1}, x_n]$ and go to step 17.

Step 14. Set $\sigma_\eta := -\sigma_\eta$ for each $\eta \in \mathbf{L}_{\rho+1}$.

Step 15. If $\rho < n-1$ go to step 9.

Step 16. If $\sigma_\eta \neq \sigma_0$ for some $\eta \in \tilde{\mathbf{L}} \setminus \mathbf{L}_n$, set $\Lambda := \Lambda \cup x_n$.

Step 17. Set $\Xi_{i,k,\ell} := \{\mathbf{q} : q_l = v_{i,l} \text{ for all } l \in \mathbf{L}, q_\ell \in \Lambda\}$.

Step 18. Stop.

3. An illustrative example

Example 1: Consider the transfer function family given by Fu [10]

$$G(s;\mathbf{q}) = \frac{N(s;\mathbf{q})}{D(s;\mathbf{q})} = \frac{N_0(s) + \sum_{k=1}^{3} q_k N_k(s)}{D_0(s) + \sum_{k=1}^{3} q_k D_k(s)} \quad (20)$$

where $(q_1, q_2, q_3)^T \in \mathbf{B} = \{\mathbf{q} : q_i \in [-3, 3], i = 1, 2, 3\}$ and

$$N_0(s) = s^2 + 4s + 20 \quad (21a)$$
$$N_1(s) = 0.4s + 1 \quad (21b)$$
$$N_2(s) = 0.2s \quad (21c)$$
$$N_3(s) = -1 \quad (21d)$$
$$D_0(s) = s^4 + 9.5s^3 + 27s^2 + 22.5s \quad (22a)$$
$$D_1(s) = 0.5s^3 + 2s^2 - s \quad (22b)$$
$$D_2(s) = -0.5s^3 + s^2 \quad (22c)$$
$$D_3(s) = 0.5s^3 + s \quad (22d)$$

To illustrate the proposed algorithm we consider the construction of the plant template $G(6j;\mathbf{B})$. For the edge $\mathbf{E}_{1,3,2}$, we have

$$\ell = 2 \quad (23a)$$
$$\mathbf{L} = \{1, 3\} \quad (23b)$$

The solutions to $f_{2,\eta}(q_2) = 0$, $\eta = 1, 3$ are

$$q_{2,1} = 2.745 \quad (24a)$$
$$q_{2,3} = 33.645 \quad (24b)$$

Since $q_{2,1} \in (q_2^-, q_2^+)$ and $q_{2,3} \notin [q_2^-, q_2^+]$, we set

$$x_1 = q_2^- \quad (25a)$$
$$x_2 = q_{2,1} \quad (25b)$$
$$x_3 = q_2^+ \quad (25c)$$
$$\mathbf{L}_0 = \{3\} \quad (26a)$$
$$\mathbf{L}_1 = \emptyset \quad (26b)$$
$$\mathbf{L}_2 = \{1\} \quad (26c)$$
$$\mathbf{L}_3 = \emptyset \quad (26d)$$

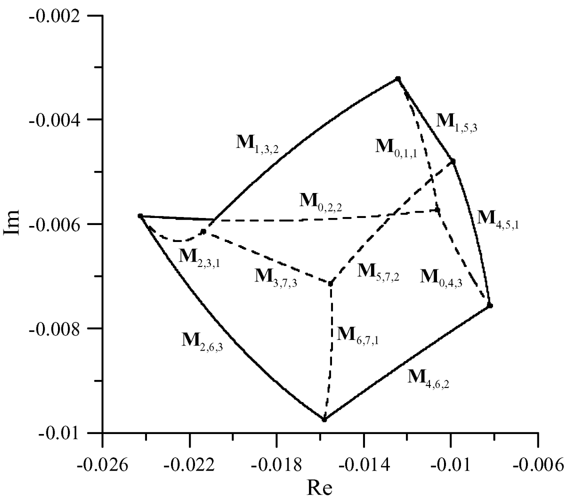

Figure1. Plant template $G(6j;\mathbf{B})$ generated from the images of the edges of the box $\mathbf{B}$, denoted by $\mathbf{M}_{i,k,\ell}$, and the image of the set $\Xi_{i,k,\ell}$ corresponding to each edge $\mathbf{E}_{i,k,\ell}$ of the box $\mathbf{B}$ (dash lines) for the system in example 1.

