Pricing Foreign-Currency Convertible Bonds And Call on Foreign-Currency Convertible Bonds Subject to Credit Risk

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This paper is the first article to price foreign-currency (or inflation-indexed) convertible bonds and their asset swaps under the consideration of risk-free and risky interest rates, stock price, exchange rate, and credit risk. We also compute the suitable swap rate for asset swap and prove that the value of a foreign-currency convertible bond is less than (equal to) the value of a synthesis straight bond plus the value a call option on foreign-currency convertible bond while the foreign-currency convertible bond is (not) embedded with the call or put provisions prior to the maturity date of asset swap. In practice, the value of a call on foreign-currency convertible bond is equal to the value of a foreign-currency convertible bond minus the value of a synthetic straight bond. It is correct only if there is not call or put provisions prior to maturity date of FCCB asset swap. From numerical analysis, we also discover the properties of foreign-currency convertible bonds, synthesis straight bonds, call options on foreign-currency convertible bonds, and the swap rate. Taking the foreign-currency convertible bond issued by Tom holdings Ltd as an example, we provide the theoretical values of the foreign-currency convertible bond, call option on foreign-currency convertible bond and the appropriate swap rate. The empirical results indicate that the numerical value is closed to the market price. Hence, our pricing model is useful for market practitioners.

Keywords: foreign-currency convertible bonds, foreign-currency convertible bond asset swap, stochastic interest rate, credit risk, foreign-currency convertible bond parity

1. Introduction

Convertible bonds (hereafter CBs) are hybrid financial instruments that entitle the investors the rights to forgo the coupon and the principle, and to convert into a pre-specified numbers of shares of common stocks instead. For CBs with call and put provisions, the issuer retains the right to call CBs back at the call prices after the call protection period, and the investors may put the CBs at the pre-specified put prices. In recent years, the commercial growth of CBs has been impressive. The worldwide CBs market is in excess of US$ 400 billion[1].

Due to the financial innovation, the convertible structure offers an ideal starting point for creating new features tailored to suit issuers and different investor groups; some of those new features are the foreign-currency CBs (henceforth FCCBs) and inflation-indexed CBs whose synthesis straight bonds link the promised payments to a foreign exchange rate or a general price index, respectively. The FCCBs or inflation-indexed CBs market has been expanding rapidly. For example, the worldwide FCCBs market grew from US$ 8.1 billion in 1990 to US$ 14 billon in 1995. In Japan, from 2000 to 2002, Sony has issued four series of U.S. dollar denominated CBs with the total amount of US$ 259 million. In Taiwan, the FCCBs market has grown from US$ 475 million in 1999 to US$ 5.278 billion in 2002. On Oct 2003 BSES, Ltd. in India has raised US$120 million through five-year FCCBs. In Israel, the coupon and principal payments of CBs traded on the Tel-Aviv Stock Exchange are linked to consumer price index (CPI) or the Dollar/Shekel exchange rate.

Since CB is a mixture of corporate bond and equity nature, this property renders it sensitive to equity, interest rate, and credit risk. Besides, the special cases of exchange rate risk arises for those FCCBs which pay coupons and face value denominated in some foreign currency but convert into some domestic equity. As shown by Moody's report, the expected default probabilities for CB issuers are higher than those without CBs in their capital structures. Consequently, credit risk is a crucial factor in pricing CBs and should not be neglected.

Another emerging market for convertibles is the CB strip market that has grown dramatically since the mid 1990s and at least 30% of the FCCBs have been stripped in 2001. As shown in Exhibit 1, FCCB asset swap dealer strips a FCCB into credit component and equity component to suit different investor groups. The credit component is viewed as FCCB asset swap. The basic FCCB asset swap transaction involves: 1) One party (credit investor) purchase a FCCB and write a American-style call option on FCCB to another party (equity investor), give the latter the right to purchase the FCCB at a strike price equal to the price of synthetic straight bond. The maturity date of call option on FCCB is the same as the maturity date of FCCB asset swap. Hence, the credit investors hold a synthesis straight bond in the transaction of FCCB asset swap. The credit investors are usually the fixed-income investors such as commercial banks, retired funds, and insurance corporations while most of equity investors are convertible arbitrage funds. 2) Interest rate swap: If default does not occur, issuer does not call the FCCB back, or the calls on FCCB are not early exercised, the FCCB asset swap dealers will pay the credit investor fixed (or floating) swap rate in return for cash flows from FCCB. Otherwise, the asset swap transaction is early terminated[2]. Hence, the credit investor owns a synthetic straight bond without equity exposure but faces credit risk. On the other hand, the equity investor owns a call option on FCCB that is free of the credit exposure of outright FCCB investment, while still benefit from the upward movement of stock prices.

The modern academic literature for pricing CBs begins with Ingersoll (1977) and Brennan and Schwartz (1977). They extend the option models of Black and Scholes (1973) and Merton (1973) to provide the analytical solution and numerical technique using finite differences method for pricing CBs as contingent claims on the firms as a whole. However, the main drawback of structural models such as Ingersoll (1977) and Brennan and Schwartz (1977) is the need to estimate the unobservable parameters of total value and the volatility of the firm's assets.

Different from the setup of Ingersoll (1977) and Brennan and Schwartz (1977), Mcconnell and Schwartz (1986) use a contingent claims pricing model for valuing Liquid Yield Option Note (LYON) with finite differences method and utilize the model to analyze a specified LYON issue. They assume that the value of LYON depends upon the issuer's stock price and avoids estimating the unobservable parameters such as the volatility of the firm's value.

Nevertheless, the above pricing models do not consider default risk of CBs and preclude the possibility of bankruptcy, and hence overstate the value of CBs. As a result, taking default risk into account, Nyborg (1996) evaluates the values of CBs by using the total values of the firms as stochastic variables and accounting for the debt obligations of the issuers in defining the random behavior of the value of the firm. However, their model also involves many parameters which are difficult to estimate in practice. Tsiveriotis and Fernandes (1998) argue that the value of CBs should have different components for different default risks. They view a CB as a contingent claim on the underlying equity and divide it into two parts by different credit levels. One is cash-only part of CB, which is subject to credit risk, and the other part is the CB related to payment in equity, which is not. This leads a pair of coupled partial differential equations that can be solved to value CBs. However, Ayache, Forsyth, and Vetzal (2003) demonstrate that the widely used CB model of Tsiveriotis and Fernandes (1998) is internally inconsistent. They provide a CB model subject to credit risk using an approach based on the numerical solution of linear complementarity problems.

Recently, an alternative for considering credit risk is the setup of hazard function. The probability of default in the next time partition is determined by a specified hazard rate.

When default occurs, some portion of the bond is assumed to be recovered. For example, following Duffie and Singleton (1999), Takahashi, Kobayashi, and Nakagawa (2001) propose a reduced-form model to price convertible debts with credit risk. They model the hazard rate as a decreasing function of stock price and assume the pre-default process of the stock price follows a diffusion process.

Hung and Wang (2002) combine Jarrow and Turnbull (1995) model involving the default-free and the risky discount rate processes with the tree for stock prices to value CBs that may be default. However, their model ignores the default hazard rate and hence underestimates the upward probability of stock price as well as the CB values. Meanwhile, their model also assumes that the risk-free interest rate is the same after the default occurs. However, since that the default event is uncorrelated with the behavior of default-free interest rate, the default-free interest rate is still stochastic after the default event occurs.

For pricing FCCBs and inflation-indexed CBs, Yigitbasioglu (2001) presents a two factor model to deal with FX sensitive cross-currency convertibles by using Crank-Nicholson technique. Landskroner and Raviv (2003) consider two sources of uncertainty allowing both the underlying stock and the CPI (or exchange rate) to be stochastic and incorporate credit risk by using a Rubinstein [1994] three-dimensional binomial tree. However, the risky interest rate is deterministic in their model.

As well as we know, our paper is the first article to explore the pricing model for valuing the values of FCCBs, call options on FCCBs and FCCB asset swaps. By modifying Hung and Wang (2002) model, we provide a lattice method for valuing callable and puttable FCCBs (or inflation-indexed CBs), call on FCCB, and FCCB asset swap for which the equity price, exchange rate (or CPI), default-free, and risky interest rates are stochastic.

If there are (not) call and put provisions prior to the maturity date of FCCB asset swap, we prove that the value of a synthetic straight bond plus the one of a call option on FCCB is higher than (equal to) the value of a FCCB. In practice, the FCCB strip is to make the value of synthetic straight bond plus the value of a call on FCCB equal to the value of FCCB. Consequently, it is correct only if there is not call or put provision prior to maturity date of FCCB asset swap.

From numerical analysis, without call and put provisions prior to maturity date of FCCB asset swap, we find that the values of FCCBs and synthesis straight bonds are increasing function of the coupon rate, and are decreasing function of hazard rate. The values of synthesis straight bonds are unrelated with the volatility of exchange rate. The relationship between value of a call option on FCCB and the level of hazard rate is a humped-shaped curve. We also find that the value of a synthetic straight bond plus the value of a call on FCCB equal to the value of a FCCB. The suitable swap rate is positive relationship with hazard rate, negative relationship with coupon rate, and unrelated with volatility of stock return. Or equivalently, the credit investor is free of equity exposure, nevertheless, they require high swap rate if they face the high degree of credit risk.

Taking the FCCBs issued by Tom holdings Ltd in Hong Kong as an example, we provide the fair prices of the FCCB, a call option on FCCB and the appropriate swap rate. The empirical results indicate that the numerical value is closed to the market price.

2. Pricing Model

In this section, we first describe the setup of cross-currency economy. Then, by modifying Hung and Wang (2002) model and incorporating the exchange rate, we develop a valuation algorithm for FCCBs.

2.1 The Economy

We assume that there are two countries, α and β, in the cross-currency economy. The corporation in country β issues a CB denominated in the currency of country α. The uncertainty of trading market in country α is described by the filtered probability space $(\Omega, F, P^*, (F_t)_{t=0}^{T^*})$, where P^* is the spot martingale measure under which discounted values of underlying assets in country α are martingales.

Under P^*, the exchange rate process $Q(t)$, which is used to convert the payoffs of country β into the currency of country α, is assumed as follows:

$$\frac{dQ(t)}{Q(t)} = (r_\alpha - r_\beta)dt + \sigma_Q dW_Q^*(t) \tag{1}$$

where W_Q^* stands for one-dimensional standard Brownian motion defined on a filtered probability space $(\Omega, F, P^*, (F_t)_{t=0}^{T^*})$. r_α and r_β are the stochastic risk-free interest rates in country α and β, respectively. σ_Q is the volatility of exchange rate.

