An Application of the Empirical-Distribution-Based Model on the Implied Volatility of Taiwan Warrants

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Theories such as jump process, stochastic volatility, the GARCH model, and implied risk-neutral distribution have been developed to account for the volatility smile. Nevertheless, none of them succeeds in solving the smile problem.

A newly creative empirical-distribution-based model (EDB model) which uses a histogram constructed from past asset prices has been applied to the S&P 500 index and it eliminates the degree of smile and the price difference. This study applies the same methodology on the TSM and UMC call warrants on the Taiwan stock market to compare its pricing and volatility smile with those derived from the Black-Scholes model. The results show that the degree of smile is not as great in the EDB model with a long historical horizon as in the BS model. Using the average value of the implied volatility as a standard deviation, the fitted prices were computed. The actual option price and both the fitted prices from the BS model and EDB model are all overpriced after examining the sell-naked profit. The profit from the EDB model is lower than the profit from the BS model and from the actual market price. This overpricing is more serious for the in-the-money than the out-of-the-money warrants and is less serious if longer historical data is used.

Keywords：Warrants, Implied Volatility, Volatility Smile, Histogram.

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台灣認購權證隱含波動的探討—實質分配模型的應用

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許多理論模型，諸如跳躍過程、隨機波動、GARCH 模型和隱含中性風險分配模型等，已被發展出來因應隱含波動的問題。然而沒有一個成功地解決了波動微笑(volatility smile)的問題。

利用過去資產歷史價格的統計圖表(histogram)所建構出的所謂實質分配模型(EDB model)，應用在 S&P 500 指數可減低微笑的程度和理論價格與市場價格的差距。EDB model 是一個創新的模型，已有多個應用此模型在債券和其它選擇權的研究正在進行中。本文應用修正過的 EDB model 在台灣股票市場的台積電與聯電認購權證上，以比較在波動微笑與價格差距上 EDB model 與 Black-Scholes model 的異同。研究結果發現，若使用長歷史水平的資產價格為基礎，則 EDB model 所導出的微笑幅度比 Black-Scholes model 的小。以隱含波動的平均值當做標準差來計算權證的理論適配值，結果發現依 EDB model 和 Black-Scholes model 所計算出的理論適配值與實際市場價格皆呈現價格過高的現象。由 EDB model 的理論適配值所得到的裸售利潤比其他二者低。這種超額利潤的現象價內權證比價外權證嚴重；同時使用短歷史水平的資產價格比使用長歷史水平的資產價格所計算出的理論適配值，其超額利潤也較高。

關鍵詞：認購權證、隱含波動、波動微笑、歷史價格的統計圖。

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1. Introduction

Since the introduction of the Black-Scholes model (1973), researchers have studied the empirical performance of the model. Early studies find that after comparing market prices and predicted prices, the model systematically miscalculates (or biases) the impact of strike prices on option prices. Starting in the early 1990's, researchers focused on the corresponding biases in implied volatility. The strike price bias, termed "volatility smile," considers the relationship between strike prices and implied volatility. The volatility smile that is generated by the Black-Scholes model can be attributed to either of the following reasons. First, the normality assumption of the return distribution of the underlying asset is inappropriate. Second, in- and/or out-of-the-money options are indeed overpriced.

Several studies attempted to resolve the strike biases in implied volatility by introducing different specifications into the distribution of the underlying asset to account for the fat tail phenomenon. Those specifications included the jump-diffusion (Naik and Lee 1990 and Bates 1991), stochastic volatility (Hull and White 1987, Johnson and Shanno 1987, Wiggins 1987, Heston 1993, Ritchen and Trevor 2000), combined jumps and stochastic volatility (Scott 1997, and Bakshi, Cao, and Chen 1997), implied risk neutral distribution (Shimko 1993, Derman and Kani 1994, Dupire 1994, Rubinstein 1994), GARCH process (Duan 1995, Kallsen and Taqqu 1998, Ritchen and Trevor 2000), and hyperbolic distribution (Eberlein, Keller, and Prause 1998). Some researchers even attributed the smile to the non-fundamental factors of the market (Longstaff (1995) Jackwerth and Rubinstein (1996), Dumas et al. (1998), Pena et al. (1999)). However, Das and Sundaram (1999) indicated that incorporating these features mitigated, but did not eliminate, the smile.