According to (18), we have

$$\sigma_1 = 1 \quad (27a)$$
$$\sigma_3 = 1 \quad (27b)$$

Following step 7 to step 16 of algorithm 1, we have

$$\Lambda = (x_2, x_3] \quad (28)$$

Therefore, the image of the set

$$\Xi_{1,3,2} = \{\mathbf{q} : q_1 = v_{1,1} = q_1^+,\ q_3 = v_{1,3} = q_3^-,\ q_2 \in \Lambda\} \quad (29)$$

lies in the interior of the plant template $G(6j;\mathbf{B})$. The set $\Xi_{1,3,2}$ thus can be omitted in the one-dimensional parameter sweeping plant template generation procedure. The set Λ corresponding to each edge of the box $\mathbf{B}$ is listed in table 1. Figure 1 shows the plant template $G(6j;\mathbf{B})$ generated from the images of the edges of the box $\mathbf{B}$, denoted by $\mathbf{M}_{i,k,\ell}$, i.e., $\mathbf{M}_{i,k,\ell} = G(6j;\mathbf{E}_{i,k,\ell})$, and the image of the set $\Xi_{i,k,\ell}$ corresponding to each edge $\mathbf{E}_{i,k,\ell}$ of the box $\mathbf{B}$. It can be seen from figure 1 that all images of the sets identified from the edges by algorithm 1 lie in the interior of the plant template. To show the efficiency of algorithm 1 the following indicator is defined

$$\phi := \frac{\sum_{\text{all edges}} \text{length of } \Lambda}{\sum_{\text{all edges}} \text{length of edge}} \qquad (30)$$

The value of ϕ indicates how much computational effort can be reduced by algorithm 1. At $\omega = 6$, the value of ϕ is 0.573. That is, algorithm 1 reduces up to 57.3% the computational effort for generating the plant template $G(6j; \mathbf{B})$. The values of ϕ at different frequencies are listed in table 2. Table 2 reveals that algorithm 1 saves at least 50% computational effort for generating the plant templates. It does reduce a lot of the computational burden.

Table 1. The set Λ corresponding to each edge of $\mathbf{B}$ for the construction of $G(6j; \mathbf{B})$ in example 1.

Edge	Λ	Edge	Λ
$\mathbf{E}_{0,1,1}$	$[-3, 3]$	$\mathbf{E}_{4,6,2}$	$\emptyset$
$\mathbf{E}_{2,3,1}$	$[-3, 3]$	$\mathbf{E}_{5,7,2}$	$[-3, 3]$
$\mathbf{E}_{4,5,1}$	$\emptyset$	$\mathbf{E}_{0,4,3}$	$[-3, 3]$
$\mathbf{E}_{6,7,1}$	$[-3, 3]$	$\mathbf{E}_{1,5,3}$	$\emptyset$
$\mathbf{E}_{0,2,2}$	$[-3, 1.977)$	$\mathbf{E}_{2,6,3}$	$\emptyset$
$\mathbf{E}_{1,3,2}$	$(2.745, 3]$	$\mathbf{E}_{3,7,3}$	$[-3, 3]$

Table 2. The values of ϕ at different frequencies associated with the transfer function family in example 1.