Furthermore, we assume that the dynamics of issuer's stock price $S_\beta(t)$ is:

$$\frac{dS_\beta(t)}{S_\beta(t)} = \left(r_\alpha - q - \rho_{SQ}\sigma_S\sigma_Q + \lambda\mu(t)[1-N(t)]\right)dt + \sigma_S dW_S^*(t) - dN(t) \tag{2}$$

where $N(t) \equiv I\left(t \geq \tau\right)$, $I(\cdot)$ is the indicator function, τ is exponentially distributed over $[0,\infty)$ with parameter λ, and $\mu(t)$ is the Poisson bankruptcy process under spot martingale probability measure P^* in country α. $\lambda\mu(t)$ represents the hazard rate. q is the continuous dividend payout rate. σ_S is the volatility of issuer's stock return in country β. ρ_{SQ} is the instantaneous correlation coefficient between the issuer's stock return and the exchange rate and satisfies:

$$E[dW_S^*(t)dW_Q^*(t)] = \rho_{SQ}dt \tag{3}$$

From the country α' s investors point of view, when they convert an foreign-currency CB into issuer's stocks, they obtain the amount equal to stock price multiplied by the concurrent exchange rate. Hence, we assume that the stock price demonstrated in the currency of country α as $S^{*}_{\beta}(t) \equiv S_{\beta}(t)Q(t)$. By Ito's Lemma, we have[3]:

$$\frac{dS^{*}_{\beta}(t)}{S^{*}_{\beta}(t)} = \left(r_{\alpha} - q + \lambda\mu(t)[1 - N(t)] \right) dt + \sigma\, dW^{*}_{3}(t) - dN(t) \tag{4}$$

where $\sigma^{2} \equiv \sigma^{2}_{S} + 2\rho_{SQ}\sigma_{S}\sigma_{Q} + \sigma^{2}_{Q}$. Let us consider a finite collection of date T_j, $j = 1,\ldots,n$, where $0 < T_1 < \ldots < T_n = T \le T^{*}$ and $T_j = j \times (T/n)$. From (4), we can see that $S^{*}_{\beta}(t)$ is a lognormal distribution and can be rewritten as follows:

$$\tilde{\xi}_{T_j} \equiv \frac{S_{\beta}(T_j)Q(T_j)}{S_{\beta}(T_{j-1})Q(T_{j-1})}$$

$$= \begin{cases} \exp\left(\sigma[W^{*}_{3}(T_j) - W^{*}_{3}(T_{j-1})] + (r_{\alpha} - q - \frac{1}{2}\sigma^{2})(T_j - T_{j-1}) + \int_{T_{j-1}}^{T_j} \lambda\mu(s)ds \right) & if \quad T_j < \tau \\ 0 & if \quad T_j \ge \tau \end{cases}$$

$$, \text{for} \quad j = 1,\ldots,n \tag{5}$$

Hence, if the underlying stock doesn't default, $\tilde{\xi}_{T_j}$ is a lognormal distribution.

Furthermore, let $p_{\alpha}(t,T)$ denotes the time t price of default-free zero coupon bond maturing at time T in country α. Let $v_{\alpha}(t,T)$ be the value at time t of a risky zero coupon bond maturing at date T, issued by the FCCB issuers[4]. If default occurs, the FCCBs investors in country α would receive not the whole face value, but rather a fraction of the face value. This fraction is usually defined as the recovery rate δ. As described in Jarrow and Turnbull (1995), we have:

$$v_{\alpha}(t,T) = p_{\alpha}(t,T)\left\{ \left[1 - \exp\left(-\int_{t}^{T} \lambda\mu(s)ds \right) \right] \times \delta + \exp\left(-\int_{t}^{T} \lambda\mu(s)ds \right) \times 1 \right\} \tag{6}$$

2.2 Lattice Methods for Default-free and Risky Interest Rates

Jarrow and Turnbull (1995) combine the default-free and risky interest rates into one tree. Hence, we adapt their model for the default-free and risky interest rates. The default-free and risky interest rates model is shown in Exhibit 2, where the pseudo-probability π is determined for the default-free interest rate process and is set to be 0.5. The recovery rate δ is given exogenously. Besides, a FCCB has positive default probabilities in each period. Given that the recovery rate δ, we can use the market data to compute $\lambda_{\mu\, j-1}$, where $\lambda_{\mu\, j-1}$ is the default probabilities prior to one period for time T_j, for $j = 1,...,n$. To demonstrate the pricing procedure of Jarrow and Turnbull (1995), we take a two-period trading economy as an example. The process of the default-free interest rate is consistent with Black, Derman, and Toy (1990). For computing the default probability at each period, at time 1, we have:

$$v_\alpha(0,T_1) = \exp[-r_\alpha(0)]\times[\lambda_{\mu 0}\times\delta + (1-\lambda_{\mu 0})\times 1]$$

$$= p_\alpha(0,T_1)\times[\lambda_{\mu 0}\times\delta + (1-\lambda_{\mu 0})\times 1] \quad (7)$$

Using the market data of $p_\alpha(0,T_1)$ and $v_\alpha(0,T_1)$ with the recovery rate δ, we can derive $\lambda_{\mu 0}$. Similarly, at time 2**,** we have:

$$v_\alpha(0,T_2) = \exp[-r_\alpha(0)]\times\{\exp[-r_\alpha(1)_u]\times\pi\times\lambda_{\mu 0}\times\delta + \exp[-r_\alpha(1)_d]\times(1-\pi)\times\lambda_{\mu 0}\times\delta$$

$$+\exp[-r_\alpha(1)_u]\times\pi\times(1-\lambda_{\mu 0})\times[\lambda_{\mu 1}\times\delta + (1-\lambda_{\mu 1})\times 1]$$

$$+\exp[-r_\alpha(1)_d]\times(1-\pi)\times(1-\lambda_{\mu 0})\times[\lambda_{\mu 1}\times\delta + (1-\lambda_{\mu 1})\times 1]\}$$

$$= p_\alpha(0,T_2)\times\{\lambda_{\mu 0}\times\delta + (1-\lambda_{\mu 0})\times[\lambda_{\mu 1}\times\delta + (1-\lambda_{\mu 1})\times 1]\} \quad (8)$$

Using the values of $p_\alpha(0,T_2)$, $v_\alpha(0,T_2)$ with the recovery rate δ and the default probability $\lambda_{\mu 0}$, we can obtain the default probability $\lambda_{\mu 1}$. According to this procedure, we can calculate $\lambda_{\mu\, j-1}$ at each time T_j recursively.

It is worth to note that if $t = 0$ and $T = T_2$, we can rewrite (6) as follow:

$$v_{\alpha}(0,T_2) = p_{\alpha}(0,T_2)\left\{\left[1-\exp\left(-\int_0^{T_1}\lambda\mu(s)ds\right)\right]\times\delta + \right.$$

$$\left. + \exp\left(-\int_0^{T_1}\lambda\mu(s)ds\right)\times\left(\left[1-\exp\left(-\int_{T_1}^{T_2}\lambda\mu(s)ds\right)\right]\times\delta + \exp\left(-\int_{T_1}^{T_2}\lambda\mu(s)ds\right)\times 1\right)\right\} \quad (9)$$

By comparing (8) with (9), we obtain that

$$\lambda_{\mu\ j-1} = 1-\exp\left(-\int_{T_{j-1}}^{T_j}\lambda\mu(s)ds\right), \text{ for } \ j=1,\ldots,n \quad (10)$$

Hung and Wang (2002) combine Jarrow and Turnbull (1995) model involving the default-free and the risky discount rate processes with the tree for stock prices to value the CBs that may be defaulted. According to their model, if default occurs, the stock price jump to zero and the default-free interest rate is the same in the successive periods. However, the stochastic behavior of default-free interest rate should be independent with the default event, and hence we can not ignore the pseudo-probability for the upward or downward movements of default-free interest rate. Hence, we modify Hung and Wang (2002) method by revising the movements of default-free interest rate after default occurs. Exhibit 3 depicts our modified four-period risky interest rate tree model.

2.3 Lattice Method for Stock Price in Unit of Another Currency

To construct a tree for stock price demonstrated in currency of country *α*, we can use a discrete process ξ_{T_j} to approximate $\tilde{\xi}_{T_j}$ in (5). We assume that ξ_{T_j} can only change to one of three possible values, 0 with probability $P_n(j)$, u with probability $P_u(j)$, or d with probability $P_d(j)$, for $j=1,\ldots,n$. The setup of lattice method for $S^*_{\beta}(T_j)$ is shown in the following theorem.

Theorem 1. Under the setup of (1) and (2), the possible values of $S^*_{\beta}(T_j)$, $j=1,\ldots,n$, are 0 with probability $P_n(j)$, $S^*_{\beta}(T_{j-1})u$ with probability $P_u(j)$, or $S^*_{\beta}(T_{j-1})d$ with probability

$P_d(j)$, where u, d, $P_u(j)$, $P_d(j)$, and $P_n(j)$ are defined as :

$$u = \frac{1}{d} = \exp(\sigma\sqrt{\frac{T}{n}}),\ \ P_n(j) = \lambda_{\mu\ j-1},\ \ P_u(j) = (1-\lambda_{\mu\ j-1})p_{r_\alpha},\ \ P_d(j) = (1-\lambda_{\mu\ j-1})(1-p_{r_\alpha}) \quad (11)$$

where

$$p_{r_\alpha} = \frac{1}{2} - \frac{\ln(1-\lambda_{\mu\ j-1})}{2\sigma}\sqrt{\frac{n}{T}} + \frac{r_\alpha - q - 0.5\sigma^2}{2\sigma}\sqrt{\frac{T}{n}} \quad (12)$$

We prove Theorem 1 in Appendix A.

In Hung and Wang's (2002) pricing model for CBs, the upward probability of stock price is as follows:

$$p_{r_\alpha}^{HW} = \frac{1}{2} + \frac{r_\alpha - q - 0.5\sigma^2}{2\sigma}\sqrt{\frac{T}{n}} = p_{r_\alpha} + \frac{\ln(1-\lambda_{\mu\ j-1})}{2\sigma}\sqrt{\frac{n}{T}} \le p_{r_\alpha} \quad (13)$$

Prior to bankruptcy, (13) doesn't take the hazard rate into consideration and hence underestimate both the upward probabilities of stock price and the CB values. For example, as shown in the numerical example of Hung and Wang (2002), since that the continuum value 86.15 is smaller than the conversion value 90, the optimal decision is to convert the CB. However, using Theorem 1 and the fact that the default-free interest rate is still stochastic even though default occurs, the continuum value should be 91.1148 and hence the optimal decision for CB investors is to hold the CB. As a result, we should apply (12) to compute the prices of CBs subject to credit risk. We illustrate the possible paths and corresponding probabilities for S_β^* in Exhibit 4.

2.4 Pricing Method for Zero-Coupon FCCBs

After that, we can incorporate these lattice methods of default-free and risky interest rates and stock price demonstrated in currency of country α into our pricing method for the T-year maturity FCCB with call and put provisions. We define that $P_{r_\alpha(t)}$ is the probability that the stock price will go up when the risk-free rate at time t is r_α. Exhibit 5 indicates a four-period pricing method for FCCBs. There are three main cases as one move through the tree. First, if the FCCB does not default at this node such as both node A and node C, we have six branches in the next period as follows:

1. Default occurs; r_α goes up, and S_β^* jumps to 0.
2. Default occurs; r_α goes down, and S_β^* jumps to 0.
3. Default doesn't occur; r_α goes up, and S_β^* goes up.
4. Default doesn't occur; r_α goes up, and S_β^* goes down.
5. Default doesn't occur; r_α goes down, and S_β^* goes up.
6. Default doesn't occur; r_α goes down, and S_β^* goes down.