Instead of proposing a theoretical return distribution, Chen and Palmon (2002) hypothesized that traders priced options using historical return distribution. Hence, they constructed a histogram from past S&P 500 daily returns and used it to price S&P 500 options. They found that their empirical-distribution-based model (EDB model) predicted option premiums considerably better than the Black-Scholes model (BS model) and it successfully eliminated the smile for the in-the-money options. They also found that out-of-the-money options were overpriced.

In this study, the same methodology was applied on the call warrants in the Taiwan stock market. First, the implied volatility was computed by the BS model and the EDB model. Using the average value of the implied volatility as a standard deviation, the fitted prices were computed. Then the problem of overpricing was examined by checking the sell-naked profit of the fitted price and the actual price.

The organization of this paper is as follows. In section 2, the warrants under study were explained and the related studies were reviewed. In section 3, the implied volatility and the fitted prices using both the BS model and the EDB model were computed. Also the volatility smile was checked and the sell-naked profit was calculated. Concluding remarks are given in section 4, and the EDB model is explained in the appendix.

2. Warrant in this study

The first warrant on the Taiwan stock market was offered on September 4, 1997. Warrants were issued by brokerage firms and traded in the stock market. By the end of 2000, 126 warrants had been issued. All warrants in the market are call warrants. Put warrants are not permitted. Among the call warrants, 110 have a single stock as its underlying asset and 16 have mixed stocks as its underlying asset. Among the former, 8 warrants have Taiwan Semiconductor Manufacturing (symbol: TSM) as their underlying asset while 6 warrants have United Microelectronics (symbol: UMC). Overall, those two are the most popular warrants in the market. In addition, the ADR of the TSM and the UMC are listed in New York Stock Exchange (NYSE). Due to the similarity of these companies and their exposure to the international market, their warrants have been chosen for analysis in this study.

All of the warrants expire in one year except for two –Jihsun 01 and Kingwha 02 that cover a year and half. Fubon 10 and Polaris 16 were dropped due to an incomplete data set since they were issued at the end of November 2000. Therefore, a total of 12 warrants are included in this study, 6 each for TSM and UMC. Table 1 shows the details of the warrants under study. Total observations are 1811 for TSM and 1600 for UMC.

The warrant in the Taiwan stock market is an American option. Yet, since the strike price is adjusted when the dividend is distributed and there is a tax disadvantage[1] on early exercise, investors usually do not exercise the warrants before the expiration date. These properties make the warrant European style. Investors can realize a profit by selling the warrant in the market.

So far, the pricing models discussed in the articles relating to the Taiwan warrants are limited to the Black-Scholes model (俞明德等 1999，李怡宗等 1999，施東河、王勝助 2001), the jump-diffusion model (俞明德等 1999，林丙輝、王明傳 2001), the CEV model (徐宗德等 1998，詹錦宏等 1999), binomial model (許溪南、張博彥 2002), and the GARCH model (巫春洲 2002). Although a few papers studied implied volatility (李進生等 2000，俞明德等 1999), they all use Black-Scholes formula to derive the option prices. 李進生等 (2000) examined the forecasting power of the five models which adopted historical volatility, Black-Scholes implied volatility, and ARCH, GARCH and random walk volatility in the model. Using the 16 warrants from September 1997 to March 1999, they found that the historical volatility model had better forecasting power than the implied volatility model. 李怡宗等(1999) had tried to find the factors which caused the difference between the market price and the theoretical price. The BS model was used to derive the theoretical price. The results showed that the Black-Scholes implied volatility affected the difference significantly. These two and other articles indicated that the Black-Scholes implied volatility was overvalued. Yet, no article has proven or discussed the presence or absence of a volatility smile.

