# A Profit-Sharing Mechanism for the Production-Distribution Alliance

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## Abstract

Important operational performance measures for a successful production-distribution alliance include not only pricing and lot-sizing policies, but also the profit sharing mechanism among members of the production-distribution alliance. This study provides a cost-based mechanism for sharing profit among members of the production-distribution alliance. The proposed mechanism share proportionally the total profit obtained from the cooperation of producer and distributor on the basis of supply costs. After formulating and solving the efficient pricing and lot-sizing algorithm to optimize the global profit for all alliance members, we illustrate the application of this mechanism using data from our previous studies. Results of the illustrated example have shown that the producer can use adequate transfer prices to support his distributors and to maximize the alliance's profit. When the market demand quantity is price dependent, the lower transfer price leads the distributor more pricing flexibility to maximize his profit. However, the cost-based mechanism may discourage the effort of alliance's members to improve their efficiency, since the profit is shared on the basis of one's existing cost rather than on that of one's operational performance.

Keywords: production-distribution alliance, profit-sharing mechanisms, profit-maximizing model, cost-based mechanism.

## 1.Introduction

The profit sharing mechanism among members is one of important operational performance measures of the production-distribution alliance. A common theme in the studies of supply chain management has been the producer's profit-maximization problem without regard to the suppliers' reactions. However, the producer is able to revise his pricing strategies for maximizing alliance's profit. Min (1992) extended the profit maximizing economic order quantity (EOQ) model for a monopolistic seller who simultaneously determines both the order quantity and quantity discount price schedule when buyers have different preferences on their purchase quantities. Chen et al. (2001) addressed a fundamental two-echelon distribution system in which a supplier distributes a single product to retailers and the sales volumes of the retailers are endogenously determined on the basis of known demand functions. They have characterized an optimal strategy, maximizing total systemwide profits in a centralized system. Bardia and Siddique (2004) explicitly modelled how flexibility can be mutually beneficial to both the producer and the suppliers. Using their model, they concurrently solve and examine the dual optimization problem for the suppliers and the producer. Burnetas and Ritchken (2005) investigated the role of option contracts in a supply chain when the demand curve is downward sloping. They have shown that options are not zero-sum games and the introduction of option contracts causes the wholesale price to increase and the volatility of the retail price to decrease. Chauhan (2005) presented a provider-retailer partnership model based on profit sharing and proposed an approach to maximize the combined profit and sharing the profit among partners proportional to their risk. Lakhal (2005) presented a framework and methodology for profit sharing and transfer-pricing between network companies, and proposed a paradigm that enables maximization of operating profits by the manufacturing-network in its larger supply chain. Under the assumption that market demand is a decreasing convex function of buyer's selling price, Lei (2006) has quantified the improvement on total supply chain profitability when moving from a non-cooperative environment to a fully cooperative environment, and show that the joint annual profit of three partners in a cooperative environment can be at least twice of what may be achieved by three independently operated companies in a leader-follower business game. Liu et al. (2006) examined the behavior of a manufacturer and a retailer in a decentralized supply chain under price-dependent, stochastic demand, and modelled a retail fixed markup (RFM) policy, which can arise as a form of vertically restrictive pricing in a supply chain.

Extending our previous work (Wu and Chen, 2006) in which an integrated pricing and lot-sizing model for a production-distribution alliance is explored, this study provides a cost-based mechanism for sharing profit among members of the production-distribution alliance. This paper is organized as follows. In the next section, the notations used throughout this paper are defined, and basic assumptions are given. Section 3 then presents a generalized mathematic model for the producer and its distributors. A general description of the model and formulates relevant revenue, cost, and profit functions for producer and each distributor is proposed. Subsequently, Section 4 presents efficient solution approaches including pricing and lot-sizing algorithm and cost-based profit sharing mechanism for each member of the alliance. In Section 5, a numeric example is conducted to illustrate the proposed algorithm and mechanism. Finally, in Section 6, conclusions for management implications are made.

# 2. Notation and assumptions

## 2.1 Notation

| | |
|---|---|
| $n$ | The number of members in the production-distribution alliance; |
| $i$ | The code name of alliance members, where the producer's $i$=0 and distributors' $i$=1, 2, …, $n$; |
| $p_{0,i}$ | The producer charged price to the $i$th distributor, \$/unit; |
| $p_i$ | Selling price of the $i$th distributor, \$/unit; |
| $q_i$ | Economic production/ordering quantity of the $i$th distributor, units/order; |
| $R_i$ | Total Revenue of the $i$th member, \$/year; |
| $C_i$ | Total cost of the $i$th member, \$/year; |
| $\Pi_i$ | Profit of the $i$th member, \$/year; |
| $Fi$ | Allocated fixed cost per year of the $i$th member, \$/year; |
| $s_i$ | Setup/ordering cost per lot, \$/order; |
| $e_i$ | Miscellaneous and administration expense, \$/unit; |
| $h_i$ | Holding cost per unit per year for the $i$th member, \$/unit·year; |
| $m$ | Direct material and labour cost for the producer, \$/unit; |
| $MP$ | Production capacity per year of the producer, units/year; |
| $T\Pi$ | Total profit of the alliance, \$/year. |