If default occurs, the tree enters into second case such as node B and node D. The stock prices will jump to zero and hence the equity component of the FCCB is zero and the debt part can receive only the frictions of the bond's face value. As a result, the value of a FCCB will be equal to the product of the recovery rate and the face value. However, unlike the setup of Hung and Wang (2002) model, after default event the default-free interest rate process still fluctuates. Hence, at node B and node D, we have two branches at each period as follows:

1. Default occurs; r_α goes up, and S_β^* jumps to 0.
2. Default occurs; r_α goes down, and S_β^* jumps to 0.

Therefore, the tree of default-free interest rate is recombined after default event. For the terminal node such as node E, the default-free interest rate as well as the stock price does not fluctuate and hence there is only one branch at this node. The third case such as node F is a special condition that $r_\alpha(3)_{uuu}$ already represents the discount rate between $t=3$ and $t=4$. Thus, the node F has three branches as follows:

1. Default occurs, and S_β^* jumps to 0.

2. Default doesn't occur, and S_{β}^{*} goes up.

3. Default doesn't occur, and S_{β}^{*} goes down.

After the construction of the tree, we should backwardly decide the payoff at each node from the terminal nodes. If default doesn't occur, three decisions at each node have to be checked. The optimal time to convert is when the conversion value exceeds its continuum value and put price, where continuum value is the sum of debt part and equity part at each node. The optimal time for the investors to put the FCCB back is when the put price exceeds the continuum value and the conversion value. For the issuer in country β, the optimal time to call the FCCB back is when the continuum value exceeds its call price. Thus, if default does not occur, the total value of the FCCB at each node can be stated as:

$$\text{Max [Min (Continuum value, Call Price), Put Price, Conversion Value]} \quad (14)$$

If default occurs, the stock price jumps to zero at this node. The FCCB investors will not exercise the conversion right to get the zero-value underlying stock. Meanwhile, because of bankruptcy, the put provision of FCCB is also invalid. Hence, the total value of the FCCB at this node is equal to the present value of recovery rate multiplied by cash flows (including the successive coupon and principal payments) from the FCCB. Finally, once having rolled back through the tree, the total value at time T_0 is the theoretical value of FCCB subject to credit risk.

3. Extensions

In this section, we extend the FCCB model to the valuation of the coupon-bearing FCCB, the coupon-bearing synthetic straight bond, the call option on FCCB, the suitable swap rate for FCCB asset swap, and the inflation-indexed CB. We also note that practitioners often regard the value of a call option on FCCB as the value of a FCCB minus the value of a synthetic straight bond. Consequently, we prove that the equality is true only if there is no call and put provisions prior to maturity date of FCCB asset swap.

3.1 Coupon-Bearing FCCBs and Synthesis Straight Bonds

We can price the coupon-bearing FCCBs by slightly changing the pricing model mentioned above. If default occurs, the call, put, and conversion provisions are invalid and hence the total value of the FCCB at this node is equal to the present value of recovery rate multiplied by the accrued interests and the principal payment in the subsequent periods. Otherwise, the total value of the FCCB at each node is determined as follows:

Max [Min (Continuum value + Coupon Payment, Call Price), Put Price, Conversion Value] (15)

Finally, once having rolled back through the tree, the total value at time T_0 is the fair price of coupon-bearing FCCB subject to credit risk.

For pricing synthesis straight bond, since it is the same as FCCB without conversion provision, by using the same FCCB model and assuming the conversion ratio equal to zero, we can obtain the value of synthesis straight bond.

3.2 Call Option on FCCB

In the transaction of FCCB asset swap, the credit investors buy the FCCB from the asset swap dealers and simultaneously sell a call option on FCCB to them. Hence, the credit investor holds a synthesis straight bond and equity investor holds a call option on FCCB. A call option on FCCB gives the equity investors the right to buy the FCCB at a strike price equal to the value of synthesis straight bond prior to the maturity of asset swap. For pricing call option on FCCB, we also have to consider whether default occurs or not. If default occurs, the value of a call option on FCCB is zero since that the value of a FCCB is equal to the value of straight bond and simultaneously the FCCB asset swap is early terminated. If default does not occur, for ease of the explanation, we first denote that:

T_m : The maturity date of FCCB asset swap (or call on FCCB).

$FCCB_i$: The value of a FCCB at time T_i, $i=1,...,m$, where $T_m \le T_n = T$.

SB_i : The value of a synthesis straight bond at time T_i, $i=1,...,m$.

$Option\_cont_i$: The continuum value of a call option on FCCB at time T_i, $i=1,...,m$.

$Option_i$: The value of a call option on FCCB option at time T_i, $i=1,...,m$.

Hence, at time T_m, the value of a call option on FCCB is as follows:

$$Option_m = \text{Max}\ [(FCCB_m - SB_m), 0\ ] \tag{16}$$

Otherwise, the option value in each node is set as:

$$Option_i = \text{Max}\ [(FCCB_i - SB_i),\ \ Option\_cont_i\ ],\ \ i=1,...,m-1 \tag{17}$$

where $Option\_cont_i$ is the discounted values of $Option_{i+1}$ in the $(i+1)^{\text{th}}$ period. Once having rolled back through the tree, the value at time T_0 is the theoretical price of a call option on FCCB subject to credit risk.

It is worth to note that the FCCB asset swap dealer usually set the maturity date of FCCB asset swap (or call on FCCB) equal to the closeness put date of FCCB. They also make the value of a FCCB be equal to the value of a call option on FCCB plus the value of a synthesis straight bond. However, it is not correct if there is call or put provisions prior to maturity date of FCCB asset swap. We demonstrate the relationship between FCCB, synthesis straight bond, and call option on FCCB in the following Theorem.

Theorem 2. With the call and put provisions prior to the maturity date of FCCB asset swap, the value of a FCCB is less than the value of a call option on FCCB plus the one of a synthesis straight bond and states as:

$$FCCB_i \le SB_i + Option_i,\ \ i=1,...,m \tag{18}$$

Otherwise, we have the FCCB Parity as follows:

$$FCCB_i = SB_i + Option_i\,,\quad i = 1,\ldots,m \tag{19}$$

We prove Theorem 2 in Appendix B.

As a result, unless there is no call and put provisions prior to the maturity date of FCCB asset swap, the equality, $Option_i = FCCB_i - SB_i$, holds. In practice, the maturity date of a call option on FCCB is set to the nearness put date of FCCB, while the issuer promises not to exercise the call right or there is no call provision prior to the maturity date of FCCB asset swap, the pricing mechanism of FCCB strip in practice work well.

3.3 Swap Rate for FCCB Asset Swap

For pricing FCCB asset swap, our goal is to find a suitable swap rate such that the FCCB asset swap is a zero sum game for both credit investors and FCCB asset swap dealer at the issue date. We denote M the nominal principal of FCCB asset swap. For time T_i, $i = 1,\ldots,m$, if default occurs, the asset swap is early terminated and the credit investors must hold the synthesis straight bond which is already defaulting. The credit investors have a loss equal to the cash inflows (including the accrued interest payment plus *M,* or equivalently, [1+swap rate]$\times M$) minus the default value of a synthesis coupon-bearing straight bond. If default does not occur, we have three following cases. First, if issuer does not call the FCCB back and equity investors do not early exercise, the payoff of credit investors at this case is equal to the accrued interest payment (swap rate multiplied by *M*) from FCCB asset swap dealer plus the continuum value of FCCB asset swap minus coupon payment of a FCCB. Second, while issuer call the FCCB back, equity investors early exercise, or the FCCB asset swap is matured, the payoff of credit investors equal the cash flows from asset swap dealer([1+swap rate]$\times M$) minus the value of a synthesis coupon-bearing straight bond.

At the date of issuance T_0, the credit investors pay M and get synthesis coupon-bearing straight bond. Consequently, the gain or loss at issue date T_0 is equal to the value of a synthesis straight bond minus M and plus the continuum value of FCCB asset swap. The suitable swap rate for FCCB asset swap is determined by the criterion that the gain or loss at time T_0 is equal to zero.

3.4 Inflation-indexed CB

Similar to a FCCB, the coupons and the principal payments of the inflation-indexed CBs are linked to the change of the consumer-price-index (CPI) during the life of the CB. We denote the CPI at time t as C_t. We also assume that the issue data of the inflation-indexed CB is at time T_0 and redefine that $Q(t) \equiv C_0 / C_t$. Then, using a cross-currency trading economy analogy, real prices correspond to prices in country α and nominal prices correspond to prices in country β. r_α becomes the real risk-free interest rate. S_β^* is the real price of underlying stock. Since that the nominal price is equal to real price at time 0, following the same valuation algorithm for pricing foreign-currency CBs, once we roll back through the tree, the total value at the node T_0 is the fair price of a inflation-indexed CB subject to credit risk.

4. Numerical Example

We use a numerical example to price FCCB, synthesis straight bond, call option on FCCB, and the swap rate of FCCB asset swap. The term sheet of the numerical example is described in Exhibit 6A. The prices of default-free and risky zero coupon bonds, default probabilities are given in Exhibit 6B.

First, we decide the payoffs of the terminal nodes, and then roll back through the tree.

Exhibit 5 gives us the four-period lattice methods for pricing FCCBs. Take nodes H and I as examples. At node H, because of the bankruptcy, the stock price jumps to zero and the conversion value becomes zero. The bondholders receive only the recovery rate multiplied by the sum of face value plus coupon payment. Hence, the debt and equity parts in node H are 44.6760 and 0, respectively.

At node I, the stock price $S^{*}_{\beta}(4)_{uuud}$ is 92.2942. If the investor converts the FCCB into three shares of underlying stock, he can receive 92.2942 × 3 = 276.8826. Otherwise, the investor still holds the FCCB; he receives the face value plus coupon payment equal to 102. It is higher than the call price 100 and hence the issuer may call back the bond at call price 100. As a result, the optimal decision for investors is to convert the bond into underlying stock, the payoff at node I is 276.8826. Using the similar pricing algorithm, we can decide the payoff at each terminal node. Subsequently, taking nodes A, C, D, E, and F as examples, we examine the details of rolling back through the tree more specifically.

For node F, there are three branches of terminal nodes. Given the payoffs of successive nodes, the equity part of node F is as follows:

$$(0.00995\times 0+0.99005\times 0.4054\times 276.8826+0.99005\times 0.5946\times 0)e^{-0.0347} = 107.3368$$

And the debt part of node F can be derived as follow:

$$(0.00995\times 44.6760+0.99005\times 0.4054\times 0+0.99005\times 0.5946\times 100)e^{-0.0347} + 2 = 59.2882$$

where we use $r_{\alpha}(3)_{uuu}$ =3.47% as the discount yield for both the equity and the debt component. Thus, the total value of the FCCB is 107.3368 + 59.2882 = 166.6250, which is higher than conversion value 53.6157 ╳ 3 = 160.8471.

We assume the maturity date of FCCB asset swap (call option on FCCB) is the nearness put date, which is the third year. Consequently, we discuss the values of a synthesis straight bond, call option on FCCB, and swap rate of FCCB asset swap. At node F, the synthesis straight bond is the same as the case of FCCB without conversion provision. By neglecting the conversion value, the continuum value of SB is 98.0544, which is lower than the put price 101, and hence the value of a synthesis straight bond is 101.

