GTMM in Analyzing Flexible Rotor System with Flexible Coupling and Foundation

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Abstract

The flexible coupling and flexible foundation in the research of flexible rotor were ignored in many studies. This will make significant error in analyzing critical speed and mode shape of flexible rotor. Traditionally, Finite Element Method (FEM) and Transfer Matrix Method (TMM) had been widely used to establish the model of the rotor system. The purpose of this paper is to establish a flexible rotor model considering the flexible coupling and flexible foundation basing on the General Transfer Matrix Method (GTMM). A more complete research was carried out in this paper. The unbalancing of shaft is considered as curve distributed in space in GTMM. The analytical solution of the flexible rotor's model could be solved with 16 state variables, and the steady state behavior could be analyzed with fixed sized matrix, which is provided with more accurate solution, lesser storage space, and faster calculation speed. In order to approach the real flexible rotor's situation, we derived the coupling model in a continuous sense with considering the mass effect, stiffness, deflection and boundary conditions; established the foundation model, including both flexible foundation-plate and vibration isolator. Owing to considering the modeling of flexible coupling, motor could be integrated into the rotor system, which could increase the reasonableness of the model. The bearing analysis will become quite complicated while considering the modeling of flexible bearing and vibration isolator, but it will increase the completeness of the model. In this paper, we discuss (1) the modeling of flexible coupling and flexible foundation; (2) constructing a complete rotor model by integrating with the modeling of flexible coupling and flexible foundation into it; (3) taking a real rotor machine to be our case study. The influence of flexible coupling and flexible foundation on the rotor system will be discussed.

1. Introduction

In TMM, it has the advantages of less storage space consumption and fast calculation speed while analyzing the steady state response with fixed sized matrix in frequency domain. Koenig [1]、Guenther and Lovejoy [2] analyze the rotating shaft system based on TMM, with considering damping and stiffness in fluid-film bearing. Lund [3]、Bansal and Kirk [4] analyze the mode Shape in rotor system with TMM, and evaluate the stability of the system according to its natural frequency of damping. Lund [5] uses TMM to evaluate how the sensitivity of critical rotating speed of rotors varies with its design factor. In the researches mentioned before, the lump model of rotor could be built according to individual centralized mass, and two adjacent points connected by a flexible shaft without considering its mass. The deflection 、angle 、inner moment, and shear stress at the mass point of shaft are considered as state variables. Lund and Orcutt [6] modified the discrete TMM as stated before, and use continuous TMM to describe the relationship between rigid disk and bearing. In this way, the degree of freedom will be decreased and the time consumption during calculating the transfer matrix will be reduced. Because of only eight state variables are applied, so it could just describe pure circular gyroscopic trajectory without considering gyroscopic moment, non-isotropic bearing, and symmetry of system. In this way, the assumed pure circular orbit will not match the pseudo-elliptical orbit actually. Gu [7] analyzes the transient response with a method which combine continuous TMM and direct integration method, this method takes advantage of both continuous TMM and direct integral method, but it only uses eight state variables and pure circular orbit. Lee, Kang, and Liu [8] describe the steady state response with 16 state variables according to the Pseudo-elliptical orbit and create a continuous shaft transfer matrix with solution, and considering the influence of gyroscope effects. Lee and Shih [9] create the GTMM which simulates the distribution of shaft unbalance in three-dimensional space. In the method, the rotor system contains rigid disk, linear bearing, and flexible shaft. There are lots of factors considered in dynamic equation of flexible shaft, such as the effect of gyroscope, bending moment, shear stress, rotating inertia, axial force, and torque etc, thus the continuous TMM will make the modeling of the shaft more complete. Lee and Shih [10,11] also create estimation equations to estimate the stiffness and damping of bearing, and the unbalance distribution of flexible shaft in the way mentioned above in the further study. In the researches about the flexible coupling and flexible foundation, [12] and [13] considering the factor of flexible support, in [12], it proposes a method which is similar to modal analysis to analyze the flexible support of the rotor system, it uses spring and damping to simulate the flexible support, it only considers the factor of vibration isolator and analyzes the flexible shaft with the technique of Bernoulli Beam, without considering the influence of flexible foundation-plate；in [13], it uses the same way as [12] to analyze the flexible shaft but uses Timoshenko beam instead. In the modeling of the flexible coupling, [14] uses spring to simulate the flexible coupling system, without considering the mass and the boundary condition of the coupling itself.