[1] Refer to 楊淑卿(2003).

3. The empirical results

In the BS model, one of the assumptions is that the underlying asset does not yield a dividend. Both TSM and UMC traditionally distributed dividends every year and the strike price is adjusted for the dividend. For example, if the ex-dividend date is on 5/9/2000 for the UMC stock and the dividend rate is 20%, therefore, the stock price and strike price would be both adjusted by 20% on 5/10/2000. Under this situation, the traditional BS model was applied to the Taiwan warrant as if it is a European option.

In this study, the historical stock prices used to construct a histogram for TSM are from 1994/9/5 to 2001/5/11 covering 1846 data set and for UMC are from 1986/11/06 to 2001/5/11 covering 4461 data set. 1994/9/5 and 1986/11/06 are the dates when TSM and UMC stocks respectively went public. Both price series had been adjusted to the dividend rate every year. The summary statistics of TSM and UMC stock prices are shown in Table 2. Table 2 shows that the average daily return and standard deviation are 0.17% and 0.0269 for TSM stock while they are 0.16% and 0.0283 for UMC stock. Two important characteristics of these historical distributions have been noted. First, the return distributions present fat tails (extra kurtosis). Second, the extra kurtosis decreases as the holding period lengthens.

Using the BS model, implied volatility (σ_{BS}) for both warrants[2] was computed. For TSM, 1132 out of a total of 1811 observations (62.51%), the calculated implied volatility is 0. For UMC, 663 out of a total of 1600 observations (41.44%), the calculated implied volatility is 0. This implies that the actual warrant prices are lower than their intrinsic values- an obvious arbitrage opportunity. This happens in the sample probably because the daily warrant and stock prices are not synchronized. This non-synchronization can be attributed to the low liquidity and the weak-form efficient market in Taiwan[3]. Although, deleting these observations may result in a selection bias in favor of observations with a relatively high implied volatility, these observations were dropped since the ratios were so high. The data left for further study is 679 for TSM and 937 for UMC. The distribution of the moneyness of these two warrants is listed in Table 3. Table 3 shows that 163 observations are in-the-money and 516 observations are out-of-the-money for TSM while 152 observations are in-the-money and 785 observations are out-of-the-money for UMC.

In the next step, the implied volatility, σ_{EDB}, from the EDB model was derived for the remaining data. The volatility in this model were allowed to vary but other moments are kept fixed. Using the model described in the appendix, the implied volatility using the past historical specification of the distribution was generated. Two time horizons, 4 years and 2 years, for TSM and 3 time horizons, 10 years, 4 years and 2 years, for UMC[4] were chosen. The summary statistics of both implied volatility, σ_{BS} and σ_{EDB}, are presented in

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[2] Price of warrants and stocks is from the Taiwan Economic Journal data bank. Interest rate is 90 days bank rate from AREMOS data bank.

[3] The trading volume of warrants in Taiwan is usually much smaller than that of stocks. Also, the brokerage firms did not perform well as a market maker even when the liquidity is very low or zero.

[4] Stock of TSM went public on 9/5/1994 and its first warrant was traded on 4/5/1999. Therefore its maximum historical horizon was 4.5 years.