## 2.2 Assumptions

1. One product with two-level vertical production-distribution framework in which a monopolistic producer distributes a single product to distributors.
2. Demand quantity is price dependent for each distribution market. Demand curves of the distributors are downward sloping and endogenously determined on the basis of known demand functions.
3. Profit-maximizing behaviour by all alliance decision makers.
4. The Producer has a capacity to satisfy the total demand from its alliance distributors whose outlets with exclusive territories.
5. Certainty of variables and functional forms.

# 3. Model formulation

## 3.1 The profit function of each distributor

Since the demand quantity is price dependent, total revenue, total cost, and profit of the $i$th distributor, where $i$=1, 2, …, $n$, in a year are given by Equation (1), (2), and (3), respectively:

$$R_i = p_i \cdot Q_i(p_i)\,. \tag{1}$$

$$C_i = F_i + p_{0,i} \cdot Q_i(p_i) + s_i \cdot \frac{Q_i(p_i)}{q_i} + h_i \cdot \frac{q_i}{2} + e_i \cdot Q_i(p_i)\text{, and} \tag{2}$$

$$\Pi_i(p_i, q_i, p_{0,i}) = R_i(p_i, q_i) - C_i(p_i, q_i) = (p_i - p_{0,i} - \frac{s_i}{q_i} - e_i) \cdot Q_i(p_i) - F_i - h_i \cdot \frac{q_i}{2}\,. \tag{3}$$

A maximum value of $\Pi_i(p_i, q_i)$ can be determined by the following conditions:

$$\frac{\partial \Pi_i(p_i,q_i)}{\partial p_i} = Q_i(p_i) + (p_i - p_{0,i} - \frac{s_i}{q_i} - e_i)\frac{dQ_i(p_i)}{dp_i} = 0\,. \tag{4}$$

$$\frac{\partial \Pi_i(p_i,q_i)}{\partial q_i} = \frac{s_i}{q_i^2} \cdot Q_i(p_i) - \frac{h_i}{2} = 0\,. \tag{5}$$

$$\frac{\partial^2 \Pi_i(p_i,q_i)}{\partial p_i\, \partial q_i} < 0\,. \tag{6}$$

$$\frac{\partial^2 \Pi_i(p_i,q_i)}{\partial p_i^2} < 0\text{, and} \tag{7}$$

$$\frac{\partial^2 \Pi_i(p_i,q_i)}{\partial q_i^2} < 0\,. \tag{8}$$

By using Cardano's formula for solving Equation (4) and (5), $p_i^*$ and $q_i^*$ will be obtained to optimize pricing and lot sizing policy for the $i$th distributor to maximize its profit. The detailed procedure has been shown in Wu and Chen (2006).

## 3.2 The profit function of the producer

Since total demand of the producer can be summated by each distributor, Quantity demanded of the producer is:

$$Q_0 = \sum_{k=1}^{n} Q_i(p_i^*)\,. \tag{9}$$

Total revenue, total cost, and profit of the producer in a year can be calculated by Equation (10), (11), and (12), respectively:

$$R_0(p_{0,i}, q_0) = \sum_{i=1}^{n} p_{0,i} \cdot Q_i(p_i^*)\,. \tag{10}$$

$$C_0(p_{0,i}, q_0) = F_0 + m \cdot Q_0 + s_0 \cdot \frac{Q_0}{q_0} + \frac{h_0}{2} \cdot \frac{q_0}{MP}(MP - Q_0) + e_0 \cdot Q_0\text{, and} \tag{11}$$

$$\Pi_0 = R_0(p_{0,i}, q_0) - C_0(p_{0,i}, q_0)\,. \tag{12}$$

Since

$$\left.\frac{d\Pi_0(q_0)}{dq_0}\right|_{q_0=q_0^*} = \left[\frac{s_0}{\left(q_0^*\right)^2} + \frac{h_0}{2\cdot MP}\right]\cdot Q_0 - \frac{h_0}{2} = 0 ,$$

we have $q_0^*$ for maximizing $\Pi_0$.