In this paper, we focus on the numerical simulation and the theoretical model construction of the flexible coupling, flexible foundation, and combine the research of rotor system before. The analysis of flexible shaft, rigid disk, linear and nonlinear bearing are included in the research of rotor system. The mass, stiffness, deflection, and boundary conditions are considered in analyzing the flexible coupling; and the flexible foundation-plate and vibration isolator are included in analyzing the flexible foundation, in this way, the modeling and simulation of flexible coupling and flexible foundation will be closer to reality. The GTMM proposed in this paper not only modifies the model of the rotor system theoretically, but also makes the numerical analysis and modeling of rotating machine more complete practically.

2. Formulation

In TMM, the transfer matrix of every component of a rotor system should be derived first. Generally speaking, a rotor system consists of Shaft, Disk (or Impeller), Bearing, and Coupling. The flexible foundation-plate and flexible support (or isolator)are considered in the paper. The transfer matrixes are derived as state below：

(1) The transfer matrix of shaft, disk, and flexible support

- The transfer matrix of both sides for a flexible shaft:

$$[Sr]_{17\times1}=[T]_{17\times17}[Sl]_{17\times1} \quad (1)$$

- The transfer matrix of both sides for a disk :

$$[Sr]_{17\times1}=[Td]_{17\times17}[Sl]_{17\times1} \quad (2)$$

- The transfer matrix of both sides for a flexible support:

$$[Sr]_{17\times1}=[Tb]_{17\times17}[Sl]_{17\times1} \quad (3)$$

Where

$$[s]=[X_c\ X_s\ Y_c\ Y_s\ \alpha_c\ \alpha_s\ \beta_c\ \beta_s\ M_{xc}\ M_{xs}\ M_{yc}\ M_{ys}\ Q_{xc}\ Q_{xs}\ Q_{yc}\ Q_{ys}\ 1]^T,$$

details and derivation of [T], [Td] and [Tb] are listed in [9].

(2) The transfer matrix of flexible coupling

For rigid coupling and flexible coupling, it uses small element to define the rotor element. The model is derived from force equilibrium, and considers the mass, rotating inertia, gyroscopic effects, bending moment, deforming by shear stress, and shaft unbalance of the coupling itself. The rigid coupling (Figure 1) and flexible coupling (Figure 2) could be modeled as Figure 3，The transfer matrix is derived as state below：