Table 4 and 5, respectively. Table 4 shows that the average volatility and its standard deviation of σ_{BS} are 0.6333 and 0.2604 for TSM warrants and are 0.6578 and 0.2055 for UMC warrants. Table 5 shows that the average volatility and its standard deviation of σ_{EDB} are 1.0939 and 0.3437 for a 4 year horizon and 1.1204 and 0.4566 for a 2 year horizon for TSM warrants. These data are 0.9016 and 0.2438 under 10 year horizon, 0.9411 and 0.2667 for a 4 year horizon, 0.9412 and 0.2920 for a 2 year horizon for UMC warrants. It can be seen that the longer the horizon the smaller the average and standard deviation of the $\sigma_{EDB.}$

To examine the smile problem, the following regressions were employed:

$$\begin{cases}\hat{\sigma}_{BS} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e \\ \hat{\sigma}_{EDB} = a + b_1 M + b_2 M^2 + b_3 M^3 + b_4 M^4 + e\end{cases} \tag{1}$$

where $\hat{\sigma}_{BS}$ and $\hat{\sigma}_{EDB}$ are the annualized implied volatilities derived from the BS model and the EDB model, respectively. The variable M is the moneyness, which is defined as $M = (S - K)/S$. The third and fourth powers of the moneyness measure were included in our regressions so not to restrict the quadratic shape of the smile. The estimates from the regression were presented in Table 6 and 7. Table 6 shows that the b_2s, the coefficients of M^2 in the $\hat{\sigma}_{BS}$ equation, are 0.8897 for TSM and 0.5347 for UMC. Table 7 shows that the coefficients in the $\hat{\sigma}_{EDB}$ are 0.7703 and 0.8951 for 4 year horizon and 2 year horizon respectively for TSM warrants, while they are 0.2202, 0.3071 and 0.6198 for 10 year, 4 year and 2 year horizon respectively for UMC. In Figure 1, the fitted volatility derived from the BS model and the EDB model was plotted against moneyness. The figure reveals that the EDB model with longer horizon, that is 10 years and 4 years, generated a smaller (flatter) smile than that from the BS model. However, the EDB model with a 2 year horizon generated a bigger smile than that from the BS model.

Next the profits generated by selling naked options using the actual prices and the option prices generated from the BS model and the EDB model were calculated. The profit of the short naked call strategy is defined as:

$$\begin{aligned}\Pi_t &= C_t - e^{-k_{T-t}^C} C_T \\ &= C_t - e^{-k_{T-t}^C} \max\{S_T - K, 0\}\end{aligned} \tag{2}$$

where the risky discount rate k_{T-t}^C is the annualized average of the $k_{t,T}$'s in the $T-t$ day sample. The results are summarized in Table 8. This table presents the average present values of the profit from selling naked options for various moneyness categories.

Positive profits were found in any moneyness for TSM and UMC in the 3 cases. The positive profit generated from the actual price implies that the warrants are overpriced. On average, TSM using BS model generated 13.64% higher profit than that from the actual price (17.6202 vs. 20.0231) while the profit generated from the EDB model is close to that from the actual price if 4 year horizon is used (17.6202 vs. 17.6885). Yet, the profit is 21.38% higher for the 2 year horizon (17.6202 vs. 21.3870). As for UMC, on average, using the BS model generated 4.75% higher profit than that from the actual price (10.8181 vs. 11.3307) while the EDB model generated 8.62% and 7.65% lower profit than that from the actual price for the 10 year and the 4 year horizon respectively (9.8852 and 9.9903). Yet the profit is 6.96% higher for the 2 year horizon (10.8181 vs. 11.5708).

The relationship between the profits and the moneyness were also examined. For the in-the-money and the out-of-the-money warrants, the BS model generated higher profit than that from the EDB model for both TSM and UMC warrants under the 10 year and 4 year horizon, but not for the 2 year horizon. The only exception is the in-the-money warrant for TSM under the 2 year horizon. Table 8 and Figure 1 show that overvaluation is much more serious for the in-the-money than the out-of-the-money warrants in the 3 cases of UMC.

From these results, a conclusion can be reached that the EDB model is better than the BS model in terms of generating the price that is close to the actual with less degree of volatility smile. In addition, the overprice problem is not as serious in the EDB model as in the BS model and even dominates the actual price. The longer the horizon the better the performance indicating that longer historical data is preferable.