At maturity date of the call option on FCCB, due to the fact that the continuum value of a call option on FCCB is zero and the value of FCCB is not less than the one of synthesis straight bond, the value of a call option on FCCB equals the value of a FCCB minus the value of a synthesis straight bond. Hence, the value of a call option on FCCB is 166.6250 – 101 = 65.6250. Similarly, the asset swap is also matured. We assume that the suitable swap rate is 3.1861%, and hence the net cash flow of the credit investor at node F is as follows:

$$(1 + 0.031861) \times 100 - 101 = 2.1861$$

If default occurs, we take nodes D and E as examples. At node E, the debt part of node E is calculated as:

$$(44.6760 \times e^{-0.0347}) + (2 \times 0.438) = 44.0283$$

where the actual cash flow received at this period becomes the original promised coupon payment multiplied by the recovery rate. The equity part of node E is equal to 0. As a result, the total value of node E is 0 + 44.0283 = 44.0283.

At node D, the stock is already jumping to zero, but the interest rate process still fluctuates. The debt node of node D is

$$(0.5 \times 44.0283 + 0.5 \times 44.1697) \times e^{-0.0276} + (2 \times 0.438) = 43.7745$$

The equity part of node D is 0, and hence the total value of node D is 0 + 43.7745 = 43.7745. Due to the fact that default occurs, the value of synthesis straight bond is equal to 43.7745, too. The value of call option on FCCB is zero. Meanwhile, the FCCB asset swap is early terminated; the credit investors have to forgo the coupon payment and face value of FCCB asset swap and hold the synthesis straight bond that have already defaulted. The net cash flow of the credit investor in node D is:

43.7745 - (1 + 0.031861) × 100 = -59.4116

Now let us consider node C, which is the most usual type of node in our model. The equity part of node C is as follows:

$$(0.5\times 0.00995\times 0+0.5\times 0.00995\times 0+0.5\times 0.99005\times 0.3928\times 244.6530$$
$$+0.5\times 0.99055\times 0.6072\times 41.1402+0.5\times 0.99005\times 0.3928\times 244.1786$$
$$+0.5\times 0.99055\times 0.6072\times 40.8327)\times e^{-0.0210}=117.2033$$

The debt part of node C is as follows:

$$(0.5\times 0.00995\times 43.7745+0.5\times 0.00995\times 44.0210+0.5\times 0.99005\times 0.3928\times 37.8300$$
$$+0.5\times 0.99055\times 0.6072\times 83.7747+0.5\times 0.99005\times 0.3928\times 38.3271$$
$$+0.5\times 0.99055\times 0.6072\times 84.2635)\times e^{-0.0210}+2=66.3876$$

The total value of FCCB at node C is 117.2033 + 66.3876 = 183.5909, which is higher than the conversion value 53.6157 × 3 = 160.8471. Hence, the optimal decision of investors is still hold the FCCB. The value of a synthesis straight bond at node C is as follows:

$$(0.5\times 0.00995\times 43.7745+0.5\times 0.00995\times 44.0210+0.5\times 0.99005\times 0.3928\times 99.7034$$
$$+0.5\times 0.99055\times 0.6072\times 99.7034+0.5\times 0.99005\times 0.3928\times 99.9614$$
$$+0.5\times 0.99055\times 0.6072\times 99.9614)\times e^{-0.0210}+2=99.2128$$

The continuum value of a call option on FCCB at node C is

$$(0.5\times 0.00995\times 0+0.5\times 0.00995\times 0+0.5\times 0.99005\times 0.3928\times 182.7796$$
$$+0.5\times 0.99055\times 0.6072\times 25.2115+0.5\times 0.99005\times 0.3928\times 182.5444$$
$$+0.5\times 0.99055\times 0.6072\times 25.1349)\times e^{-0.0210}=84.3781$$

If call option on FCCB early exercises, its value equals 183.5909 – 99.2128 = 84.3781 and is the same as the continuum value of a call option on FCCB. Consequently, the optimal decision of equity investors is not early exercised, and the value of call option on FCCB is 84.3781.

Since that the call option on FCCB is not early exercised, default does not occur, and no call provision at this period, the credit investors receive the swap coupon payment in return for the coupon payment of the FCCB. The net cash flow of credit investor is:

(0.031861 × 100 – 2) + 2.0616 = 3.2477 where 2.0616 is the continuum value of FCCB asset swap at node C. Following the pricing algorithm, we can derive the equity and the debt parts of each node. Finally, we show the results at node A to obtain the theoretical values of FCCB, synthesis straight bond, call option on FCCB. The equity part of node A is as follows:

$$\begin{aligned}&(0.5\times 0.00995\times 0 \ +0.5\times 0.00995\times 0 \ +0.5\times 0.99005\times 0.3872 \ \times 117.2033\\&+0.5\times 0.99055\times 0.6128\times 15.6063 \ +0.5\times 0.99005\times 0.3872\times 116.7182\\&+0.5\times 0.99055\times 0.6128\times 15.4528 \ )\times e^{-0.0150} \ =53.4518\end{aligned}$$

The debt part of node A is as follows:

$$\begin{aligned}&(0.5\times 0.00995\times 43.8618 \ +0.5\times 0.00995\times 44.1779 \ +0.5\times 0.99005\times 0.3872 \ \times 66.3876\\&+0.5\times 0.99055\times 0.6128\times 93.1914 \ +0.5\times 0.99005\times 0.3872\times 67.0590\\&+0.5\times 0.99055\times 0.6128\times 93.7268)\times e^{-0.0150} \ =81.4861\end{aligned}$$

Hence, the total value of FCCB is 53.4518 + 81.4861 = 134.9379, which is higher than conversion value 31.1465 × 3 = 93.4395.

Following the same method, the value of a synthesis straight bond, the continuum value of a call option on FCCB, and the continuum value of FCCB asset swap are 97.4069, 37.5310, and 2.5931, respectively. At node A, FCCB – SB = 134.9379 - 97.4069 = 37.5310 is just equal to the continuum value of call option on FCCB, and hence the equity investor will not early exercise the call option on FCCB. Hence, the theoretical value of a call option on FCCB is 37.5310 and this verifies the FCCB Parity. At the date of issuance, the credit investor should pay the face value of FCCB asset swap and receive a synthesis straight bond. The net cash flows of credit investor at node A is as follows:

$$97.4069 + 2.5931 - 100 = 0$$

Since that the value of FCCB asset swap is zero, the suitable swap rate is 3.1861%.

5. Numerical Analysis

In this section, we first discuss the characteristics of FCCBs, synthesis straight bonds, and call options on FCCB under the following situation: (1) FCCBs without the call and put provisions. (2) Without call or put provisions prior to the maturity date of FCCB asset swap. (3) With call or put provisions prior to the maturity date of FCCB asset swap. We also show that the relationship of FCCBs, synthesis straight bonds, and call options on FCCBs coincides with Theorem 2. Finally, we also investigate the characteristics of suitable swap rate in FCCB asset swap.

In the following section, we assume that a German corporation issues a ten-year maturity FCCB with face value equal to US$ 100. Each FCCB can convert to 3 shares of the underlying stock. The initial stock price is EUR 25.4. Initial exchange rate is: USD 1 = EUR 0.8155. Hence, the stock price is US$ 31.1465 (equal to 25.4/0.8155). The volatility of the stock return and the instantaneous correlation coefficient of stock return and exchange rate are 20% and 15%, respectively. We also assume that the time-*t* price function of default-free zero coupon bond in America is $p_{\alpha}(0,t) = \exp[-0.015-0.025*(t-1)]$, where $t \geq 1$. The volatility of the default-free interest rate in America is equal to 5% and the maturity date of FCCB asset swap (call options on FCCBs) is at the third year.

5.1 FCCBs without Call and Put Provisions

Using the pricing method for coupon-bearing FCCBs without call or put provisions, we report the numerical values of FCCBs, synthesis straight bonds, and call options on FCCBs by varying the different levels of coupon rate, hazard rate, and volatility of exchange rate (or equivalently, the volatility of stock return) in Exhibit 7.

Since that the higher volatility of exchange (or stock return) makes the conversion right more valuable and the higher level of coupon rate makes the debt component of FCCBs more costly, it is reasonable that the values of FCCBs are increasing function of volatility of exchange (or stock return) and coupon rate in Exhibit 7A. Meanwhile, due to the fact that the higher level of hazard rate results in higher default probability, and hence the values of FCCBs are decreasing function of hazard rate. Similarly, from Exhibit 7B, the values of synthesis straight bonds are increasing function of coupon rate, and are decreasing function of the hazard rate. Nevertheless, the synthesis straight bonds are debt part of FCCBs, their values are irrelevant with the volatility of exchange rate (or stock return).

The results from Exhibit 7C indicate that the values of three-year maturity call options on FCCBs have a positive relationship with the volatility of exchange rate (stock return). The influence of coupon rate is also clear. If the coupon rate increases, and hence the values of synthesis straight bonds (the strike price) increase, the values of call options on FCCBs decrease. Meanwhile, the higher level of hazard rate causes the higher credit risk and results in the higher option value. However, the higher credit risk makes the stock price jump to zero with a higher probability. The tradeoff causes the relationship between values of call options on FCCBs and the level of hazard rate to be a humped-shaped curve. By the way, from Exhibit 7A, 7B, and 7C, we can see that (19), the FCCB Parity, holds.

5.2 Without Call and Put Provisions Prior to Maturity Date of FCCB Asset Swap

We assume that FCCBs are embedded with call and put provisions. The call prices at the 4^{th}, 7^{th}, and 9^{th} years are at par, and the put price at the 3^{rd}, 6^{th}, and 10^{th} year are US$ 107, US$ 110, and US$ 123, respectively. In Exhibit 8, the numerical results are given for different levels of hazard rate, coupon rate, and the volatility of exchange rate. General speaking, the properties are similar to the case of FCCBs without call and put provisions. For example, the values of FCCBs and synthesis straight bonds are increasing function of the coupon rate, and are decreasing function of hazard rate. The values of synthesis straight bonds are also indifferent with the volatility of exchange rate. However, the call and put provisions make the relationship between coupon rate and the values of call options on

FCCBs uncertain. For example, from Exhibit 8C, we can see that their relationship is V-shaped curve at the case that hazard rate is equal to 0.4. Otherwise, they are positive relationship for different levels of hazard rate. Since that the call or put provisions will influence the strike price of call options on FCCBs, it is reasonable for those results. Similarly, if there is no call and put provisions prior to maturity date of FCCB asset swap, the FCCB parity also holds even though the FCCBs are embedded with call and put provisions.

5.3 Call and Put Provisions Prior to Maturity Date of FCCB Asset Swap

Exhibit 9A, 9B, and 9C report the numerical results of FCCBs, synthesis straight bonds, and call options on FCCBs for the case that call and put provisions are prior to maturity date of FCCB asset swap. The properties of them are the same as the above case. However, the only difference is the FCCB parity is invalid and (18) holds. The numerical results verify Theorem 2.