$$[S_r]_{17\times1}=[T_c]_{17\times17}[S_l]_{17\times1} \qquad (4)$$

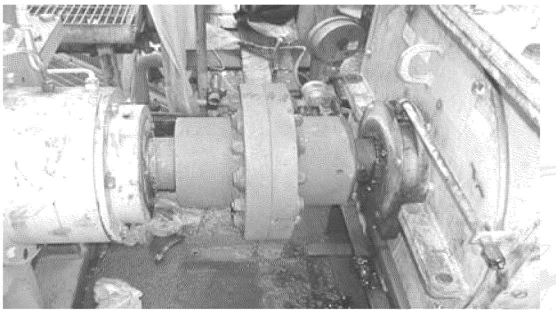

Figure 1: the rigid coupling

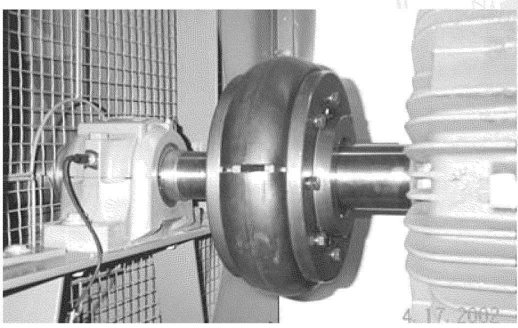

Figure 2: the flexible coupling

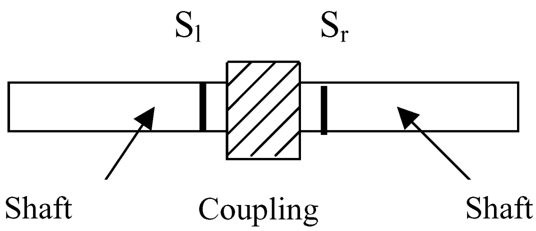

Figure 3: the states at the two end of the coupling

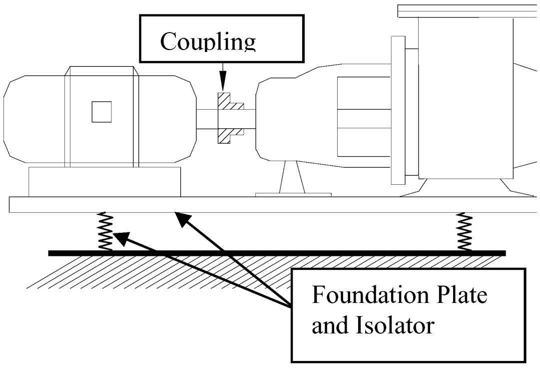

Fig. 4 The Flexible Rotor System

(3) The transfer matrix of flexible foundation-plate connected with bearing

The flexible rotor system refer to Figure 4. and the states variables at bearing are marked as Figure 5. The states transfer in flexible foundation-plate connected with bearing differs from the traditional transfer matrix, because the flexible foundation-plate is not fixed. And then the number of transferred state variable becomes 32 and add a non-state-variable, so the transferred vector is 33×1. The transfer matrix is shown as below：

$$[S_r]_{33\times1}=[T_{bf}]_{33\times33}[S_l]_{33\times1} \qquad (5)$$

where

$$[S_r]_{33\times1} == \begin{bmatrix} [S_{sr}]_{16\times1} \\ [S_{fr}]_{16\times1} \\ 1 \end{bmatrix} \qquad (6)$$

and

$$[S_l]_{33\times1} == \begin{bmatrix} [S_{sl}]_{16\times1} \\ [S_{fl}]_{16\times1} \\ 1 \end{bmatrix} \qquad (7)$$

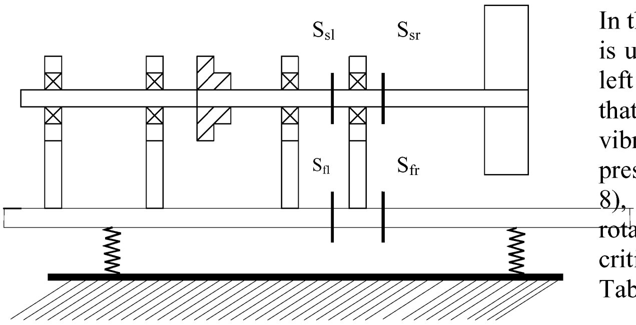

Figure 5: the states of flexible foundation-plate

After the transfer matrixes between the left and right sides are derived for each component described before, the overall transfer matrix of the rotor can be obtained from on end step by step to another end. The states of two free ends in rotor system are shown in Figure 6, and the overall transfer matrix is shown as below：

$$[S_{rn}]_{33\times1}=[T_{total}]_{33\times33}[S_{l0}]_{33\times1} \tag{8}$$

where

$$[S_{rn}]_{33\times1} == \begin{bmatrix} [S_{sn}]_{16\times1} \\ [S_{fn}]_{16\times1} \\ 1 \end{bmatrix} \tag{9}$$

$$[S_{l0}]_{33\times1} == \begin{bmatrix} [S_{s0}]_{16\times1} \\ [S_{f0}]_{16\times1} \\ 1 \end{bmatrix} \tag{10}$$

The right end and the left end of the system are free ends, so the shear stress and bending moment in $[S_{rn}]$ and $[S_{l0}]$ are equal to 0 and apply to Equation (8), and the state variable of $[S_{l0}]$ could be obtained. The state variable of any point in the rotor system could be derived by the relationship between that point and $[S_{l0}]$