4. Conclusion

In this study, the same methodology as in the Chen and Palmon (2002) is applied on the TSM and UMC warrants in the Taiwan stock market. The implied volatility was computed by the BS model and the EDB model. The results from the regression show that the degree of smile is not so great in the EDB model with a long historical horizon as in the BS model. Using the average value of the implied volatility as a standard deviation, the fitted prices were computed. After checking the sell-naked profit, it is found that the actual option price and both the fitted prices from the BS model as well as the EDB model are all overpriced and that the profit generated from the EDB model with a long horizon is less than that from the BS model. Moreover, it is even lower than the profit from the actual price. The degree of overprice is more serious for the in-the-money than the out-of-the-money warrants. The overprice phenomenon that existed in the actual price can attribute to the tax disadvantage to the issuer of the warrants and the weak-form efficient stock market in Taiwan. The results show that longer historical data proves to be more useful.

Table 1: Warrants with TSM and UMC as its underlying asset

TSM					UMC				
code	period	Issued price	Exercise price	# of obs.	code	period	Issued price	Exercise Price	# of obs.
日盛 01	1999/4/15-2000/10/14	36.36	65.48	373	元富 01	1998/3/19-1999/3/18	19.85	70.48	268
日盛 04	1999/8/3-2000/8/2	31.2	95.70	266	大華 07	1999/6/11-2000/6/10	13.68	41.31	264
京華 02	1999/9/18-2001/3/17	47.54	108.59	392	大華 10	1999/11/1-2000/3/18	21.87	67.50	267
中信 02	1999/12/1-2000/11/30	34.88	121.09	268	寶來 11	1999/11/30-2000/11/29	20.70	75.00	268
寶來 13	2000/5/2-2001/5/1	38.75	191.95	263	富邦 05	2000/1/26-2000/1/25	26.52	114.00	265
元大 21	2000/5/31-2001/5/30	34.88	121.09	249*	建弘 07	2000/2/10-2000/2/9	21.20	126.67	268

* Data in this study is cut off on 5/11/2001.

Table 2: Statistics Summary of Daily Returns for TSM and UMC Stocks

	TSM 1994/9/5-2001/5/11	UMC 1986/11/6-2001/5/11
Maximum	0.06989	0.07246
Minimum	-0.06987	-0.14716
Average	0.00172	0.00158
Standard Deviation	0.02692	0.02827
Kurtosis	3.54491	3.31634
Skewness	0.29757	0.00809
# of observations	1846	4461

Table 3: Moneyness of the Call Warrants

Moneyness	TSM	UMC
> 50% in-the-money	36	2
≤ 50% in-the-money	127	150
≤ 50% out-of-the-money	198	568
> 50% out-of-the-money	318	217
Total	679	937

Table 4: Summary Statistics of the Implied Volatility Derived from the BS Model

	TSM	UMC
Maximum	1.8500	1.7632
Minimum	0.0206	0.1496
Average	0.6333	0.6578
Standard Deviation	0.2604	0.2055
Kurtosis	3.1095	3.0045
Skewness	1.2395	0.8625
# of observations	679	937

Table 5: Summary Statistics of the Implied Volatility derived from the EDB Model

	TSM		UMC		
Horizon	4 years	2 years	10 years	4 years	2 years
Maximum	2.3534	2.7975	1.7655	1.9045	2.0394
Minimum	0.0425	0.0405	0.2104	0.1998	0.2214
Average	1.0939	1.1204	0.9016	0.9411	0.9412
Standard Deviation	0.3437	0.4566	0.2438	0.2667	0.2920
Kurtosis	0.5501	0.1389	0.6805	0.7640	0.8064
Skewness	0.0294	0.4354	-0.1900	-0.1219	0.3905
# of observations	679		937		