5.4 Swap Rate for FCCB Asset Swap

For simplicity, we assume that the maturity dates of FCCBs and their asset swap are five and three years, respectively. We discuss three cases as described as follows: (1) Without call and put provisions (2) Put prices at third and fifth years are equal to 110% of the principal, and call price at the fourth and fifth years are at par. 3) Put prices at third and fifth years are equal to 110% of the principal, and call price at the second and fifth years are at par. Hence, there is no call and put provisions prior to maturity date of FCCB asset swap for the first and second cases. In Exhibit 10, we summarize the suitable swap rates of FCCB asset swaps by varying the different levels of coupon rate, hazard rate, and the volatility of stock return (exchange rate) for the above three cases.

In the first and second cases, as the level of coupon rate increases, the present value of synthesis straight bond also increases, and hence the initial cash outflow paid by credit investor decreases. As a result, it is clear that the swap rate is decreasing function of coupon rate. For the impact of hazard rate, since that the higher level of hazard rate means the high default probability, the swap rate is increasing function of hazard rate. It is deserve to be mentioned that the volatility of stock return is irrelevance for the suitable swap rate, and hence the credit investors only face the credit risk and is free of equity exposure. For the third case, the higher level of hazard rate also makes the suitable swap rate increase. However, the call provision prior to maturity of FCCB asset swap makes the influence of coupon rate and volatility of stock return uncertain.

In practice, the maturity date of FCCB asset swap is the nearness put date of FCCBs, if the issuer do not call or there is no call provision prior to maturity, the suitable swap rate is positive relationship with hazard rate, negative relationship with coupon rate, and unrelated with volatility of stock return. Or equivalently, the credit investor is free of equity exposure, nevertheless, they require high swap rate for the high credit risk exposure.

It is worth to note that the above examples can be explained as a German CB with principals linked to CPI in Germany. The initial CPI is set as 0.8155, the real prices of default-free and risky zero-coupon bonds correspond to the prices of default-free and risky zero coupon bonds in America. Hence, using the same pricing algorithm, we can obtain the theoretical values of inflation-indexed CBs.

6. Empirical Results

In this section, we use the pricing model to provide the realistic computations for the FCCBs that are launched by Tom Holdings Ltd in Hong Kong. We first calibrate the parameters of the model according to market prices. Then, we compare the numerical values with the real issue price of those securities. Moreover, we also provide the value of a call options on FCCB and the suitable swap rates for the FCCB asset swap.

In 2003, Tom holdings Ltd, the greater china media group in Hong Kong, issued a five-year zero-coupon FCCB. On the date of issuance, the stock price of Tom holding Ltd was HK$ 2.55 and the exchange rate was US$ 1.00 = HK$ 7.77. The volatilities of the stock return and the exchange rate are 31.8597% and 1.3585%, respectively. The correlation coefficient between the stock return and exchange rate was 0.190352. The put price was US$ 102.31 on the third anniversary and the call price is US$ 103.86 at the maturity date. Prior to maturity, the investors may convert the FCCB into 235.2941 shares of underlying stock of Tom Holdings Ltd per US$ 100 nominal. Exhibit 11A illustrates the term sheet of the FCCB.

Besides, our pricing model also needs the prices of default-free and risky zero coupon bonds at each period. Taking US Treasury as the proxy for the default-free interest rate, and can be obtained from Bloomberg. The S&P credit rating of this FCCB is BB; we apply the yield spreads of the same rating BB from Goldman Scachs' daily spreads of corporate bond index to Treasuries of rating BB[5] as a proxy for risky interest rate. The default-free and risky yield curves on the date of issuance are shown in Exhibit 11B. Then, we can compute the implied prices of default-free and risky zero coupon bonds, both shown in Exhibit 11C. The recovery rate for the senior unsecured bond that is estimated by Moody's corporation is 41.2%[6]. Given the prices of default-free and risky zero coupon bonds, and the recovery rate, the default probabilities of Tom Holdings Ltd at each period are summarized in Exhibit 11D.

According to our five-period pricing model, the theoretical value at the date of issuance is US$100.2963 which is close to the issue price US$ 100. We can adjust the implied volatility of the return of the stock price equal to 31.401% such that the numerical value equal issue price US$ 100. We also assume that the first put date is the maturity date of a call option on FCCB or FCCB asset swap. Using our pricing model, the value of synthesis straight bond on the date of issuance is US$ 80.0045. Given the values of FCCB and synthesis straight bond, the call option on FCCB is US$ 19.9955 (100 – 80.0045). Because of the high credit risk exposure, the required swap rate for FCCB asset swap of Tom holding Ltd is estimated as 17.35%.

7. Conclusions

This article provides a new pricing method for valuing FCCBs, call options on FCCBs, and FCCB asset swap under the consideration of credit risk. We incorporate the default-free and the risky interest rates, the exchange rate, and foreign stock price into one tree. The Hung and Wang (2002) model for pricing CBs is the special case when the volatility of exchange is zero and exchange rate equals one. However, prior to bankruptcy, Hung and Wang (2002) model ignores that the expected stock return should be adjusted for the hazard function, and hence undervalue the upward probability of stock price and the values of FCCBs. After default occurs, their model also ignores that the default-free interest rate possibly goes up or down.

This paper is the first article to price foreign-currency (or inflation-indexed) convertible bonds and their asset swaps under the consideration of risk-free and risky interest rates, stock price, and exchange rate. We also compute the suitable swap rate for asset swap and prove that the value of a FCCB is less than (equal to) the value of a synthesis straight bond plus the value of a call option on FCCB while the FCCB is (not) embedded with the call or put provisions prior to the maturity date of FCCB asset swap. From numerical analysis, we also discover the properties of FCCBs, synthesis straight bonds, call options on FCCBs, and the suitable swap rates. Taking the FCCB issued by Tom holdings Ltd. as an example, we provide the fair prices of the FCCB, a call option on FCCB and the appropriate swap rates. The empirical results indicate that the numerical value is closed to the market price. As a result, our model can price not only FCCBs (or CBs if we assume that the exchange rate and the volatility of exchange rate equal to 1 and 0 in our pricing model, respectively) but also other cross-currency credit derivatives, as long as they depend on the stochastic default-free and risky interest rates, the exchange rate, and the foreign stock price. Hence, our pricing model is useful for market practitioners.