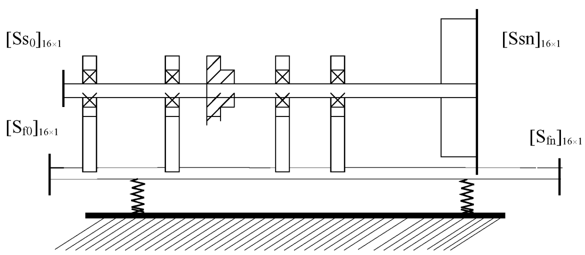

Figure 6: the states of rotor system

In this paper, a centrifugal pump shown as Figure 7 is used for numerical analysis. One impeller at the left side of the rotor (refer to Figure 7) and it seems that a cantilever beam hung with a mass. The vibration isolator pad is aided into the pump. It presents a real sized simulation (shown as Figure 8), and discusses the error in analyzing the critical rotating speed (displayed in Figure 9). The exact critical speeds of the numerical results are listed in Table 1.

Table 1. The Exact Critical Speeds of numerical case

	Coupling Effects are ignored	Flexible Coupling
Critical Speeds	7323 RPM (1st) 19956(2nd)	7361 RPM (1st) 19973 RPM (2nd) 23512 RPM (3rd)

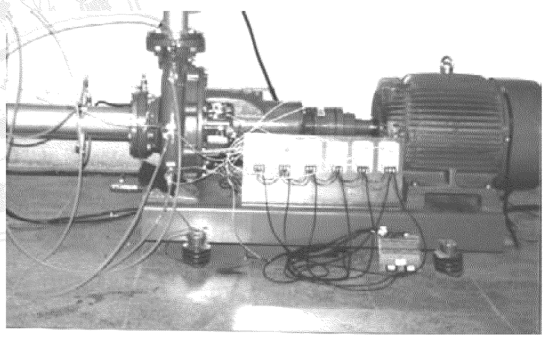

Figure 7: centrifugal pump

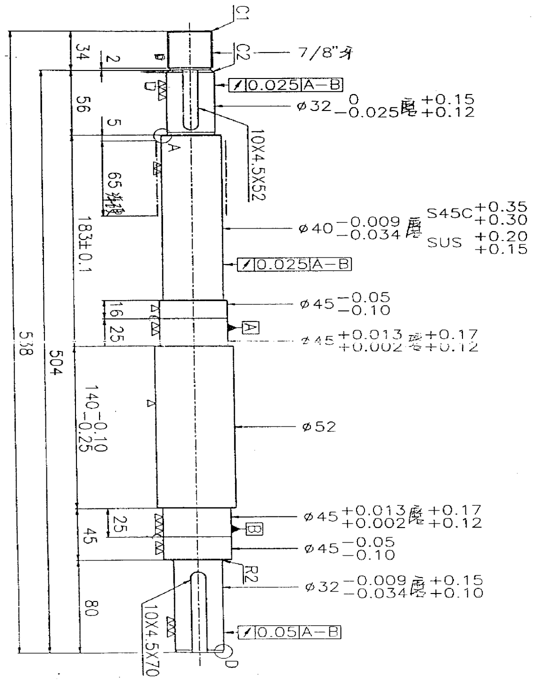

Figure 8: the dimensions of centrifugal pump

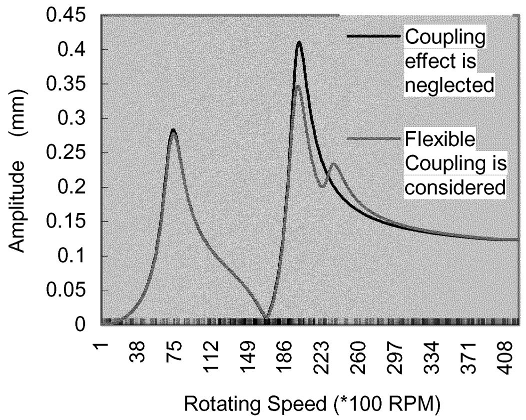

Figure 9 The Vibration Response at the Impeller

3. Conclusion

In this case, after the flexible coupling is adding into the formulation, the results are shown that first mode and second mode are little higher for about less 1%. It is reasonable for the phenomena, due to the coupling end of the shaft becomes stiffer. But it is worth our attention that the third critical speed (23512 RPM) appears and adds in the result. When coupling is considered into the rotor system, it is equivalent to that the shaft of motor is involved and be formulated into the rotor system. That is, the rotor system has been changed. Any error for neglecting the coupling effect will be probably happened.

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