Table 6: Smile Generated by the BS Model

	TSM	UMC
Const	0.3907[a]	0.5172[a]
	(41.6955)	(71.7152)
M	-0.2783[a]	-0.3492[a]
	(-7.0054)	(-10.5647)
M^2	0.8897[a]	0.5347[a]
	(26.8365)	(7.9615)
M^3	1.1193[a]	0.4991[a]
	(16.9038)	(3.4822)
M^4	0.3538[a]	0.0738
	(8.8654)	(0.8365)
R^2(adjusted)	0.6616	0.5268
F test	332.4056[a]	261.4580[a]
# of obs.	679	937

[a] significant at the 1% level
[b] significant at the 5% level
[c] significant at the 10% level

Table 7: Smile Generated by the EDB Model

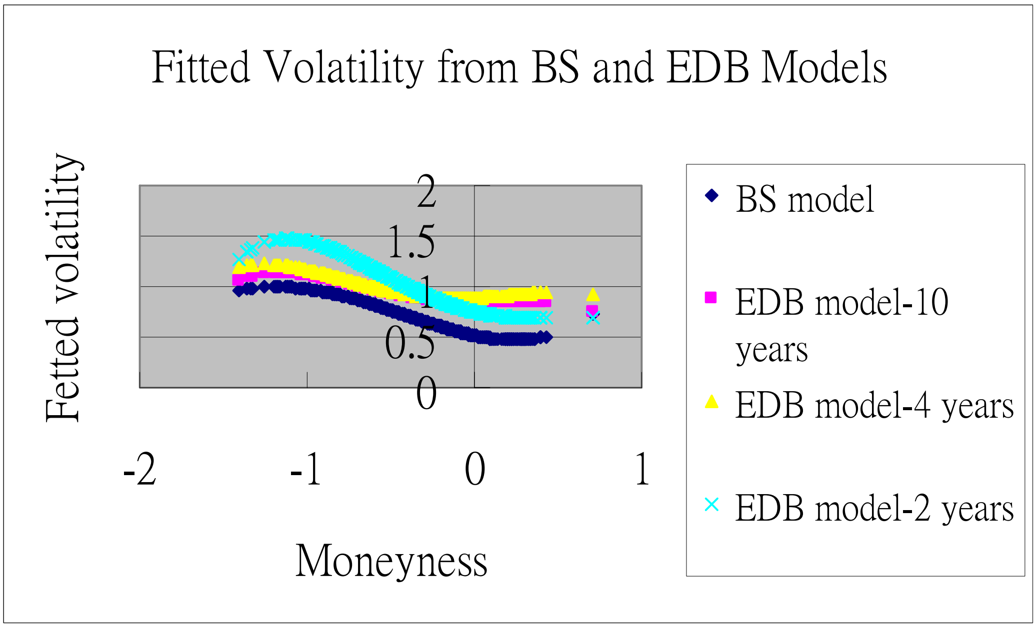
	TSM		UMC		
Horizon	4 years	2 years	10 years	4 years	2 years
Const	0.9004 [a]	0.7124 [a]	0.8439 [a]	0.8903 [a]	0.7525 [a]
	(49.1944)	(51.5327)	(71.8246)	(68.7278)	(76.6060)
M	-0.1541 [b]	-0.2681 [a]	-0.0271	0.0652	-0.3848 [a]
	(-2.2416)	(-5.1626)	(-0.5031)	(1.0987)	(-8.5461)
M^2	0.7703 [a]	0.8951 [a]	0.2202	0.3071 [b]	0.6198 [a]
	(11.8949)	(18.3001)	(0.1094)	(2.5456)	(6.7755)
M^3	0.8537 [a]	0.1327	-0.3081	-0.3067	-0.0257
	(6.5999)	(1.3583)	(-1.3195)	(-1.1913)	(-0.1318)
M^4	0.2493 [a]	-0.1587 [a]	-0.2845 [b]	-0.2722 [c]	-0.3342 [a]
	(3.1981)	(-2.6982)	(-1.9801)	(-1.7185)	(-2.7818)
R^2(adjusted)	0.2589	0.7604	0.1080	0.0935	0.5653
F test	60.2176 [a]	539.0000 [a]	29.3196 [a]	25.1239 [a]	305.3025 [a]