### 3.3 The profit function of the alliance

The total profit of the alliance is

$$T\Pi = \sum_{i=0}^{n} \Pi_i . \qquad (13)$$

## 4. Solution approaches

### 4.1 Pricing and lot-sizing algorithm

The Pricing and Lot-sizing (P-Ls) algorithm proposed in this section is used to solve the integrated pricing and lot-sizing model, mentioned above, for the production-distribution alliance. The following steps are the pricing and lot-sizing algorithm:

*Step* 1. Initialize $T\Pi^*$, $p_{0,i}^*$, $q_0^*$, $p_i^*$, and $q_i^*$ $i$=1, 2, …, $n$.

*Step* 2. Compute the feasible range of $p_{0,i}$.

*Step* 3. For all feasible values of $p_{0,i}$,
calculate corresponding values of $p_i$ and $q_i$,
calculate $D\Pi_i(p_i, q_i, p_{0,i})$, $P\Pi_0$, $q_0$, and $T\Pi$.
If $T\Pi^* = \max\{T\Pi\}$, $p_{0,i}^* = p_{0,i}$, $q_0^* = q_0$, $p_i^* = p_i$, and $q_i^* = q_i$.

### 4.2 Profit-Sharing Mechanism

Since the mechanism is based on the existing cost of each member, the total profit obtained from the cooperation of producer and distributors can be shared as follows. The shared profit of the $i$th member is

$$\overline{\Pi}_i = \frac{C_i}{\sum_{i=0}^{n} C_i} T\Pi . \qquad (14)$$

The adjusted profit for each member is

$$\Delta\Pi_i = \overline{\Pi_i} - \Pi_i . \qquad (15)$$

## 5. Computational Example

A computational example including one producer and three distributors was used to illustrate the profit-sharing mechanism mentioned above. Parameters presented for each member of the alliance were shown in Table 1 and a solution procedure for this computational example was explored in this section. Table 2 presents the optimal decisions and results obtained by using the proposed Pricing and lot-sizing algorithm. Table 3 show the result of the proposed profit-sharing mechanism.

Results of the illustrated example shown in Table 2 reveal the fact that the producer can use adequate transfer prices to support his distributors and to maximize the alliance's profit. The lower the transfer price is, the more flexible is the pricing of distributors. With the pricing flexibility, a distributor can maximize the revenue or profit in the market that demand quantity is price dependent.

## 6. Conclusions

Most studies of supply chain management have been the producer's profit-maximization problem without regard to the suppliers' reactions. However, our model have characterized an optimal strategy for maximizing total alliance profits with optimal pricing and lot-sizing policies and reasonable profit-sharing mechanism. This study has not only presented an effective algorithm for all members of the production-distribution alliance to make the integrated pricing and lot-sizing decisions, but also proposed a cost-based mechanism for sharing the profits. However, the cost-based mechanism may discourage the effort of alliance's members to improve their efficiency, since the profit is shared on the basis of one's cost rather than on that of one's performance.


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**Table 1 Parameters of the illustrated example**

| Producer | | Distributor | I | II | III |
|---|---|---|---|---|---|
| $FC_0$ | 2000 | $FCi$ | 500 | 500 | 500 |
| $C_0$ | 10 | $s_i$ | 50 | 50 | 50 |
| $S_0$ | 200 | $e_i$ | 0.4 | 0.4 | 0.4 |
| $h_0$ | 1.5 | $h_i$ | 2.0 | 2.0 | 2.0 |
| $MP$ | 16500 | $Q_i(p_i)$ | 2500-50$p_1$ | 6000-200$p_2$ | 8000-450$p_3$ |

**Table 2 Results of the illustrated example**

| i / variables | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $p_{0,i}$ | — | 11 | 11 | 11 |
| $p_i$ | — | 30.8142 | 20.7823 | 19.1076 |
| $q_i$ | 1397.1442 | 219.0083 | 303.6059 | 336.7289 |
| Cost | 59227.75 | 11873.95 | 22123.47 | 27025.54 |
| Profit | -3451.6899 | 17685.83194 | 16189.43204 | 16305.2219 |

**Table 3 Profits shared by using the proposed mechanism**

| i / variables | | 0 | 1 | 2 | 3 | Total |
|---|---|---|---|---|---|---|
| Cost | Value | 59227.75 | 11873.95 | 22123.47 | 27025.54 | 120250.71 |
| | Percentages | 49.25% | 9.87% | 18.40% | 22.47% | 100% |
| Profit | obtained | -3451.69 | 17685.83 | 16189.43 | 16305.22 | 46728.80 |
| | shared | 23015.59 | 4614.15 | 8597.06 | 10501.98 | 46728.80 |
| | adjustment | 26467.28 | -13071.68 | -7592.37 | -5803.24 | 0.00 |