Exhibit 1

The FCCB Strip

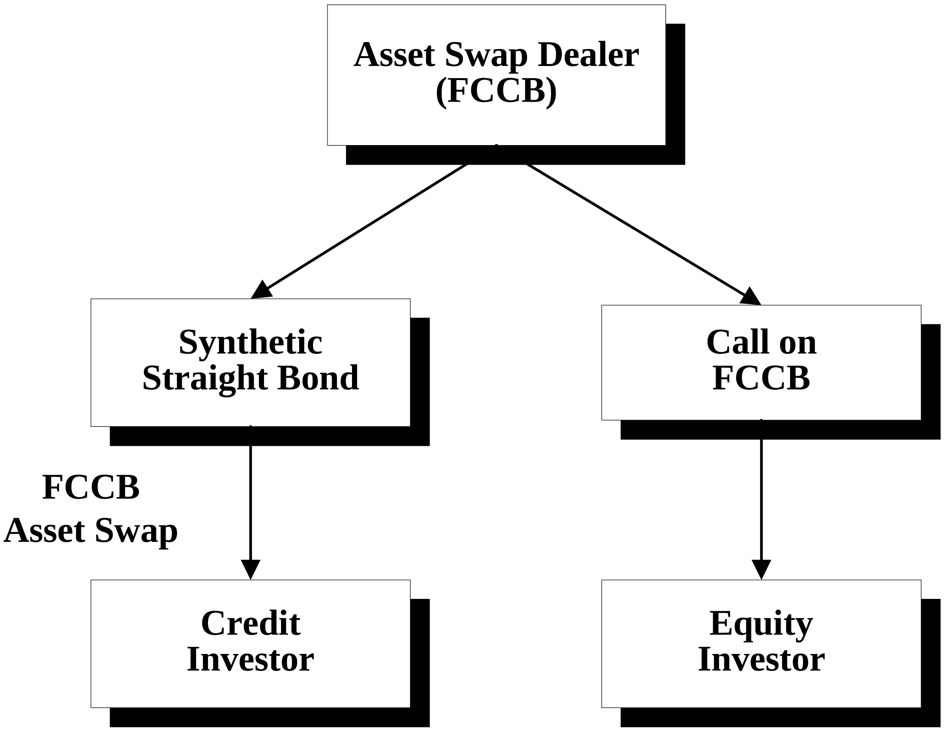

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Exhibit 2

Two-Period Risky Interest Rate Tree Method

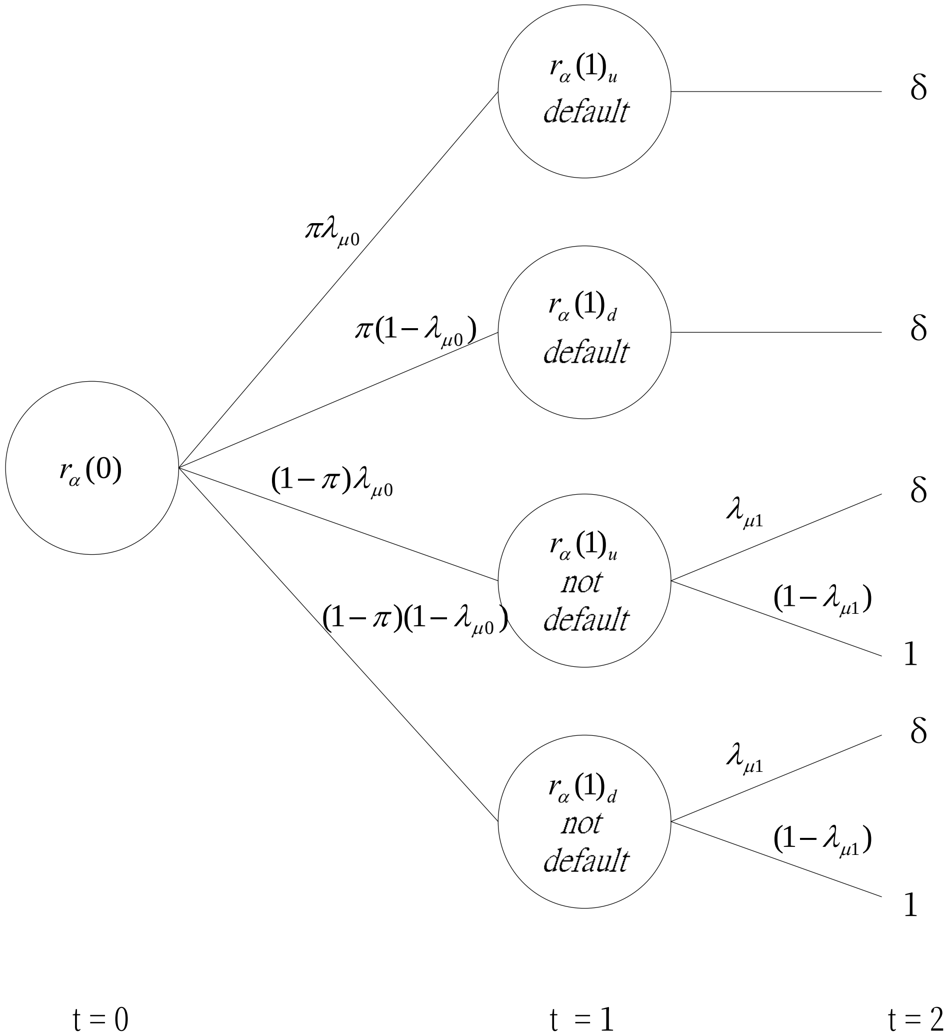

Exhibit 3

Four-Period Risky Interest Rate Tree Method

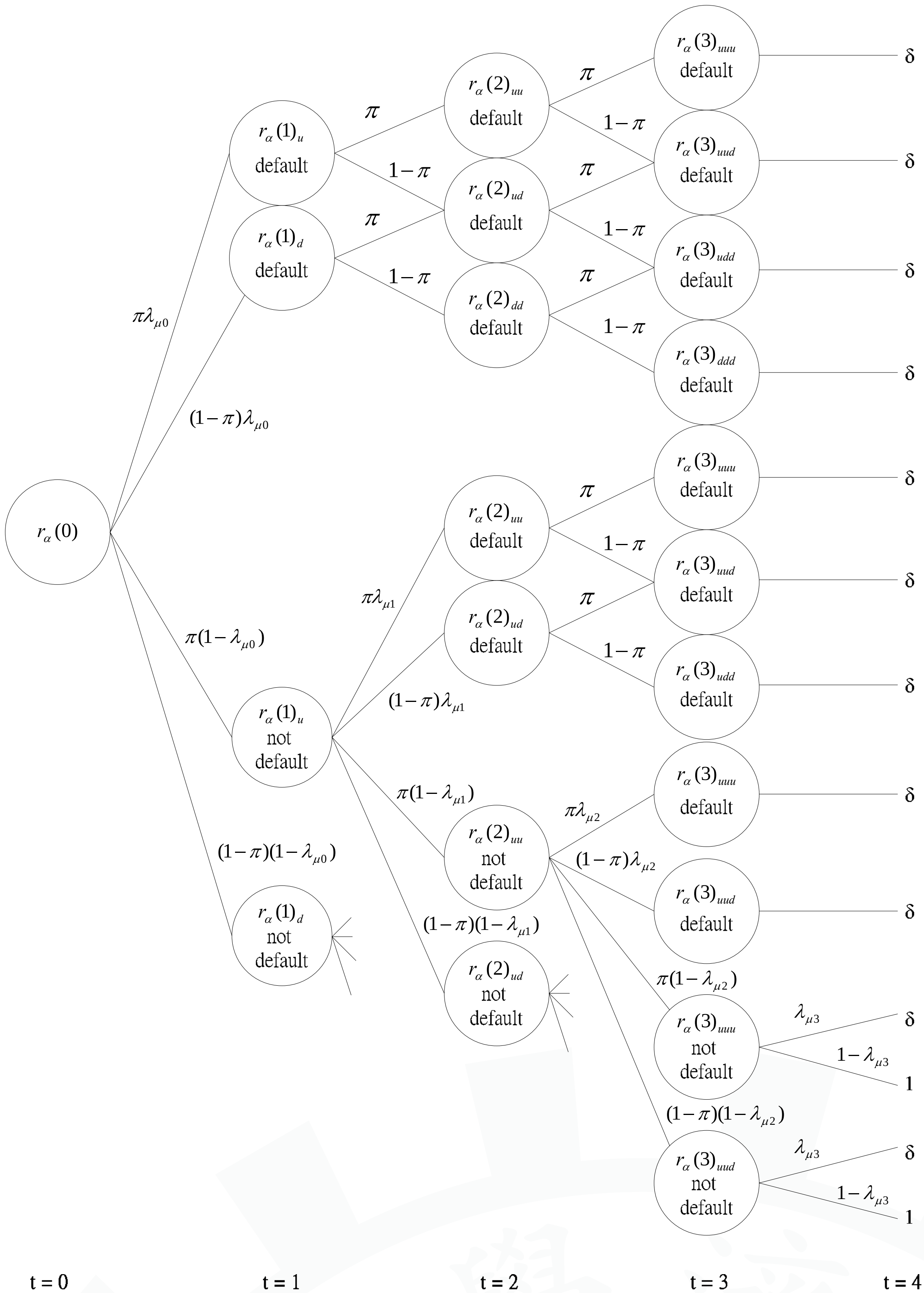

Exhibit 4

Possible Values of S_β^*

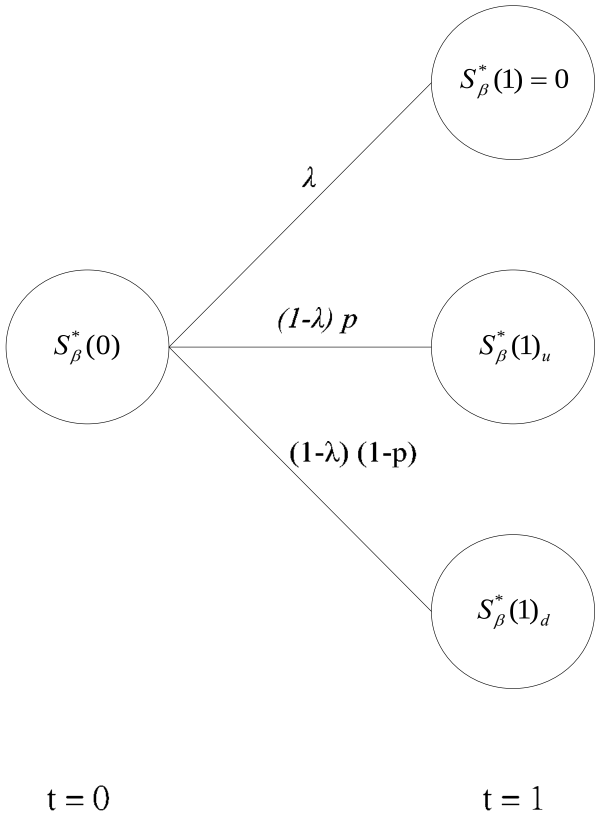

Exhibit 5

Pricing Model for FCCBs (or Inflation-indexed CBs)

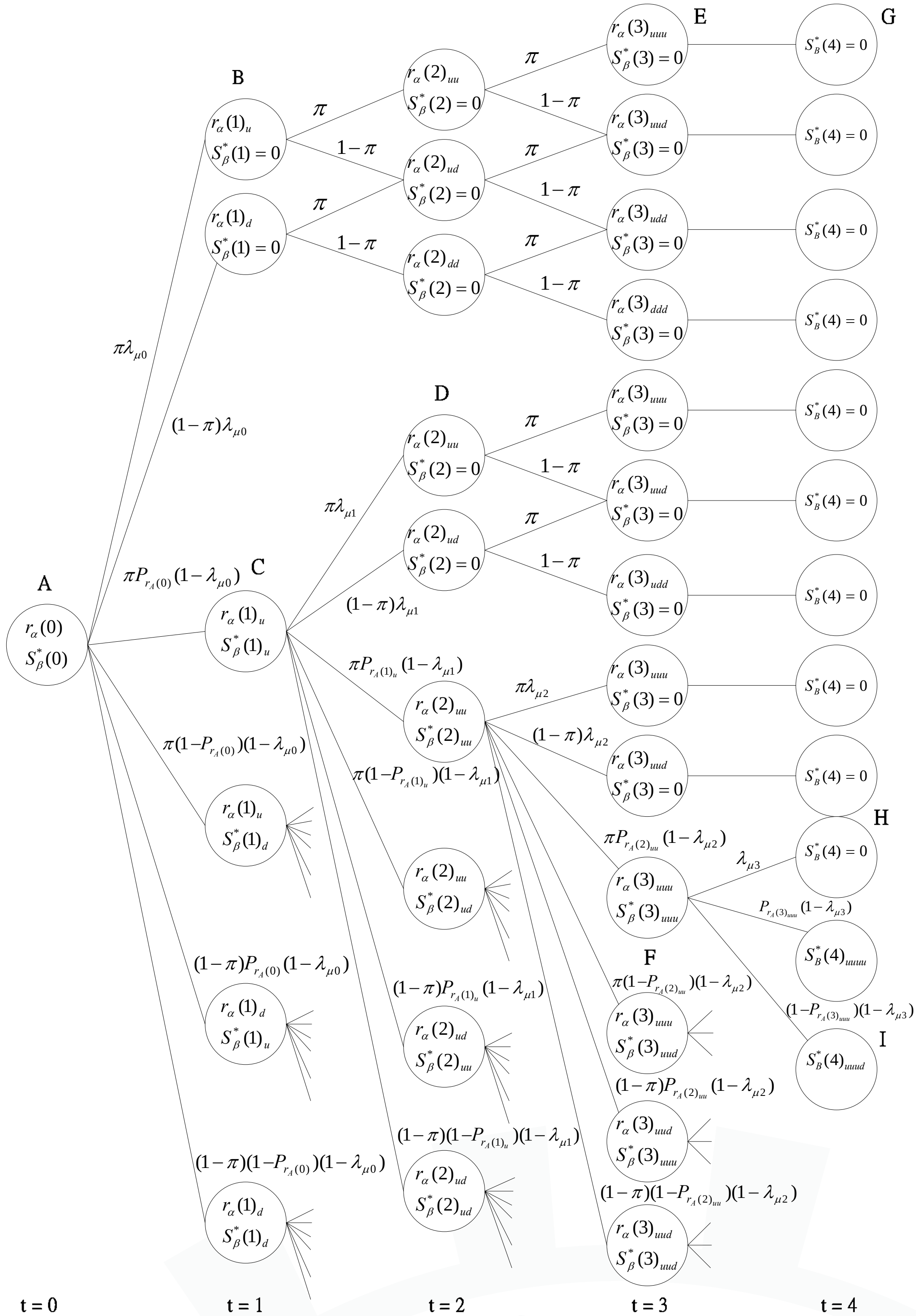

Exhibit 6A

Term Sheet of Numerical Example

TERM SHEET			
Stock price $S_{\beta}^{*}(0)$	31.1465	***Volatility of Stock Return*** $\sigma_{S_{\beta}^{*}}$	50%
Conversion Ratio	3	***Asset Swap Maturity***	3
Coupon Rate	2%	***Recovery Rate***	0.438
Face Value of FCCB	100	***Face Value of Asset Swap***	100
Maturity Date of FCCB	4	***Maturity Date of Asset Swap***	3
Call Date	4^{th} year	***Put Date***	3^{rd} year
Call Price	100	***Put Price***	101
Correlation between Stock Return and Exchange Rate	15%	***Volatility of Exchange Rate*** σ_{Q}	15%

Exhibit 6B

The Price of Default-Free and Risky Zero Coupon Bonds and Default Probabilities

Maturity	**1**	**2**	**3**	**4**
Prices of Default-free Zero Coupon Bonds	98.5112	96.5605	94.1765	9.13931
Prices of Risky Zero Coupon Bonds	75.2545	73.7644	71.9431	69.8169
Default Probability $\lambda_{\mu t}$	0.0099502	0.0099502	0.0099502	0.0099502

Exhibit 7A

Ten-Year Maturity FCCB without Call and Put Provisions

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%	111.8908	118.3493	124.6167
	2%	128.3069	134.8717	141.1619
	5%	153.1985	159.9138	166.3165
0.01	0%	110.8970	116.9301	122.8402
	2%	126.8833	133.0200	138.9606
	5%	151.0443	157.3366	163.3926
0.1	0%	108.0561	110.8208	114.0605
	2%	120.7645	123.6014	126.9183
	5%	140.1699	143.0896	146.4862
0.15	0%	106.8360	108.7852	111.2365
	2%	118.1192	120.1595	122.7269
	5%	135.6560	137.7341	140.3898
0.2	0%	105.0681	106.7028	108.7334
	2%	115.0233	116.8223	119.0341
	5%	130.8641	132.7507	135.1094
0.4	0%	98.3080	99.4139	100.8075
	2%	104.2889	105.7401	107.4464
	5%	115.2231	116.9527	119.0893

This table reports the influence of ten-period FCCB without call or put provisions by varying the volatility of return of exchange rate σ_Q, coupon rate, and hazard rate $\lambda\mu(t)$. The results indicate the value of FCCB has a positive relationship with σ_Q and coupon, but a negative relationship with $\lambda\mu(t)$. From Exhibit 6A, 6B, and 6C, The FCCB Parity holds.