ω	ϕ	ω	ϕ	ω	ϕ
0.5	0.500	3.0	0.667	5.5	0.570
1.0	0.500	3.5	0.667	6.0	0.573
1.5	0.500	4.0	0.647	6.5	0.572
2.0	0.502	4.5	0.591	7.0	0.571
2.5	0.500	5.0	0.585	7.5	0.570

4. Conclusions

In this paper, an effective algorithm has been presented for reducing the computational burden in the generation of QFT plant template for systems with affine linear uncertainties. The algorithm is based on a new sufficient condition established by the principal point notion for the image of a point on an edge of the parameter domain box to lie in the interior of the plant template. By eliminating, from the set of edges of the parameter domain box, the set of the points identified by the proposed algorithm, the computational burden for generating the plant templates of systems with affine linear uncertainties can be greatly reduced. It facilitates applying the QFT technique to systems having affinely dependent uncertainties.

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References

[1] I. M. Horowitz. Survey of quantitative feedback theory. *International Journal of Control,* 53(2): 255-291, 1991.

[2] I. M. Horowitz. *Quantitative Feedback Design Theory* QFT Publications, Boulder, Colorado, U.S.A., 1992.

[3] I. M. Horowitz, and M. Sidi. Synthesis of feedback systems with large plant ignorance for prescribed time-domain tolerance. *International Journal of Control,* 16(2): 287-309, 1972.

[4] F. N. Bailey, and C. H. Hui. A fast algorithm for computing parametric rational functions. *IEEE Transactions on Automatic Control,* 34(11): 1209-1212, 1989.

[5] F. N. Bailey, D. Panzer, and G. Gu. Two algorithms for frequency domain design of robust control systems. *International Journal of Control,* 48(5): 1787-1806, 1988.

[6] A. C. Bartlett. Nyquist, Bode, and Nichols plots of uncertain systems. *Proceedings of the American Control Conference,* San Diego, USA, pp. 2033-2036, 1990.

[7] A. C. Bartlett, A. Tesi, and A. Vicino. Frequency response of uncertain systems with interval plants. *IEEE Transactions on Automatic Control,* 38(6): 929-933, 1993.

[8] M. Brown, and I. R. Petersen. Exact computation of the Horowitz bound for interval plants. *Proceedings of the 30th Conference on Deci-*

sion and Control, Brighton, England, pp. 2268-2273, 1991.

[9] J. J. Chen, and C. Hwang. Computing frequency responses of uncertain systems. *IEEE Transactions on Circuits and Systems – I: Fundamental Theory and Applications,* 45(3): 304-307, 1998.

[10] M. Fu. Computing the frequency response of linear systems with parametric perturbation. *Systems & Control Letters,* 15(1): 45-52, 1990.

[11] C. Hwang, and J. J. Chen. Computation of the frequency response of interval systems. *Circuits, Systems, and Signal Processing,* 15(11): 1-12, 1996.

[12] A. Karamancioğlu, and V. Dzhafarov. A sectoring algorithm for exact computation of the frequency response of linear interval systems. *Circuits, Systems, and Signal Processing,* 17(4): 483-493, 1998.

[13] S. K. Shen, B. C. Wang, and T. T. Lee. An improved algorithm for computing the boundary of parametric rational functions. *IEEE Transactions on Automatic Control,* 44(1): 227-231, 1999.

[14] A. Karamancioğlu, V. Dzhafarov, and C. özemir. Frequency response of PID-controlled linear interval systems. *Circuits, Systems, and Signal Processing,* 15(6): 735-748, 1996.

[15] N. Tan, and D. P. Atherton. Frequency response of uncertain systems: A 2q-convex parpolygonal approach. *IEE Proceedings - Control Theory and Applications,* 147(5): 547-555, 2000.

[16] J. Kogan, and A. Leizarowitz. Frequency domain criterion for robust stability of interval time-delay systems. *Automatica,* 31(3): 463-469, 1995.

[17] B. T. Polyak, and J. Kogan. Necessary and sufficient conditions for robust stability of linear systems with multiaffine uncertainty structure. *IEEE Transactions on Automatic Control,* 40(7): 1255-1260, 1995.