[a] significant at the 1% level
[b] significant at the 5% level
[c] significant at the 10% level

Table 8: Actual Profits of Selling Naked

		Actual	BS Model	EDB Model		
				10 years	4 years	2 years
TSM	All	17.6202	20.0231		17.6885	21.3870
	Out-of-the Money	7.8339	9.0577		7.2661	9.5010
	In-the-Money	48.6002	54.7357		50.6824	44.1533
UMC	All	10.8181	11.3307	9.8852	9.9903	11.5708
	Out-of-the Money	7.4140	7.2380	6.3106	6.3030	7.3486
	In-the-Money	28.3985	32.4671	28.8127	29.0331	33.3763

Figure 1: Fitted Volatility from BS and EDB Models



Appendix[5]

The risk neutral pricing theory pioneered by Cox and Ross (1976) indicates that any deflated asset price should be a martingale. Hence one can write the option pricing model as:

$$C_t = e^{-r(T-t)}\hat{E}_t[\max\{S_T - K, 0\}] \qquad (3)$$

where S_T represents the underlying asset price at the maturity time T, K is the strike price of the option, r is the risk free rate, t is the current time, and $\hat{E}_t[\cdot]$ represents the conditional expectation (at t) under the risk neutral probability measure. However, this pricing methodology is valid only continuous trading is possible in a complete market. In the absence of continuous trading and a complete market, the risk neutral expectation is not tractable and thus the option price is computed by:

$$C_t = e^{-k_{t,T}} E_t[\max\{S_T - K, 0\}] \qquad (4)$$

where $E_t[\cdot]$ is the conditional expectation under the real measure and $k_{t,T}$ is the risk-adjusted discount rate.

In this study, options on the warrant of TSM/UMC are evaluated. Thus, the realizations of TSM/UMC returns are used to form histograms that are used to compute option values. The option price at any given time t is calculated using a histogram of TSM/UMC stock price returns for a holding period of $T-t$ taken from a fixed time window immediately preceding time t. For example, for the 10 year horizon case, the

[5] The model is revised from Chen and Palmon (2002).

36-calendar-day (or roughly 25 trading day) option price on any date is evaluated using a histogram of 25-trading-day holding period returns taken from a window that starts 7560 ($=30\times 252$, assuming an average of 252 trading days a year) trading days before the valuation date and ends the day before the valuation date. Thus, this histogram contains 7535 ($=7560-25$) realizations. Note that this distribution is not risk neutral and thus the options are evaluated using Equation (4). Furthermore, the distribution does not follow a nice functional form and thus the option value cannot be valued by a closed form formula. Therefore, the expectation of Equation (4) is evaluated numerically.

To facilitate the numerical valuation of Equation (4) using the return distribution, the variables are normalized as follows:

$$C_t^* = \frac{C_t}{S_t}$$

$$R_{t,T} = \frac{S_T}{S_t} \qquad (5)$$

$$K^* = \frac{K}{S_t}$$

where S_t represents the current ex-dividend TSM/UMC stock price. Thus, Equation (4) turns into:

$$C_t^* = e^{-k_{t,T}} E_t[\max\{R_{t,T} - K^*, 0\}] \qquad (6)$$

Given that the European option valuation is a single period valuation, the Capital Asset Pricing Model can be used to estimate the risk premium and the risk-adjusted discount rate:[6]

$$k_{t,T} = r(T-t) + \beta_{t,T}(E_t[R_{t,T}] - r(T-t)) \qquad (7)$$

where $E_t[R_{t,T}]$ is the market expected rate of return for the period [t, T] which is also approximated by the expected rate of return on the TSM/UMC. The variable r is the risk free rate for which the 90-day bank rate is used as a proxy. The systematic risk β for the option is defined as:

$$\begin{aligned}\beta_{t,T} &= \frac{\mathrm{cov}[R_{t,T}, \frac{C_T}{C_t}]}{\mathrm{var}[R_{t,T}]} \\ &= \frac{\mathrm{cov}[R_{t,T}, \frac{1}{C_t^*}\max\{R_{t,T} - K^*, 0\}]}{\mathrm{var}[R_{t,T}]}\end{aligned} \qquad (8)$$