Exhibit 7B

Ten-Year Maturity Synthesis Straight Bonds without Call and Put Provisions

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%		68.7287	
	2%		85.7997	
	5%		111.4062	
0.01	0%		65.0530	
	2%		81.6437	
	5%		106.5298	
0.1	0%		44.3127	
	2%		57.7373	
	5%		77.8742	
0.15	0%		38.7217	
	2%		51.0155	
	5%		69.4562	
0.2	0%		35.3306	
	2%		46.7760	
	5%		63.9442	
0.4	0%		30.8106	
	2%		40.3919	
	5%		54.7637	

This exhibit reports the values of ten-period synthesis straight bond without call or put provisions with different levels of the volatility of exchange rate σ_Q, coupon rate, and default hazard rate $\lambda\mu(t)$. The results indicate the value of straight bond has a positive relationship with coupon rate and a negative relationship with $\lambda\mu(t)$. However, σ_Q has no impact on the value of the synthesis straight bond.

Exhibit 7C

Three-Year Maturity Call Options on FCCB without Call and Put Provisions

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%	43.1620	49.6205	55.8880
	2%	42.5071	49.0720	55.3621
	5%	41.7923	48.5075	54.9102
0.01	0%	45.8439	51.8770	57.7872
	2%	45.2395	51.3762	57.3168
	5%	44.5145	50.8068	56.8627
0.1	0%	63.7434	66.5081	69.7477
	2%	63.0272	65.8641	69.1810
	5%	62.2957	65.2154	68.6120
0.15	0%	68.1143	70.0635	72.5148
	2%	67.1036	69.1439	71.7114
	5%	66.1998	68.2779	70.9335
0.2	0%	69.7375	71.3722	73.4027
	2%	68.2472	70.0463	72.2581
	5%	66.9199	68.8065	71.1652
0.4	0%	67.4973	68.6032	69.9968
	2%	63.8970	65.3482	67.0545
	5%	60.4594	62.1890	64.3256

This table reports the influence of call option on FCCB without call or put provisions by varying the volatility of return of exchange rate σ_Q, coupon rate, and hazard rate. The results indicate the value of call option on FCCB option has a positive relationship with volatility of exchange rate, but a negative relationship with coupon. The relationship between the hazard rate and the value of call option on FCCB is a humped-shape curve.

Exhibit 8A

Ten-Year Maturity FCCB without Call and Put Provisions Prior to Maturity Date of FCCB Asset Swap

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%	119.3472	124.9639	130.7038
	2%	124.0618	129.5958	135.2418
	5%	131.1454	136.5651	142.0998
0.01	0%	118.2935	123.8714	129.5657
	2%	123.1965	128.6879	134.2872
	5%	130.5510	135.9134	141.3910
0.1	0%	111.0249	115.8856	120.9419
	2%	117.3038	122.0075	126.9300
	5%	126.7222	131.1902	135.9125
0.15	0%	108.0807	112.3157	116.8972
	2%	114.9316	118.9877	123.4091
	5%	125.2080	129.0027	133.1982
0.2	0%	105.5463	109.0853	113.2025
	2%	112.8252	116.1593	120.0419
	5%	123.7639	126.8415	130.4889
0.4	0%	98.6680	99.4804	101.6434
	2%	104.7321	105.7441	107.9779
	5%	116.2551	116.7355	118.9064

By varying the volatility of return of exchange rate, coupon rate, and hazard rate, this table reports the values of ten-period FCCB. The issuer may call at 4^{th}, 7^{th}, and 9^{th} year at par, and the bondholders may put at 3^{rd}, 6^{th}, and 10^{th} with 107, 110, and 123, respectively. The results indicate the value of FCCB is the increasing function of the volatility of exchange rate and coupon rate, and is decreasing function of hazard rate. From 8A, 8B, and 8C, the FCCB Parity holds.

Exhibit 8B

Ten-Year Maturity Synthesis Straight Bond without Call and Put Provisions Prior to Maturity Date of FCCB Asset Swap

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%		100.7686	
	2%		104.6701	
	5%		110.5222	
0.01	0%		98.6801	
	2%		102.7195	
	5%		108.7786	
0.1	0%		82.4534	
	2%		87.5478	
	5%		95.1893	
0.15	0%		75.1615	
	2%		80.7176	
	5%		89.0519	
0.2	0%		68.8852	
	2%		74.8307	
	5%		83.7489	
0.4	0%		51.3872	
	2%		58.3568	
	5%		68.8113	

This exhibit reports the value of ten-period synthesis straight bond with call or put provisions prior to maturity of FCCB asset swap with different levels of the volatility of exchange rate, coupon rate, and default hazard rate. The results indicate the value of synthesis straight bond has a positive relationship with coupon rate and a negative relationship with hazard rate. However, the value of the synthesis straight bond is unrelated with the volatility of exchange rate.

Exhibit 8C

Three-Year Maturity Call Option on FCCB without Call and Put Provisions Prior to FCCB Asset Swap

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%	18.5786	24.1953	29.9352
	2%	19.3918	24.9258	30.5717
	5%	20.6232	26.0429	31.5776
0.01	0%	19.6134	25.1913	30.8855
	2%	20.4770	25.9684	31.5677
	5%	21.7724	27.1348	32.6124
0.1	0%	28.5715	33.4322	38.4885
	2%	29.7561	34.4597	39.3823
	5%	31.5329	36.0009	40.7232
0.15	0%	32.9192	37.1542	41.7357
	2%	34.2140	38.2701	42.6915
	5%	36.1562	39.9508	44.1464
0.2	0%	36.6611	40.2001	44.3173
	2%	37.9945	41.3286	45.2112
	5%	40.0150	43.0926	46.7400
0.4	0%	47.2808	48.0932	50.2562
	2%	46.3753	47.3872	49.6210
	5%	47.4438	47.9242	50.0951

The results indicate the value of call option on FCCB has a positive relationship with hazard rate and the volatility of exchange rate. However, the relationship between the value of call option on FCCB and coupon rate is uncertain. For example, when the hazard rate is 0.4, their relationship is V-shaped curve. Otherwise, the value of call option on FCCB is increasing function of coupon rate.

Exhibit 9A

Ten-Year Maturity FCCB with Call and Put Provisions Prior to FCCB Asset Swap

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%	110.7063	114.4261	118.3274
	2%	112.6765	116.3963	120.2977
	5%	115.6318	119.3517	123.2530
0.01	0%	110.1378	113.8399	117.7220
	2%	112.2279	115.9300	119.8121
	5%	115.3632	119.0653	122.9474
0.1	0%	105.7585	109.0085	112.4850
	2%	109.0902	112.4333	115.9971
	5%	113.7181	117.0724	120.6469
0.15	0%	104.0045	106.8371	109.9497
	2%	107.7167	110.6576	113.8746
	5%	113.2040	116.2593	119.5869
0.2	0%	102.6870	105.1041	107.8486
	2%	106.7313	109.2502	112.0999
	5%	112.7973	115.4688	118.4758
0.4	0%	98.6138	99.4392	101.0407
	2%	103.4532	104.7127	106.3422
	5%	111.3569	112.5912	114.2945

By varying the volatility of return of exchange rate, coupon rate, and hazard rate, this table reports the values of ten-year maturity FCCB. The issuer may call at 2nd, 7th, and 9th year at par, and the bondholders may put at 3rd, 6th, and 10th with 107, 110, and 123, respectively. The results indicate the value of FCCB is the increasing function of the volatility of exchange rate and coupon rate, and is decreasing function of hazard rate. From 9A, 9B, and 9C, since that the there is a call date prior to maturity date of FCCB asset swap, the FCCB Parity do not hold.

Exhibit 9B

Ten-Year Maturity Synthesis Straight Bond with Call and Put Provisions Prior to FCCB Asset Swap

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q		
		20%	30%	40%
0	0%	96.5602		
	2%	98.5304		
	5%	101.4858		
0.01	0%	95.2443		
	2%	97.3344		
	5%	100.4697		
0.1	0%	82.4534		
	2%	87.3543		
	5%	92.1731		
0.15	0%	75.1615		
	2%	80.7176		
	5%	88.1560		
0.2	0%	68.8852		
	2%	74.8307		
	5%	83.7420		
0.4	0%	51.3872		
	2%	58.3568		
	5%	68.8113		

The results indicate the value of synthesis straight bond has a positive relationship with coupon rate and a negative relationship with hazard rate. However, the value of the synthesis straight bond is unrelated with the volatility of exchange rate.

Exhibit 9C

Three-Year Maturity Call Option on FCCB with Call and Put Provisions Prior to FCCB Asset Swap

Hazard Rate $\lambda\mu(t)$	Coupon Rate	σ_Q 20%	30%	40%
0	0%	20.2052	26.3378	33.1530
	2%	24.3292	31.1363	38.3280
	5%	27.0631	34.2101	41.2215
0.01	0%	21.0557	27.1318	33.4489
	2%	24.7490	31.3129	38.3976
	5%	28.9058	35.8398	42.6649
0.1	0%	28.6416	33.8398	39.1875
	2%	32.2449	36.8739	41.7840
	5%	38.1036	42.3836	47.3685
0.15	0%	32.0273	36.5506	41.4510
	2%	35.6104	39.5093	43.8800
	5%	41.2163	44.2314	48.0861
0.2	0%	34.7983	38.6817	43.0436
	2%	37.9488	41.2559	45.2396
	5%	43.2394	45.6968	48.9631
0.4	0%	44.3484	45.2306	47.4728
	2%	43.4385	44.7507	47.0194
	5%	43.9358	45.5826	48.0167

With call at 2^{nd}, 7^{th}, and 9^{th} year at par, and put at 3^{rd}, 6^{th}, and 10^{th} at 107, 110, and 123, respectively, the results indicate the value of call option on FCCB has a positive relationship with coupon, $\lambda\mu(t)$, and σ_Q. However, the relationship between the value of call option on FCCB and coupon rate is uncertain. For example, when the hazard rate is 0.4, their relationship is V-shaped curve. Otherwise, the value of call option on FCCB is increasing function of coupon rate.

Exhibit 10

Suitable Swap Rate for Three -Year Maturity FCCB Asset Swap

Call and Put Provisions	Coupon Rate	$\sigma_{S^*_\beta}$	$\lambda\mu(t)$				
			0	0.01	0.1	0.15	0.2
Without Call and Put Provisions	0%	0.2	2.01%	3.26%	16.54%	25.89%	37.19%
		0.5					
	2%	0.2	2.01%	3.20%	15.73%	24.55%	35.21%
		0.5					
	5%	0.2	2.01%	3.09%	14.53%	22.55%	32.24%
		0.5					
Put: 3rd and 5th year at 110% Call: 4th and 5th year at 100%	0%	0.2	2.01%	3.26%	16.54%	25.89%	37.19%
		0.5					
	2%	0.2	2.01%	3.20%	15.73%	24.55%	35.21%
		0.5					
	5%	0.2	2.01%	3.09%	14.53%	22.55%	32.24%
		0.5					
Put: 3rd and 5th year at 110% Call: 2nd and 5th year at 100%	0%	0.2	4.02%	5.07%	16.87%	25.89%	37.19%
		0.5	4.44%	5.50%			
	2%	0.2	5.11%	6.11%	16.89%	24.66%	35.21%
		0.5					
	5%	0.2	7.07%	7.59%	16.94%	24.05%	32.76%
		0.5					

Form the results in this exhibit, since that there are no call and put provisions prior to maturity of FCCB asset swap in the case 1 and case 2, the swap rate has a positive relationship with hazard rate, a negative relationship with coupon, and has no relationship with the volatility of stock return. If there is call or put provision prior to maturity date of FCCB asset swap, the volatility of exchange rate may have impacts on the swap rate in low level of hazard rate.

Exhibit 11A

The Term Sheet of Tom Holdings Ltd Zero coupon FCCB

TERM SHEET	
Issuer	Tom Holdings Ltd
Offer Size	US＄150 million
Issuer Credit Rating	"BB" (Standard & Poor)
Maturity	28, November, 2008 (5 years)
Denomination	US＄1000
Coupon	0 %
Issue Price	100 %
Conversion Ratio	2352.941 shares
Put Price	On the 3rd anniversary: 102.31 (November 28, 2006)
Redemption Price	On the 5th anniversary: 103.86 (November 28, 2008)

Exhibit 11B

Risk-free and Risky Yield Curves on Date of Issuance

Maturity	**Risk-free Yield Curve (%)**	**Risky Yield Curve of BB (%)**
1	1.2822	6.6985
2	1.8070	7.2232
3	2.3300	7.7463
4	2.7874	8.2037
5	3.2448	8.6611

Exhibit 11C

Default-free and Risky Zero-coupon Bonds on Date of Issuance

Maturity Date *T*	Prices of Default-free Zero Coupon Bonds with Face Value US $ 100	Prices of Risky Zero Coupon Bond with Face Value US $ 100
1	98.7260	93.2096
2	96.4506	86.4853
3	93.2487	79.6380
4	89.4495	72.2570
5	85.0237	64.5258

Exhibit 11D

Default Probabilities at Each Period

Time period	0-1	1-2	2-3	3-4	4-5
Default Probability $\lambda_{\mu t}$	0.07988	0.08224	0.08489	0.08787	0.09125

APPENDIX A

Proof of Theorem 1

By using equation (5) and (10), we match the first two moments as follows:

$$E[\ln(\xi_{T_j})] = P_u(j)\ln u + P_d(j)\ln d \qquad = \exp(-\int_{T_{j-1}}^{T_j} \lambda\mu(s)ds)\left[(r-q-\frac{1}{2}\sigma^2)\frac{T}{n} + \int_{T_{j-1}}^{T_j} \lambda\mu(s)ds\right]$$

$$= (1-\lambda_{\mu\, j-1})\left[(r-q-\frac{1}{2}\sigma^2)\frac{T}{n} - \ln(1-\lambda_{\mu\, j-1})\right] \tag{A.1}$$

$$E[\ln(\xi_{T_j})]^2 = P_u(j)(\ln u)^2 + P_d(j)(\ln d)^2 = \sigma^2(1-\lambda_{\mu\, j-1})\frac{T}{n} + \left[E[\ln(\xi_{T_j})]\right]^2 \tag{A.2}$$

Defining that $P_n(j) = \lambda_{\mu\, j-1}$, $P_u(j) = (1-\lambda_{\mu\, j-1})p_r$, $P_d(j) = (1-\lambda_{\mu\, j-1})(1-p_r)$, and the equal jump size condition: $\ln u = -\ln d$, we have:

$$u = \exp(\sigma\sqrt{\frac{T}{n}}),\ \ d = \exp(-\sigma\sqrt{\frac{T}{n}}),\ \ p_r = \frac{1}{2} - \frac{\ln(1-\lambda_{\mu\, j-1})}{2\sigma}\sqrt{\frac{n}{T}} + \frac{r-q-0.5\sigma^2}{2\sigma}\sqrt{\frac{T}{n}} \tag{A.3}$$

This completes the proof of Theorem 1.

APPENDIX B

Proof of Theorem 2

Case 1: With the call and put provision prior to maturity date of a FCCB asset swap

First, if default occurs, the FCCB asset swap is early terminated and the equity investors should not exercise since that the value of a FCCB and stock price are equal to the value of straight bond and zero, respectively. We have:

$$Option_i = 0 = FCCB_i - SB_i \tag{B.1}$$

If default does not occur and $FCCB_i - SB_i \geq Option\_cont_i$, we have

$$Option_i = Max(FCCB_i - SB_i, Option\_cont_i) = FCCB_i - SB_i \tag{B.2}$$

Therefore, the equity investors should early exercise.

If default does not occur and $FCCB_i - SB_i < Option\_cont_i$, we obtain

$$Option_i = Option\_cont_i > FCCB_i - SB_i \tag{B.3}$$

The equity investors should hold the call on FCCB. Briefly, we have

$$Option_i \geq FCCB_i - SB_i, \quad i = 1,...,m \tag{B.4}$$

This completes the proof of (18).

Case 2: Without the call and put provision prior to maturity date of a FCCB asset swap

Similarly, if default occurs, (B.1) holds again. If default does not occur, we can use the mathematical induction to prove that (18) holds. We denote that:

r_i: The default-free interest rate between time T_i and T_{i+1} for the j^{th} node.

$P_{i,j}$: The probability for the j^{th} node at time T_i.

$FCCB\_cont_{i,j}$: The continuum value of a FCCB for the j^{th} node at time T_i.

$FCCB\_conv_{i,j}$: The conversion value of a FCCB for the j^{th} node at time T_i.

$SB\_cont_{i,j}$: The continuum value of a synthesis straight bond for the j^{th} node at time T_i.

More specificity, we also define that the subscript i, j indicate the j^{th} node at time T_i. At the maturity date of call on FCCB (or FCCB asset swap), the value of call on FCCB at y^{th} node at time T_m is equal to

$$Option_{m,y} = Max(FCCB_{m,y} - SB_{m,y}, 0) = FCCB_{m,y} - SB_{m,y} \tag{B.5}$$

where we use the fact that the value of a FCCB is not less than the one of synthesis straight bond. Let (18) holds for all nodes at time T_k, i.e., $Option_{k,j} = FCCB_{k,j} - SB_{k,j}$. At time T_{k-1}, since that each node has six branches if default does not occur, we have

$$Option\_cont_{k-1,x} = \exp(-r_{k-1,x})(\sum_j Option_{k,j} \times P_{k,j}), \quad j = 1,\ldots,6 \tag{B.6}$$

The value of a call on FCCB at x^{th} node at time T_{k-1} is equal to

$$\begin{aligned} Option_{k-1,x} &= Max(FCCB_{k-1,x} - SB_{k-1,x}, Option\_cont_{k-1,x}) \\ &= Max[FCCB_{k-1,x} - SB_{k-1,x}, \exp(-r_{k-1,x})(\sum_j P_{k,j} \times Option_{k,j})] \\ &= Max\left(FCCB_{k-1,x} - SB_{k-1,x}, \exp(-r_{k-1,x})[\sum_j P_{k,j} \times (FCCB_{k,j} - SB_{k,j})]\right) \end{aligned} \tag{B.7}$$

Because of no call and provisions prior to the maturity date of call on FCCB, we have

$$SB_{k-1,x} = SB\_cont_{k-1,x} \tag{B.8}$$

First, if $FCCB\_cont_{k-1,x} \geq FCCB\_conv_{k-1,x}$, we obtain

$$\begin{aligned} FCCB_{k-1,x} - SB_{k-1,x} &= FCCB\_cont_{k-1,x} - SB\_cont_{k-1,x} \\ &= \exp(-r_{k-1,x})[\sum_j P_{k,j} \times FCCB_{k,j} - \sum_j P_{k,j} \times SB_{k,j}] = Option\_cont_{k-1,x} \end{aligned} \tag{B.9}$$

or equivalently

$$Option_{k-1,x} = FCCB_{k-1,x} - SB_{k-1,x} = Option\_cont_{k-1,x} \tag{B.10}$$

If $FCCB\_cont_{k-1,x} < FCCB\_conv_{k-1,x}$, we have

$$FCCB_{k-1,x} - SB_{k-1,x} = FCCB\_conv_{k-1,x} - SB\_cont_{k-1,x}$$

$$\geq FCCB\_cont_{k-1,x} - SB\_cont_{k-1,x} = Option\_cont_{k-1,x} \qquad \text{(B.11)}$$

As a result, it is clear that

$$Option_{k-1,x} = Max(FCCB_{k-1,x} - SB_{k-1,x}, Option\_cont_{k-1,x}) = FCCB_{k-1,x} - SB_{k-1,x} \qquad \text{(B.12)}$$

By mathematical induction, this completes the proof of (19).

ENDNOTES

1. Yigitbasioglu, A. (2001), ISMA Centre Discussion Papers In Finance, 2001-14, November, 2001.
2. If default occurs, the credit investors hold the FCCBs. On the other hand, while the issuer redeem the FCCBs or the equity investors early exercise, the asset swap dealer immediately pays the accrued interest and nominal principal of the asset swap to credit investors in return for the synthesis straight bonds.
3. Similar to Yigitbasioglu [2001], we also use the domestic price of foreign equity to reduce the required state variables. However, we use a reduced-form approach and consider the hazard rate process in the setup of equity price.
4. If FCCB issuer doesn't issue zero-coupon bonds in units of currency of country α, we can use the risky zero coupon bonds in the same credit class as a proxy.
5. For reference, please see the article "*Treasury Yields, Equity Returns, and Credit Spread Dynamics",* 2003.
6. For reference, please see the article "*Default and Recovery rates of corporate bond issuers*", January 2004.

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