Note that $\beta_{t,T}$ depends also on K^* and C_t^*. The expected option payoff is calculated as the average payoff where all the realizations in the histogram are given equal weights. Thus, Equation (6) is numerically calculated as:

$$C_t^* = e^{-k_{t,T}} \frac{\Sigma_{j=1}^{N} \max\{R_{t,T,j} - K^*, 0\}}{N} \qquad (9)$$

[6] Using the CAPM for the risk-adjusted discount rate implicitly assumes a quadratic utility function for the risk.

where N is the total number of realizations in the histogram and $R_{t,T,j}$ is the j-th realized return.

The performance of this model is compared with that of the Black-Scholes model. The Black-Scholes model assumes a log normal diffusion for the TSM/UMC stock price:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \tag{10}$$

Where μ is the expected rate of return on the TSM/UMC, σ is the instantaneous standard deviation of the TSM/UMC return, and W_t represents the Wiener process whose differential has 0 mean and dt variance. The Black-Scholes call option formula on the TSM/UMC warrant is:

$$C_t = S_t N(h) - e^{-r(T-t)} K N(h - \sqrt{V}) \tag{11}$$

where

$$h = \frac{\ln(S_t / K) + r(T-t) + V/2}{\sqrt{V}}$$

$$V = \sigma^2 (T-t).$$

To facilitate the comparison between the Black-Scholes model and this model, the similar normalization is conducted:

$$C_t^* = N(h) - e^{-r(T-t)} K^* N(h - \sqrt{V}) \tag{12}$$

where

$$h = \frac{\ln(1/K^*) + r(T-t) + V/2}{\sqrt{V}}.$$

To compute the implied volatility of the Black-Scholes model, we substitute the market price of the call option into the pricing equation and solve for the volatility that is symbolized as $\hat{\sigma}$.

The model is calibrated to the market price by choosing the volatility (second moment) of the distribution as follows:

$$\hat{R}_{t,T,j} = \frac{\hat{v}_{t,T}}{v_{t,T}} (R_{t,T,j} - \overline{R}) + \overline{R} \qquad j = 1, \ldots, N. \tag{13}$$

where $R_{t,T,j}$ is the raw return defined in Equation (9), $\overline{R}$ is the mean return, $v_{t,T}$ is the standard deviation of the histogram, $\hat{v}_{t,T}$ is the target volatility, and $\hat{R}_{t,T,j}$ is the adjusted value. In switching from the distribution of $R_{t,T}$ to the distribution of $\hat{R}_{t,T}$, we change the standard deviation from $v_{t,T}$ to $\hat{v}_{t,T}$. Note that this scaling does not change the mean, skewness, or kurtosis. The preservation of the high moments (skewness and kurtosis) is a constraint on this model that simplifies the solution for the implied volatility $\hat{v}_{t,T}$. In short, a proper volatility, $\hat{v}_{t,T}$, such that the resulting histogram, $\hat{R}_{t,T,j}$, can produce the market price of the option is desired and searching.

Note that the pricing equation, Equation (9), relies also on the correct risk adjusted discount rate $k_{t,T}$, which in turn relies on the knowledge of the option price, described in Equation (7) and Equation (8). Hence, $\hat{v}_{t,T}$ is solved numerically by finding the solution to the simultaneous equation system that includes Equation (7), Equation (8), and Equation (9), where the variable $R_{t,T,j}$ in these equations is replaced by $\hat{R}_{t,T,j}$ as given by Equation (13).

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